Effect of the full wave forms to the parameter estimation accuracy by the network of ground based detectors

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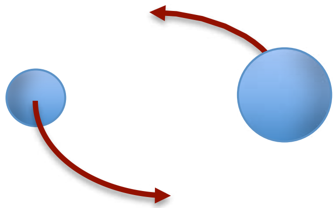
Purpose



A promising target of the ground based GW detectors
: compact binary coalescence

We investigate the parameter estimation accuracy of the inspiral signals of coalescing compact binaries using the network of ground based laser interferometers.

Especially, we investigate the effect of the full wave form (FWF) of the post-Newtonian waveform, and compare it with the restricted waveform (RWF).

We consider the 2nd generation detectors (aLIGO (L, H) , aVirgo (V), KAGRA (K), and a detector at Australia (A)) .

We also consider the future Einstein telescope(ET).

Post-Newtonian full wave form

2.5PN Full Wave Form formula (FWF) for plus and cross modes (Arun et al. ('04))

(note: 3PN FWF was already computed (Blanchet et al.('08)))

$$h_{+,\times} = \frac{2GM\eta x}{c^2 r}\left\{H^{(0)}_{+,\times} + x^{1/2}H^{(1/2)}_{+,\times} + xH^{(1)}_{+,\times} + x^{3/2}H^{(3/2)}_{+,\times} + x^2 H^{(2)}_{+,\times} + x^{5/2}H^{(5/2)}_{+,\times}\right\}$$

$$x = \left(\frac{GM\Omega}{c^3}\right)^{2/3}, \Omega = \frac{2\pi}{P_{\rm orbit}} \qquad M = m_1 + m_2, \eta = m_1 m_2/M^2, r = \text{distance to source}$$

$$H_a^{(n/2)} = \sum_{k=1}^{7}\left\{c_{a,k}^{(n/2)}\cos(k\Psi(t)) + s_{a,k}^{(n/2)}\sin(k\Psi(t))\right\}$$

$\Psi(t)$: Orbital phase formula (3.5PN)

$c_{a,k}^{(n/2)}, s_{a,k}^{(n/2)}$: functions of inclination angles, mass

Waveform at the detector

$$h(t) \quad \left(=\frac{\Delta L}{L}\right) = F_+ h_+ + F_\times h_\times = \sum_{a=+,\times} F_a h_a(t)$$

$F_+, \quad F_\times$: detector's response function

$$= \frac{2GM\eta x}{c^2 r}\sum_{a=+,\times}\sum_{n=0}^{5} F_a H_a^{(n/2)} x^{n/2}$$

$$= \frac{2GM\eta x}{c^2 r}\sum_{a=+,\times}\sum_{n=0}^{5}\sum_{k=1}^{7} x^{n/2}\left[F_a c_{a,k}^{(n/2)}\cos(k\Psi(t)) + F_a s_{a,k}^{(n/2)}\sin(k\Psi(t))\right\}$$

FWF and RWF

$$h(t) = \frac{2GM\eta x}{c^2 r} \sum_{a=+,\times} \sum_{n=0}^{5} \sum_{k=1}^{7} x^{n/2} \left[F_a c_{a,k}^{(n/2)} \cos(k\Psi(t)) + F_a s_{a,k}^{(n/2)} \sin(k\Psi(t)) \right\}$$

FWF(Full Wave Form) contain all modes (n=0..5, k=1..7)

On the other hand, the waveform used traditionally is called RWF (Restricted Wave Form) which contain n=0, k=2 modes. It also ignores the amplitude correction term occurs due to the stationary phase approximation of the Fourier transformation.

Previous works

FWF related works

Sintes and Vecchio ('00,'00) initial LIGO, LISA
Hellings and Moore ('02,'03) LISA
van den Broeck, Sengupta ('07,'07) FWF, aLIGO, EGO, Parameter estimation
Trias and Sintes ('07) LISA
Arun et al. ('07,'07) LISA

Parameter estimation accuracy by the network of ground based detectors

Jaranowski, Krolak et al. ('94,'96) RWF, Fisher matrix
Pai,Dhurandhar,Bose('01) RWF, Fisher matrix
Röver, Meyer, Christensen ('07) FWF, MCMC, (one case)
Wen, Chen ('10)
Nissanke et al. ('11)
Klimenko et al. ('11)
Fairhurst ('11)
Schutz ('11)
Nissanke et al. ('11)

FWF

$$h(t) = \frac{2GM\eta x}{c^2 r} \sum_{a=+,\times} \sum_{n=0}^{5} \sum_{k=1}^{7} x^{n/2} \left[F_a c_{a,k}^{(n/2)} \cos(k\Psi(t)) + F_a s_{a,k}^{(n/2)} \sin(k\Psi(t)) \right\}$$

FWF for ground based detectors:

