Numerical Relativity and Modeling of GW signals

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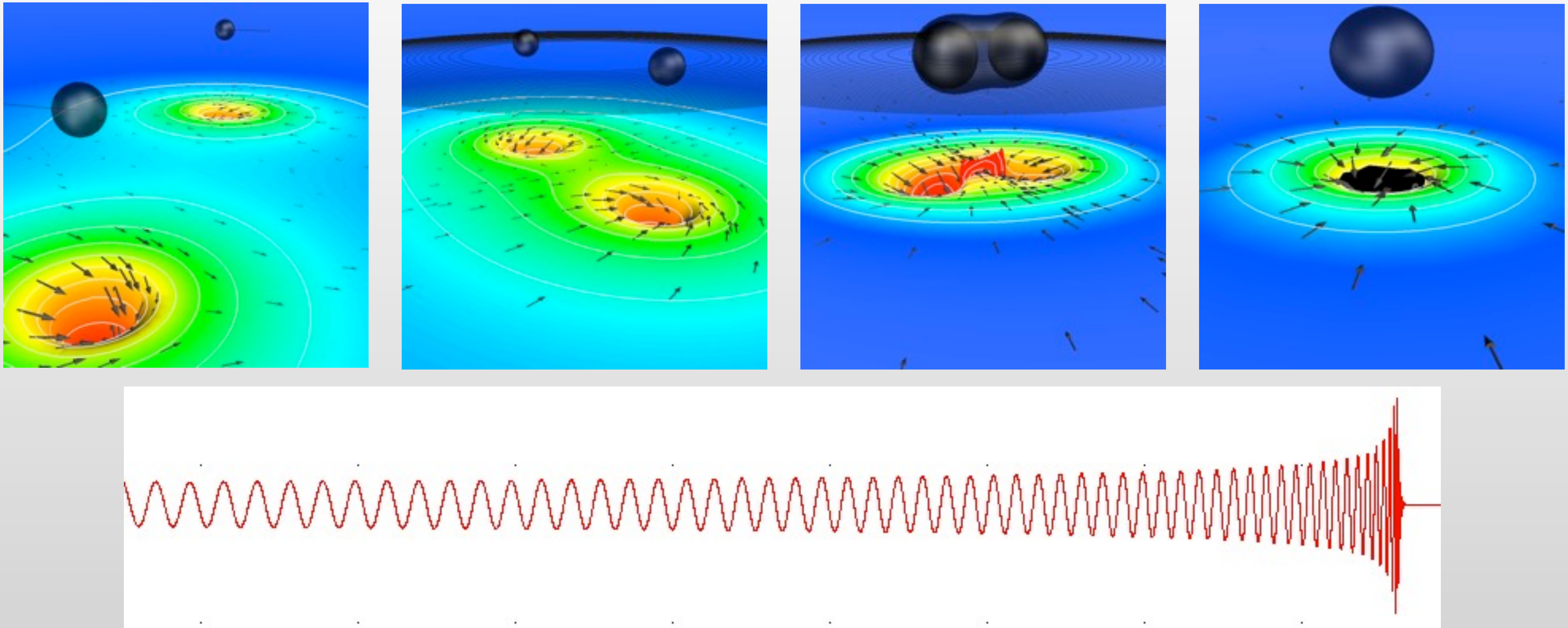
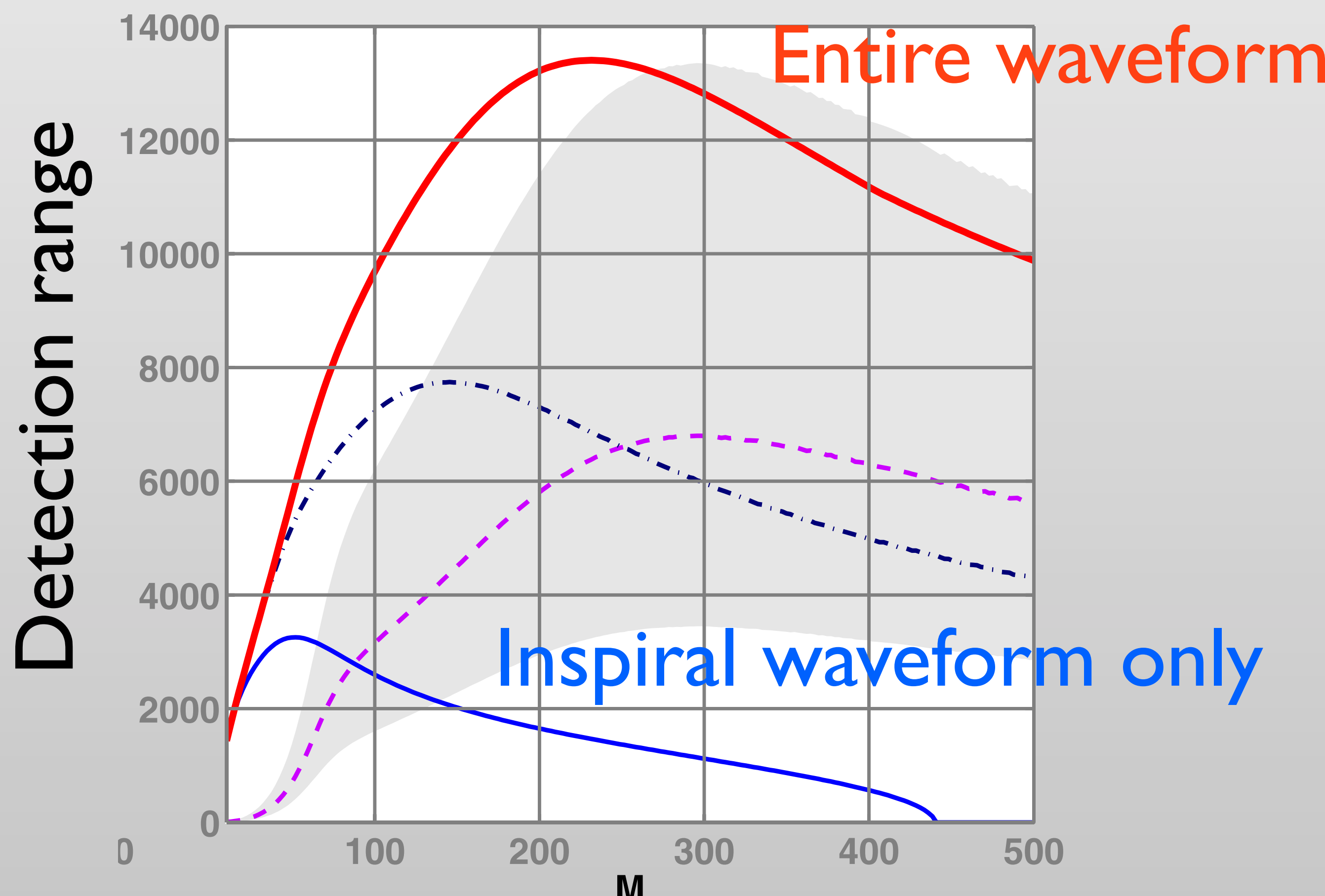


