Mesh Adaptive Direct Search for Continuous Gravitational Waves

in a small parameter space

Systematic follow-up of Einstein@Home candidates

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Stating the problem

- Consider an interesting parameter space point selected in an Einstein@Home gravitational wave search, e.g.

$$\lambda_c = \left\{ \alpha, \delta, f, \dot{f} \right\} \ , \sigma\lambda_c$$

with α - right ascension, δ - declination, f - frequency, $\dot{f}$ (phase parameters) and $\sigma\lambda_c$ associated uncertainties.

What should we do next?!

Outlook

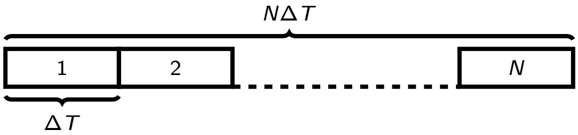
- Follow-up strategy
- Monte Carlo study
- Summary

Follow-up strategy

Follow-up strategy - reminder

- Search target: unknown sources of continuous gravitational waves, e.g. unknown pulsars.
- Search technique: computation of a matched filter ($\mathcal{F}$-statistic, StackSlide).
- Even simple wide band search for unknown pulsars is a search over very large 4D- parameter space
$$\left\{\alpha, \delta, f, \dot{f}\right\}.$$
- Fully-coherent integration is computationally limited, thus semi-coherent techniques are applied (e.g. Einstein@Home).

Follow-up strategy - reminder



- We can divide the data in N segments of duration ΔT and combine the individual statistics of the segments to a new semi-coherent statistic.
- The expectation value of the individual segments is

$$E[2\mathcal{F}] = 4 + \rho^2 \ ,$$

where ρ^2 is the squared signal to noise ratio (SNR).

- The combined expectation can be expressed in terms of averaged over segments quantities

$$E[\overline{2\mathcal{F}}] = 4 + \overline{\rho^2} \ .$$

Follow-up strategy - mismatch and metric

- The fractional loss of the expected statistical value for a template λ_c due to the unknown signal parameters λ_s

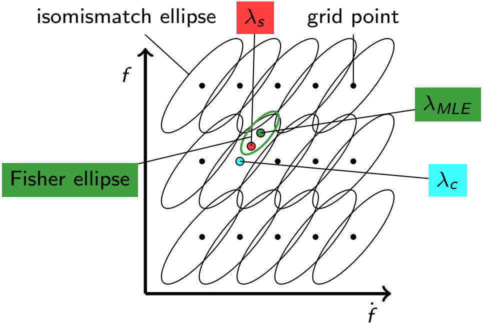
$$\begin{aligned} m^* &\equiv \frac{E[2\mathcal{F}(\lambda_s)] - E[2\mathcal{F}(\lambda_c)]}{E[2\mathcal{F}(\lambda_s)] - 4} \\ &\approx g_{ij}(\lambda_s)\Delta\lambda^i\Delta\lambda^j + \mathcal{O}(3)\ , \end{aligned}$$

  can be Taylor-expanded around λ_s, where g_{ij} is the metric, i, j are running over the phase parameters and $\Delta\lambda = \lambda_c - \lambda_s$.
- Neglecting higher order terms in the expansion we use $m^* \in [0, 1]$ as a distance measure.
- Similarly, we define the loss of squared SNR

$$m \equiv \frac{2\mathcal{F}(\lambda_s) - 2\mathcal{F}(\lambda_c)}{2\mathcal{F}(\lambda_s) - 4}\ .$$

- With $m \in [-\epsilon, 1]$ we measure our ability to recover the signal SNR, where ϵ is a small number.

Follow-up strategy - search grid and Fisher information



- λ_c - semi-coherent candidate
- λ_{MLE} - maximum likelihood estimator (MLE)
- λ_s - true signal location (TSL)
- The Fisher ellipse represents the statistical uncertainty of the MLE for the TSL.

- Follow-up strategy for semi-coherent candidates:
  1. *refinement* - find the semi-coherent MLE.
  2. *zoom* - increase the coherent integration time using the semi-coherent Fisher-ellipse and ideally all the data.

CGW searches - follow-up strategy for semi-coherent candidates

However:

- Grid-based *refinement*, even if possible, becomes computationally expensive for small false dismissals.
- Grid-based *zoom* using all the data remains computationally prohibitive.

Thus we wonder:

Could a grid-less search algorithm do better?!