1. Observable mass range is wider than RWF (due to k>2 modes).
2. At lower mass range, S/N of FWF is smaller than RWF.
3. Parameter estimation errors of masses and coalescence time t_c is better in the FWF case than RWF, especially at larger mass $(M_{tot} > 50 M_{\rm solar})$ because of the complicated structure of the waveform formula. (one detector case)

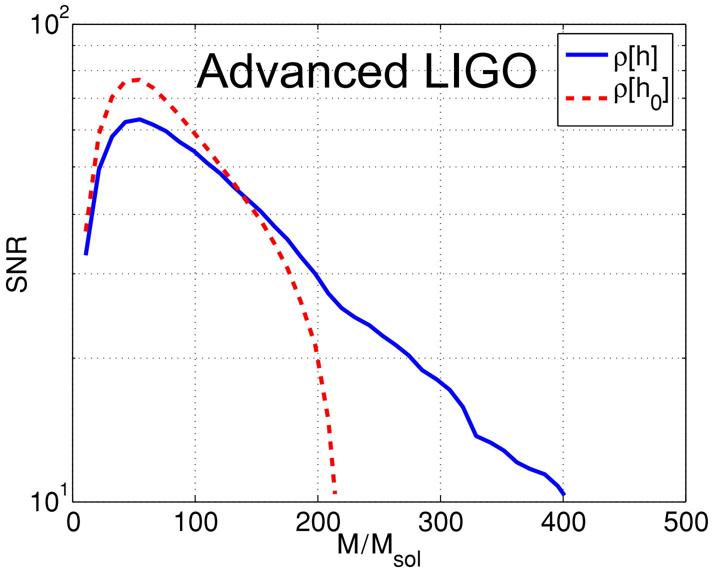
This work (difference from the previous works)

- We consider the parameter estimation error for the network of ground based detectors, including ET.
- We consider a larger parameter space of direction, inclination, polarization angles by simulation.
- Main interest is the accuracy of the angle parameters and distance.

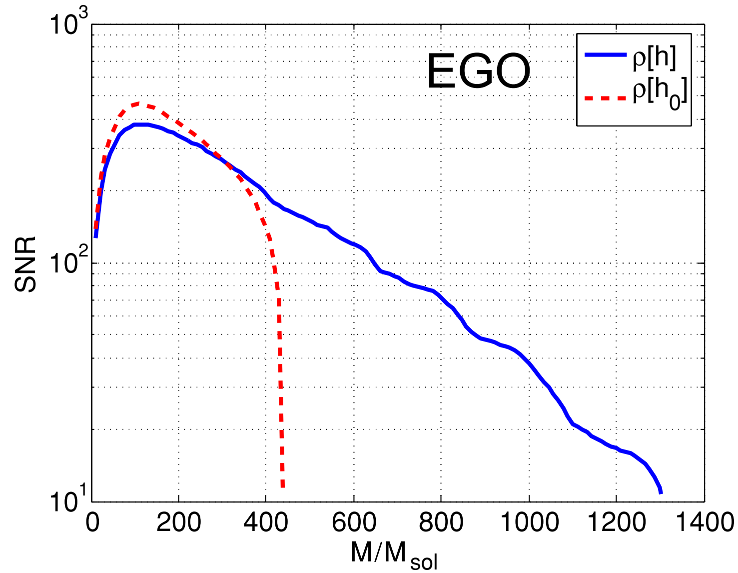
FWFとRWF, Mass reach

Van Den Broeck and Sengupta (‘07)