❖ **Goal** of this talk: Describe efforts to model CBC waveforms

- Compute the waveform *quickly* for *any* relevant parameters

Importance for GW detectors

Event detection



Ajith et al, PRD 2008

Event Characterization

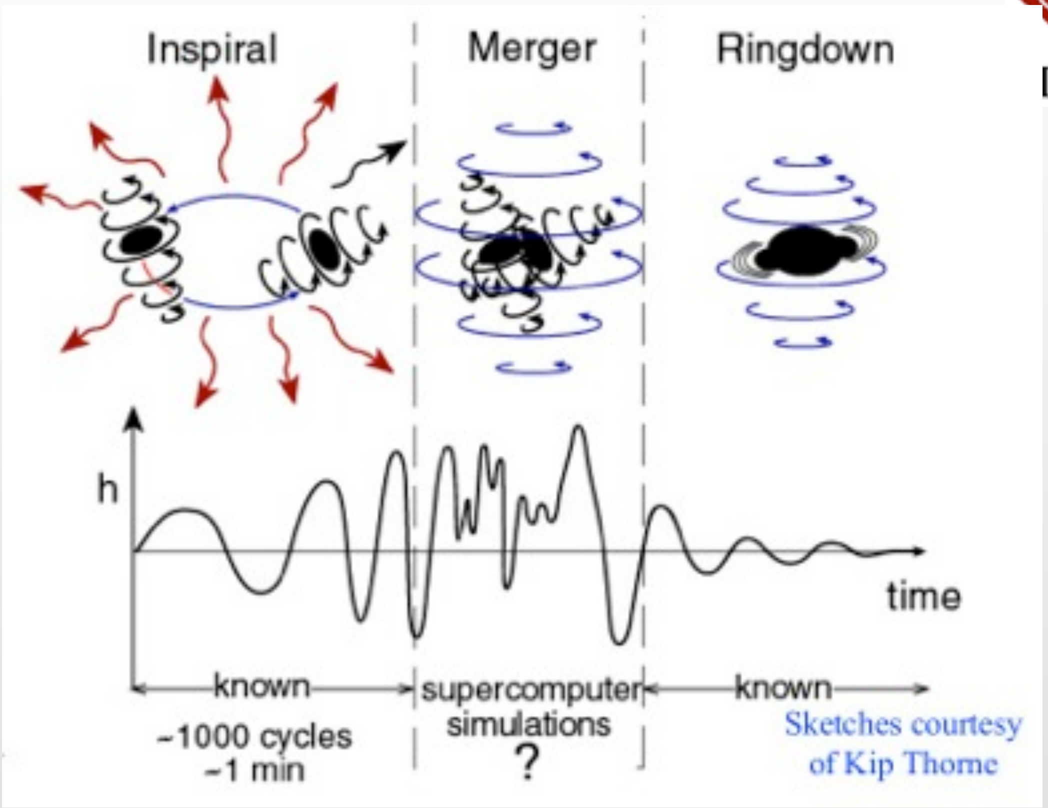
MCMC codes

Have we seen a BH or a NS?

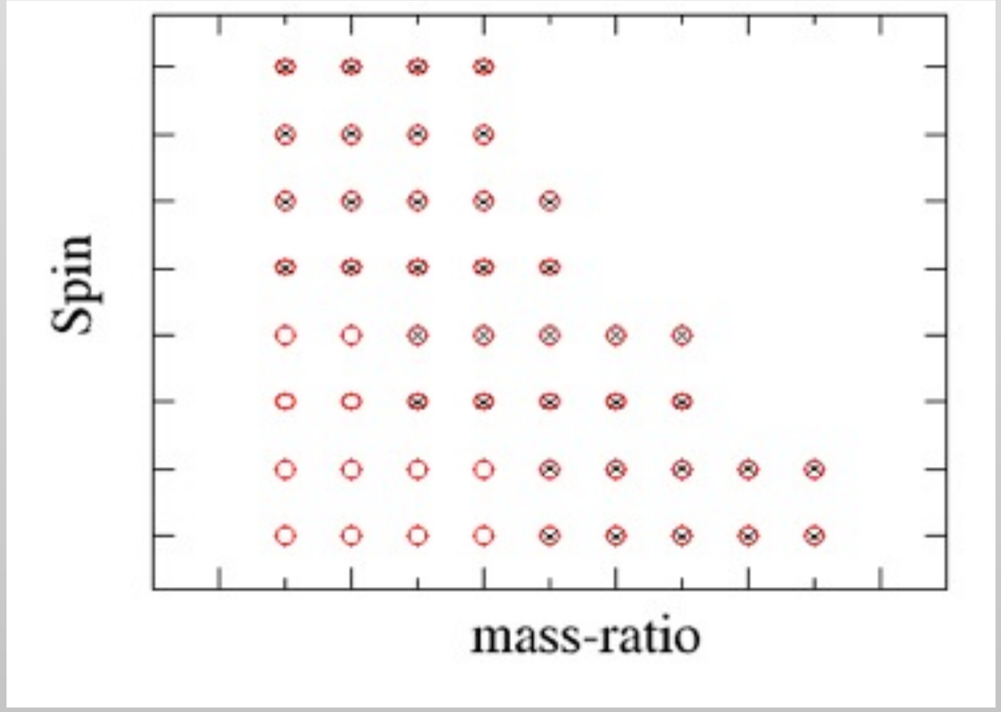
Was Einstein right?