Follow-up strategy - Non-smooth Optimization by Mesh Adaptive Direct Search

- **M**esh **A**daptive **D**irect **S**earch is a derivative-free class of algorithms for black-box optimization.
- NOMAD is LGPL C++ implementation of MADS

- Mark A.Abramson
- Charles Audet,
- Sebastien Le Digabel
- Gilles Couture
- John E. Dennis
- Jr., Quentin Reynaud

`http://www.gerad.ca/nomad/Project/Home.html`

Follow-up strategy - using NOMAD

- In the *refinement* step we keep the setup of the semi-coherent search, i.e. N and ΔT and use NOMAD to find the the maximum likelihood estimator.
- In the *zoom* step we estimate the search box using the SNR of the MLE and apply NOMAD to a fully-coherent $\mathcal{F}$-statistic search using all of the available data.
- We use the SNR of the MLE to estimate the SNR of the fully-coherent *zoom* search

$$\rho_s^2 \approx N\rho_{MLE}^2 \ ,$$

  and with this the expected $E[2\mathcal{F}_s]$ value.
- After the fully-coherent optimization, the expectation $E[2\mathcal{F}_s]$ is used to distinguish between:
  I. detection,
  II. candidate not-consistent with Gaussian noise,
  III. candidate consistent with Gaussian noise .

Monte Carlo study

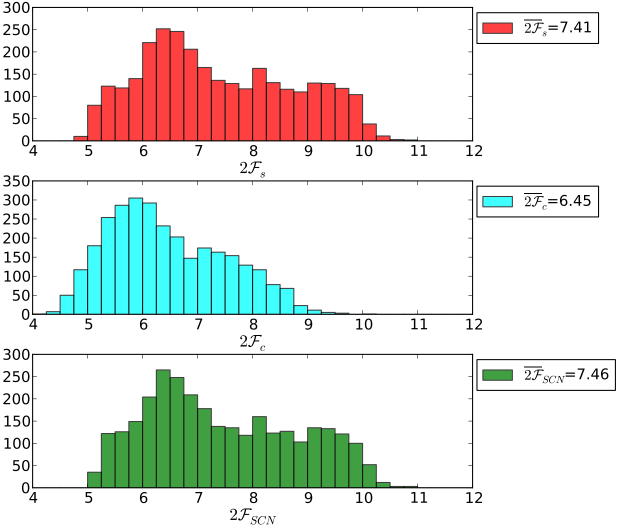
Monte Carlo study

- 3000 trials with different noise realization and pulsar parameters.
- The parameters of the candidate to follow-up are drawn from a box around the injection and accepted if $m^* \in [0.2, 0.8]$.
- The follow-up chain consist of two steps
  1. semi-coherent $\mathcal{F}$-statistic NOMAD (SCN) *refinement*.
  2. fully-coherent $\mathcal{F}$-statistic NOMAD (FCN) *zoom*.
- In every step we set an upper limit $\mathcal{N}_{max}$ on the number of explored parameter space points.

step	N	ΔT [days]	NOMAD runs	$\mathcal{N}_{max}$
refinement	200	1	80	1.6×10^5
zoom	1	200	200	4×10^6

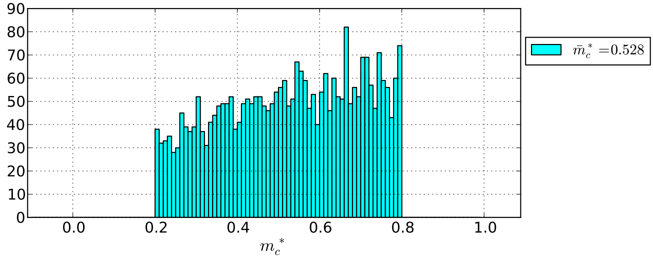
Monte Carlo study

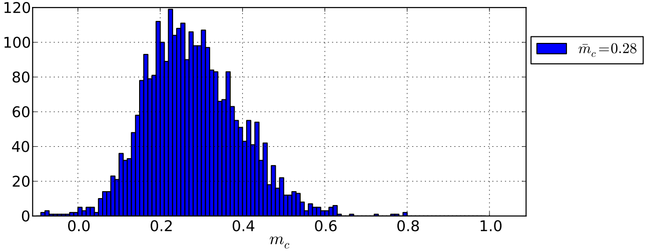
- $2\mathcal{F}$ distributions - average $\overline{2\mathcal{F}}_s$ of the injections, average $\overline{2\mathcal{F}}_c$ of the candidates and average $\overline{2\mathcal{F}}_{SCN}$ after semi-coherent NOMAD optimization.