ρ



ρ



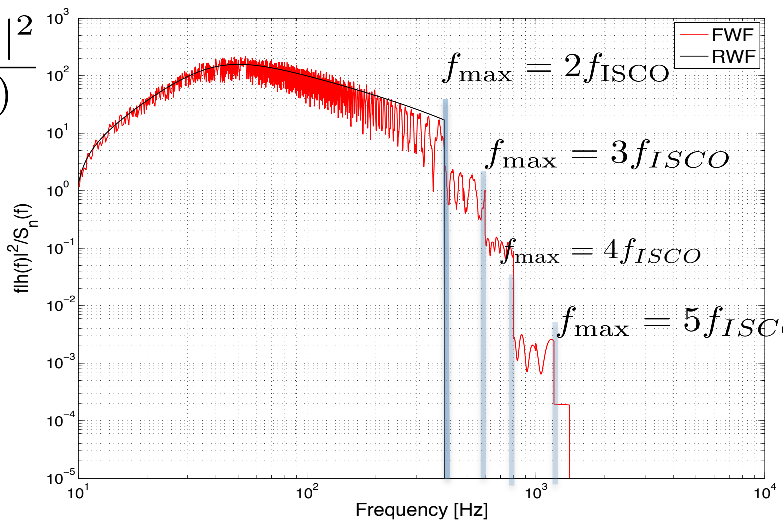
$m_1+m_2=10:1$

$$\rho = 4\int_0^\infty \frac{|\tilde{h}(f)|^2}{S_h(f)}df$$

$$\frac{f|h(f)|^2}{S_n(f)}$$

aLIGO

(10,1)Msolar



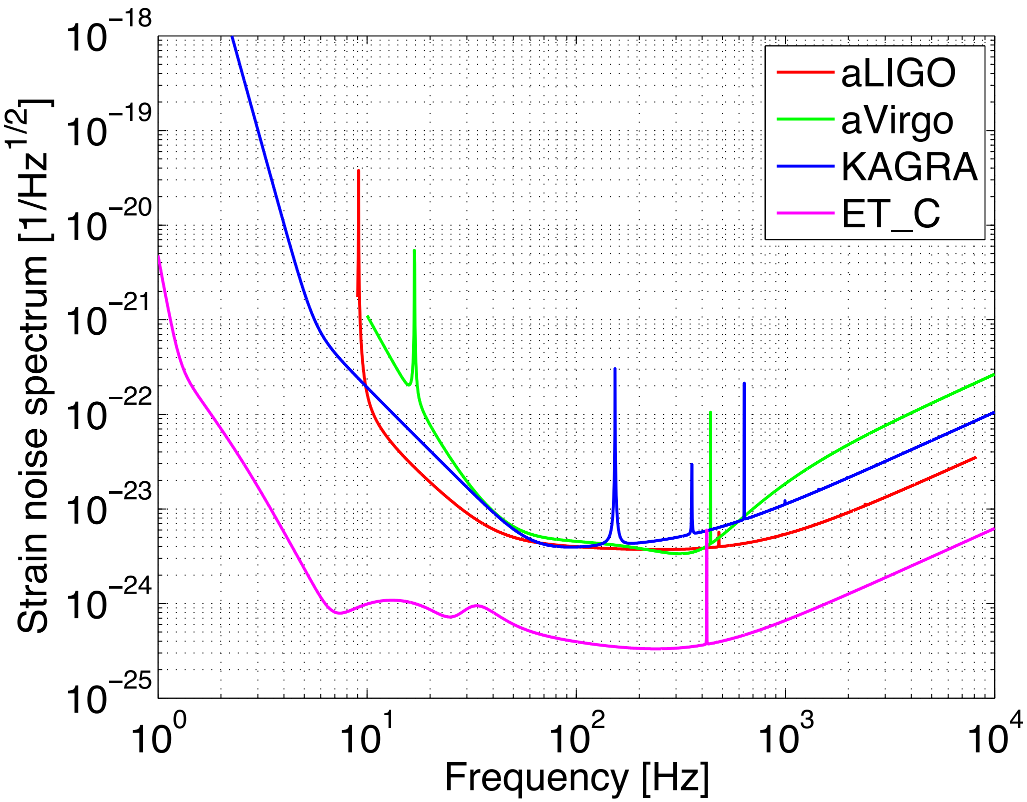
$f_{\rm ISCO}$: Orbital frequency at ISCO

Detectors

Detector location : 2 LIGO (L, H), Virgo (V), KAGRA (K) and Austraria (A)

(I will include IndIGO soon)

Noise curve: aLIGO, aVirgo, KAGRA and Einstein telescope (ET_C)



aLIGO: ZERO_DET_high_P

aVirgo: from the webpage

KAGRA: varBRSE

ET_C: arXiv:0906.2655

The cases presented here

Detector combination

3 detectors (location): L-H-V
4 detecotrs (location): L-H-V-K
5 detectors (location): L-H-V-K-A

Noise curve

case 1. 2nd generation case: L: aLIGO, H: aLIGO, V: aVirgo, K: KAGRA, A: aLIGO

case 2. 2nd gen. + ET : Virgo are replaced by ET.
L: aLIGO, H: aLIGO, V: ET-C, K: KAGRA, A: aLIGO

case 3. ET case: all 5 detectors are ET-C noise curve.

Mass

We concentrate on a non-equal mass cases, (10, 1.4) and (100,10) Msolar.
The effect of FWF is largest in the non-equal mass case.