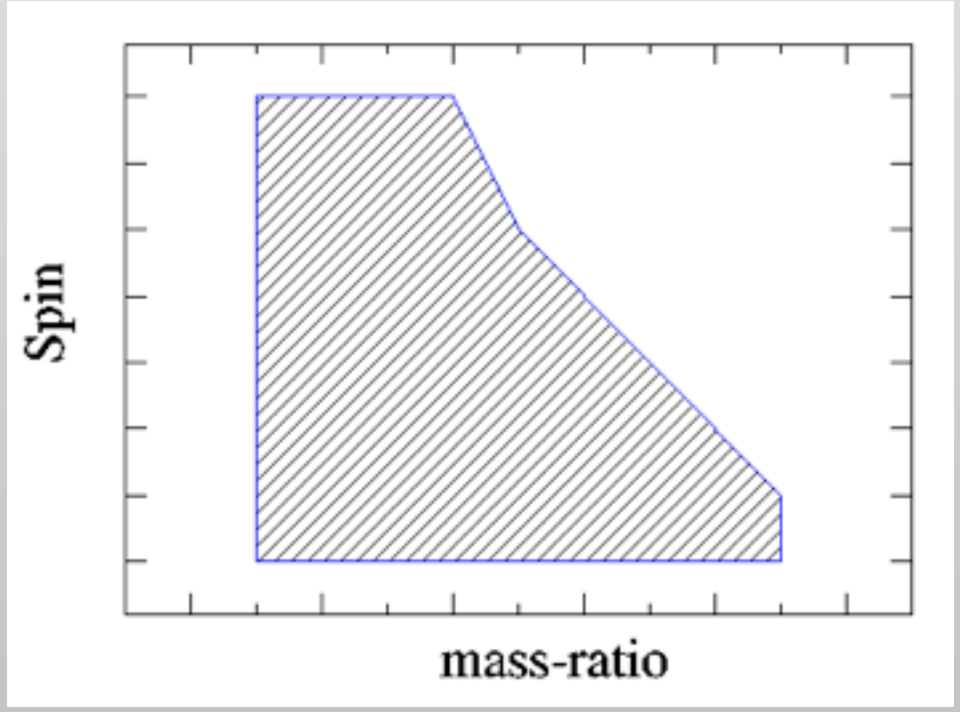
Basic ingredients

- ❖ Analytical results for early inspiral
- ❖ Numerical Relativity (NR) for late inspiral, merger, ringdown



- ❖ Combine. Interpolate to continuous parameters.