Monte Carlo study

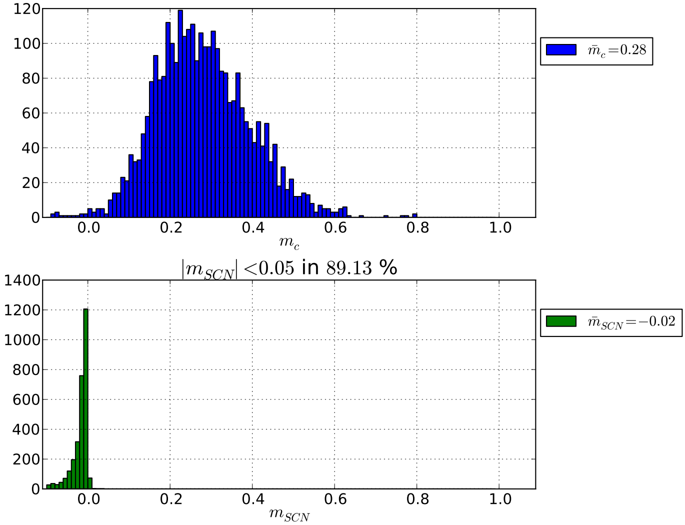
- Mismatch m^* of the injections and corresponding relative SNR^2 loss m.





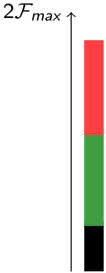
Monte Carlo study

- Relative SNR^2 loss *m* of the candidate before and after NOMAD optimization.



Monte Carlo study

- Depending on the maximal found $2\mathcal{F}_{max}$ value in the *zoom* step, the expected $E[2\mathcal{F}_s]$ value and the expected maximal $E[2\mathcal{F}_n]$ value in Gaussian noise, where n represents confidence factor, we distinguish three possible outcomes:



Zone:

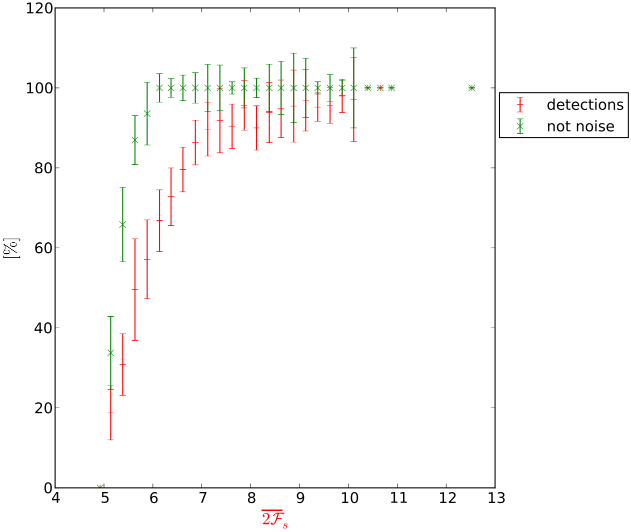
I. $2\mathcal{F}_{max} \geq E[2\mathcal{F}_s] - n\sigma(E[2\mathcal{F}_s])$ - detection.

II. $2\mathcal{F}_{max} > E[2\mathcal{F}_n] + n\sigma(E[2\mathcal{F}_n])$ - not consistent with Gaussian noise.

III. $2\mathcal{F}_{max} < E[2\mathcal{F}_n] + n\sigma(E[2\mathcal{F}_n])$ - consistent with Gaussian noise.

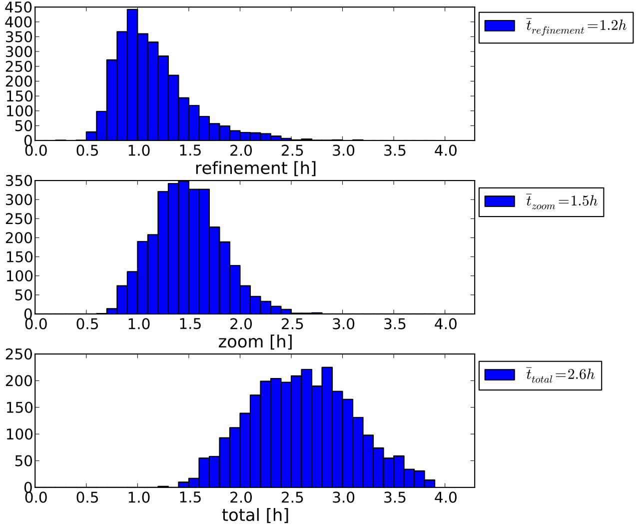
Monte Carlo study

- Detection and not Gaussian noise confirmation as function of the strength of the injected signal $\overline{2\mathcal{F}}_s$.



Monte Carlo study

- Computing cost measured in hours.



Summary

The systematic fully-coherent follow-up of candidates from semi-coherent searches, e.g. Einstein@Home

- seems to be possible
- at acceptable computing cost

even for rather weak candidates.

The End.

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