Computation

Parameters

$$(\ln(r), \ln(M_c), \delta \text{ or } \ln(\eta), t_c f_0, \phi_c, \theta_s, \phi_s, \psi, \cos(\epsilon))$$

polarization angle (ψ)

direction (θ_s, ϕ_s)

Inclination angle (ϵ)

$$M_c = M\eta^{3/5}, \quad \delta = (m_1 - m_2)/M$$

$$M = m_1 + m_2, \eta = m_1 m_2 / M^2$$

Effect taken

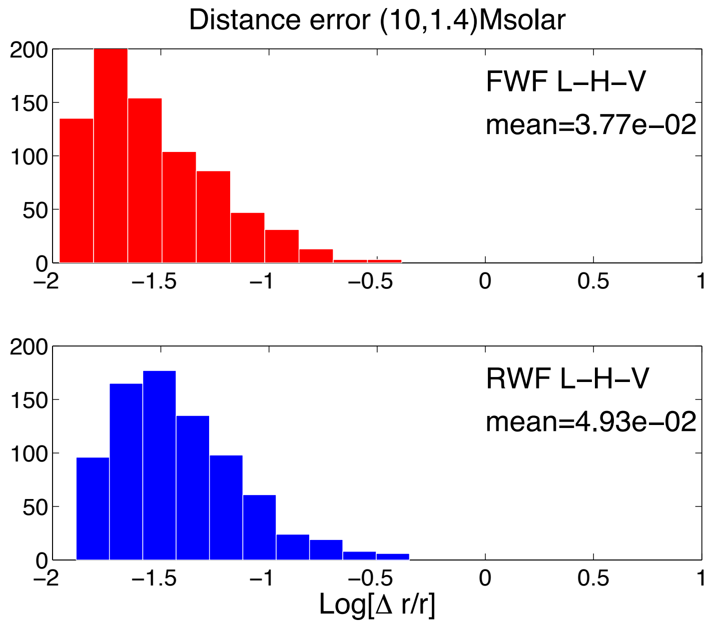
- Different orientation of each detectors
- Arrival time difference (which depend on the direction)
- Earth rotation (only for low mass and ET case) (This is irrelevant to today's results)

Give $\theta_s, \phi_s, \psi, \epsilon$ randomly, and compute Fisher matrix and covariance matrix.

Computation should be straightforward, but it's not so easy.
Sometimes, Fisher matrix for RWF becomes "ill-conditioned" due to some technical reasons (about 1% cases) (although FWF cases are OK!).
In the results below, all of such cases are just ignored.
The results below should be considered as preliminary.
This problem should be solved soon.

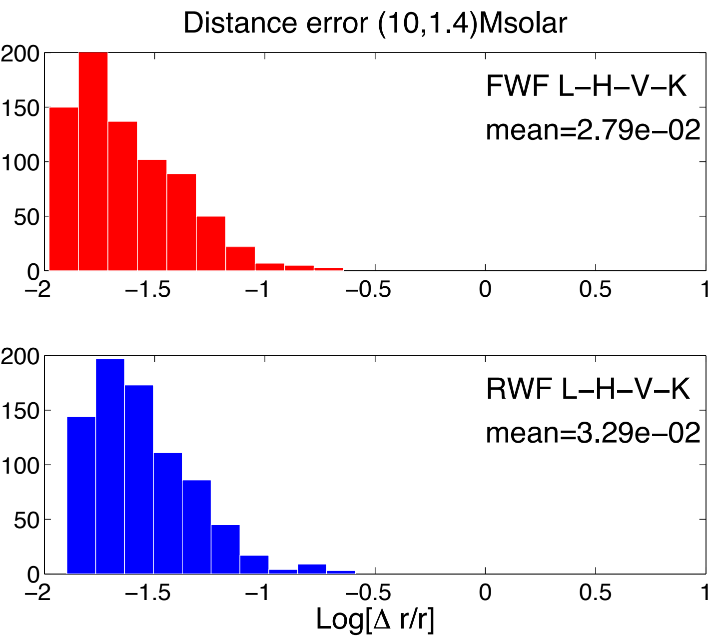
Distance error distribution (2nd gen)