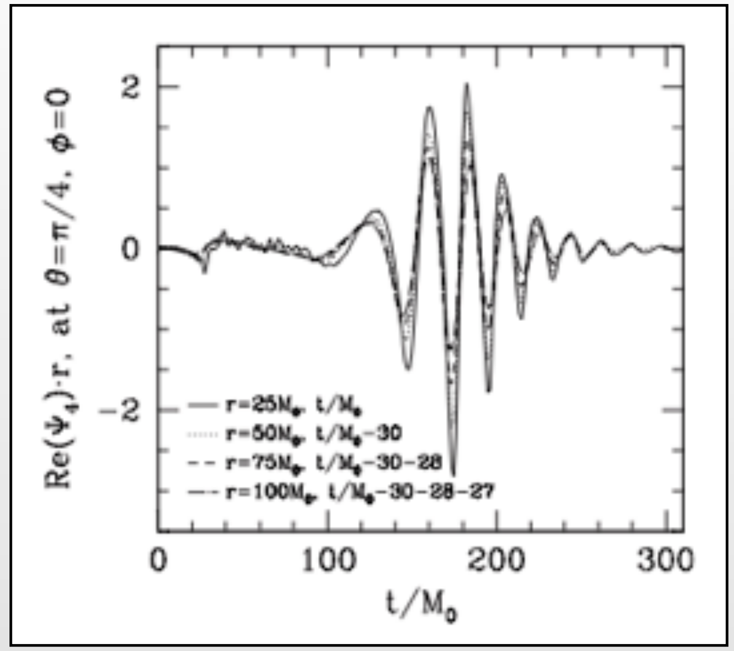


Stages of Scientific Discovery

CITA–ICAT

❖ Breakthrough

Pretorius '05



❖ Early results

- BH-BH kicks
- 1-config. PN-NR comparisons
- 1-parameter waveform models

❖ Breadth & Depth

- Cover parameter space
- Improve quality
- Understand systematic errors

BH-BH

NS-NS

BH-NS

Outline

1. Introduction
2. Numerical Relativity
3. Current analytical waveform families
4. Going forward: Pessimistic view
5. Going forward: Optimism
6. BH-NS, NS-NS
7. Summary

Numerical Relativity

The two approaches to BH–BH

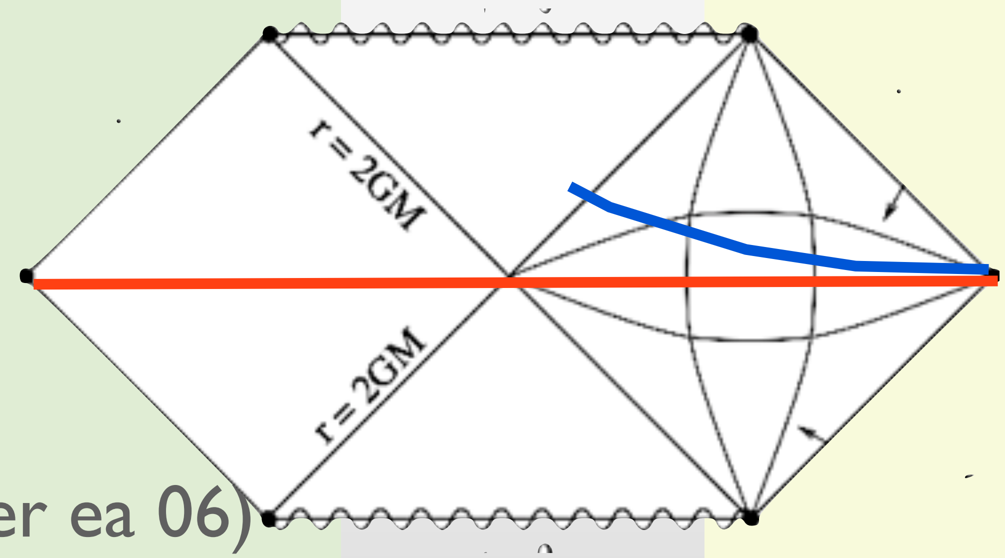
CITA–ICAT

Puncture initial-data
(Brandt&Brügmann 97)