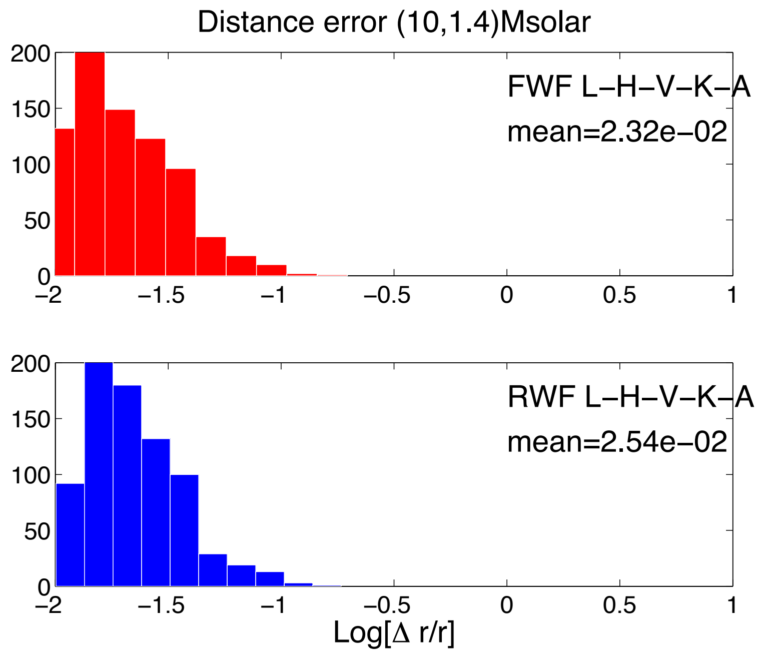
(10,1.4) Msolar
@100Mpc



FWF: red

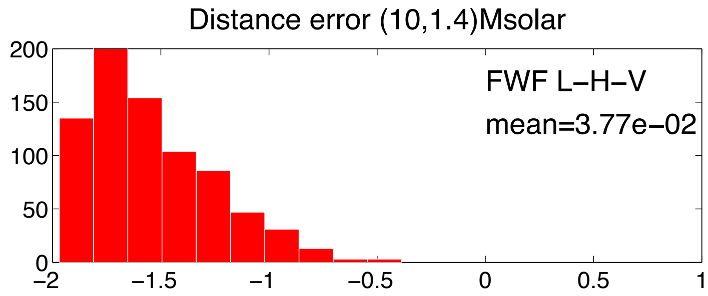
RWF:blue





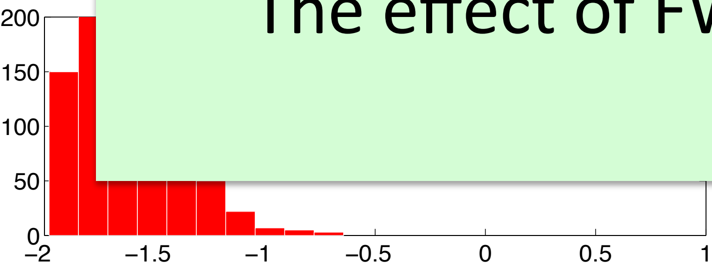
Distance error distribution (2nd gen)

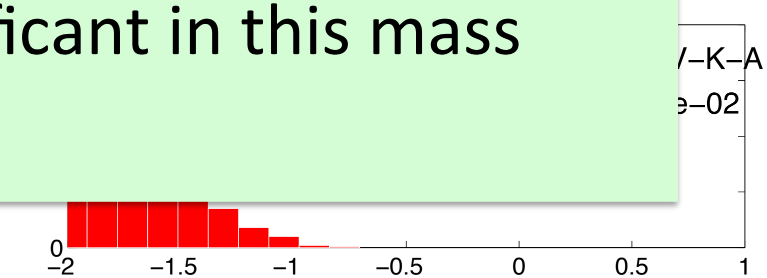
(10,1.4) Msolar
@100Mpc

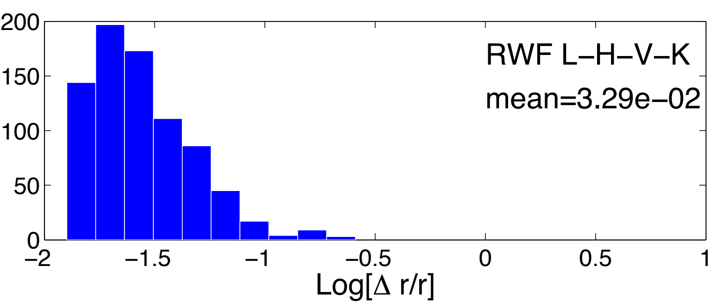


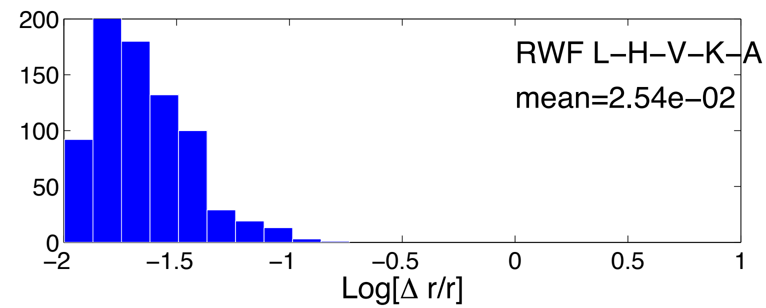
FWF: red

Accuracy of the distance is slightly better in FWF case, but the difference is small.
The effect of FWF is not significant in this mass



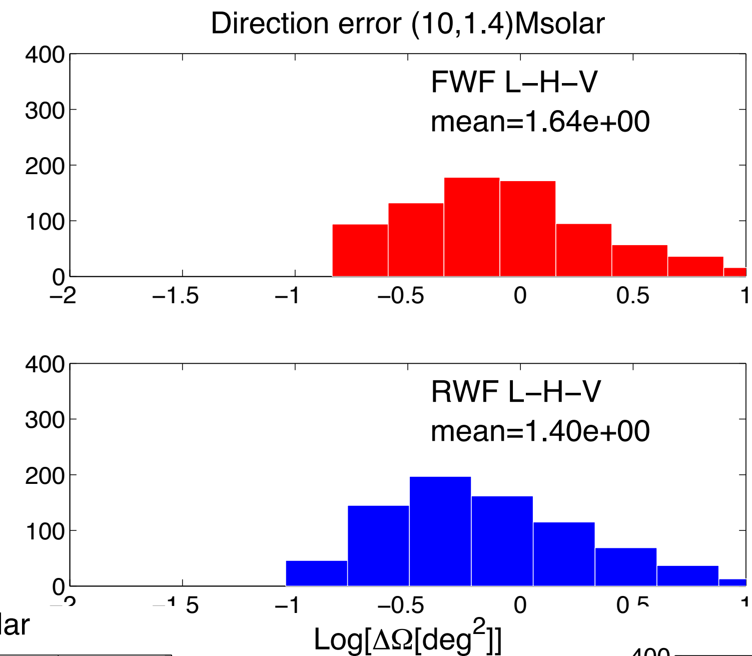


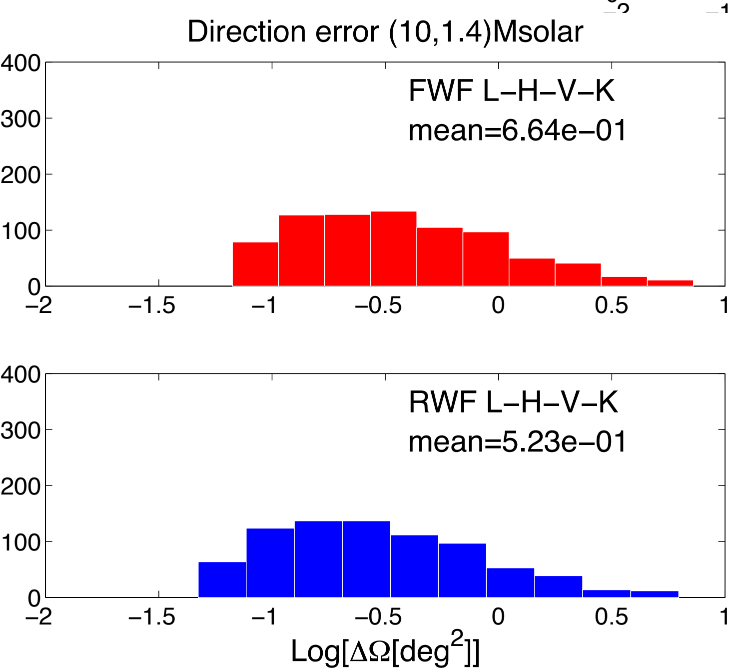


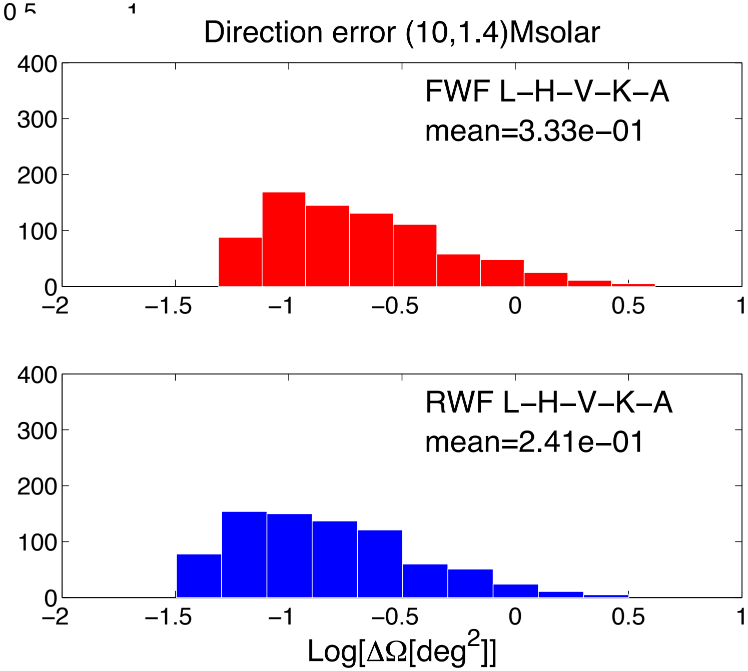


Direction error distribution(2nd gen)

(10,1.4) Msolar
@100Mpc

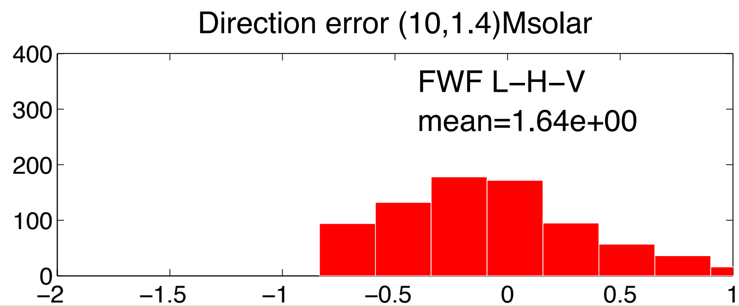




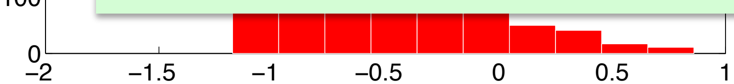


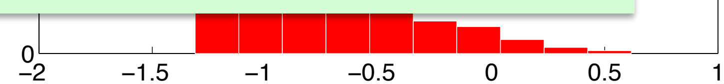
Direction error distribution(2nd gen)

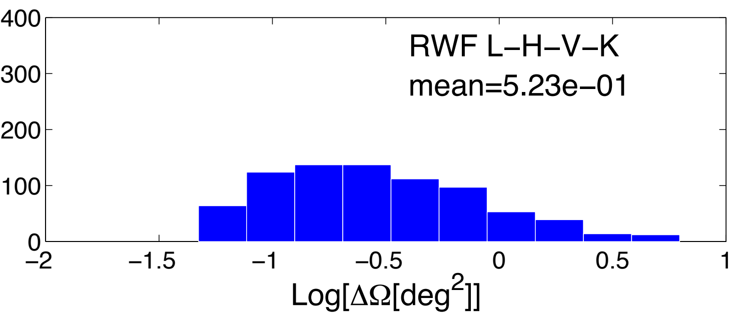
(10,1.4) Msolar
@100Mpc

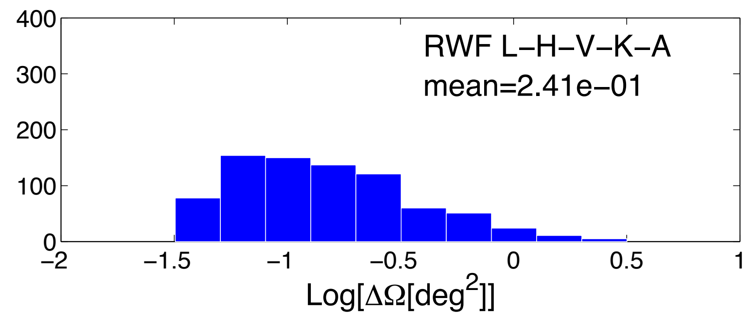


Accuracy of the direction is nearly the same in both cases. This is because the accuracy of the direction are determined by the accuracy of the coalescence time which are similar in both cases.









Parameter estimation errors

2nd generation detectors

Average of 800 simulations

SNR	L	H	V	K	A
FWF	29.2	31.0	23.3	24.1	29.5
RWF	31.6	33.6	25.9	26.3	32.0

$(10, 1.4)M_{\mathrm{solar}}$

@100Mpc

Parameter estimation errors

about 0.3-1.4 sq-degree

		ln(r)	ln(Mc)	delta	tc[msec]	Omega[sr]	minor axis [arcmin]	Major axis [arcmin]
FWF	LHVKA	0.023	3.8×10^{-5}	3.1×10^{-4}	0.16	1.0×10^{-4}	10	73
	LHVK	0.027	4.5×10^{-5}	3.7×10^{-4}	0.19	2.0×10^{-4}	14	93
	LHV	0.037	5.3×10^{-5}	4.4×10^{-4}	0.23	5.0×10^{-4}	20	184
		ln(r)	ln(Mc)	eta	tc[msec]	Omega[sr]	minor axis [arcmin]	Major axis [arcmin]
RWF	LHVKA	0.025	3.1×10^{-5}	9.8×10^{-4}	0.12	7.3×10^{-5}	8.5	61
	LHVK	0.032	3.6×10^{-5}	1.1×10^{-3}	0.16	1.5×10^{-4}	12	82
	LHV	0.049	4.1×10^{-5}	1.3×10^{-3}	0.022	4.2×10^{-4}	18	163

Summary: Direction and distance

5detector, FWF, average value

2^{nd} Gen.	(10,1.4)	(100,10)
$\Delta(\ln(r))$	0.023	0.010
$\Delta\Omega$	0.27 deg^2	0.35 deg^2

2^{nd} Gen. +1ET	(10,1.4)	(100,10)
$\Delta(\ln(r))$	8.3×10^{-3}	3.6×10^{-3}
$\Delta\Omega$	0.081 deg^2	0.14 deg^2

5ET	(10,1.4)	(100,10)
$\Delta(\ln(r))$	9.4×10^{-4}	1.9×10^{-4}
$\Delta\Omega$	5.5 $arcmin^2$	2.0 $arcmin^2$

Summary

- We considered parameter estimation accuracy for many combination of angle parameters.

- (10, 1.4) , (100,10)Msolar binary @100Mpc.

- In this case, S/N is smaller in FWF than in RWF.
- Nevertheless, parameter estimation accuracy is slightly better in FWF in many cases.

- FWF is most effective in the accurate determination of the distance. Distance is determined more accurately with FWF.

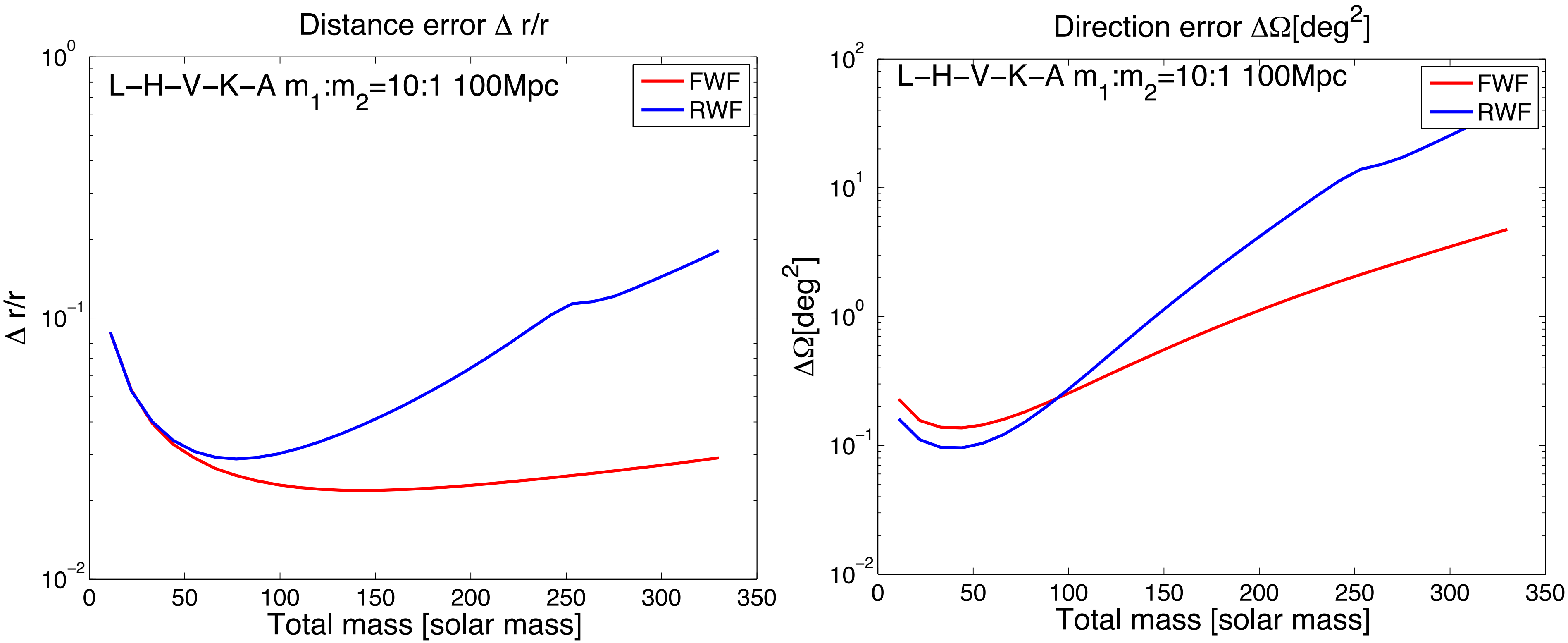
- However, the effect of FWF is not very large for these cases.

- In much larger mass cases, effect of FWF becomes more apparent.

distance and direction v.s. mass

$\theta_s = \pi/9, \phi_s = 8\pi/9,$ L-H-V-K-A (100Mpc)

$\psi = \pi/4, \epsilon = \pi/3$



Summary

- Accuracy of the direction is not improved in FWF case.

Accuracy of the direction is almost determined by the accuracy of t_c, and the accuracy of the t_c is worse in FWF case, probably because of the low S/N.

- In any case, the accuracy obtained by ET is very good and impressive.

- In the case of ETx5 and (10,1.4)Msolar @100Mpc case,

distance is determined less than 1 % accuracy, and direction is determined around 5 square arc minutes. The value is even better in the large mass case (100,10)Msolar.

- Even one ET combined with other 2nd[nd] generation detectors improve the accuracy significantly.