BSSN w/ moving punctures
(Campanelli ea 06, Baker ea 06)

$$
\begin{aligned}
g_{ij} &= e^{4\phi}\tilde{g}_{ij}, \\
\tilde{\Gamma}^i &= \tilde{g}^{jk}\tilde{\Gamma}^i_{jk} \\
\partial_t \phi &= \ldots \\
\partial_t \tilde{g}_{ij} &\approx -\tilde{A}_{ij} \\
\partial_t \tilde{A}_{ij} &\approx -\Delta \tilde{g}_{ij} \\
\partial_t \tilde{\Gamma}^i &= \partial_t\left(\tilde{g}^{jk}\tilde{\Gamma}^i_{jk}\right)
\end{aligned}
$$

Finite differences w/ AMR
(RIT, AEI, GATech, Goddard, Jena, Palma, Cardiff, Perimeter)



Quasi-equilibrium excision initial-data
(Cook 02, Cook&HP 04)

Generalized Harmonic w/ constraint damping
(Gundlach ea 05, Pretorius 05)

$$
\Box g_{ab} = -2\nabla_{(a}H_{b)} + \gamma_0\left[t_{(a}C_{b)} - \frac{1}{2}g_{ab}t^c C_c\right] + \text{lower o}
$$

Multi-domain spectral methods
SpEC (Cornell-Caltech-CITA-Wash.)

The two approaches to BH–BH

Finite differences w/ AMR
(RIT, AEI, Georgia Tech, Jena, Palma, Cardiff, Perimeter)

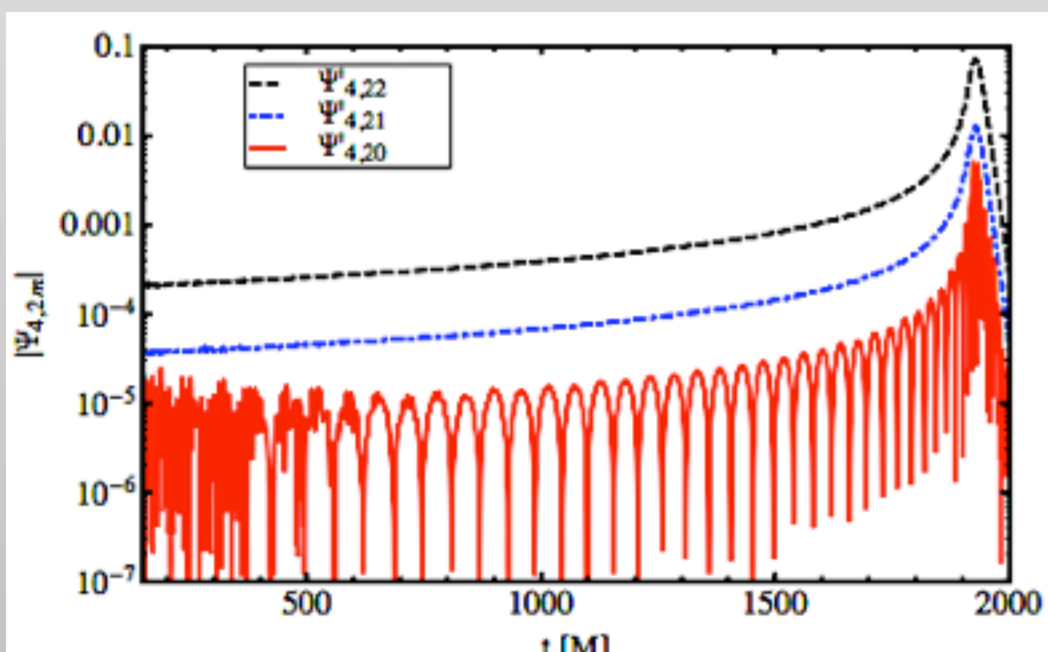
Conventional wisdom:
- -- Robust, "easy"
- -- Many short simulations
- -- Lower accuracy, higher cost

Currently:
- -- about 10 orbits
- -- accuracy ok for GW detection

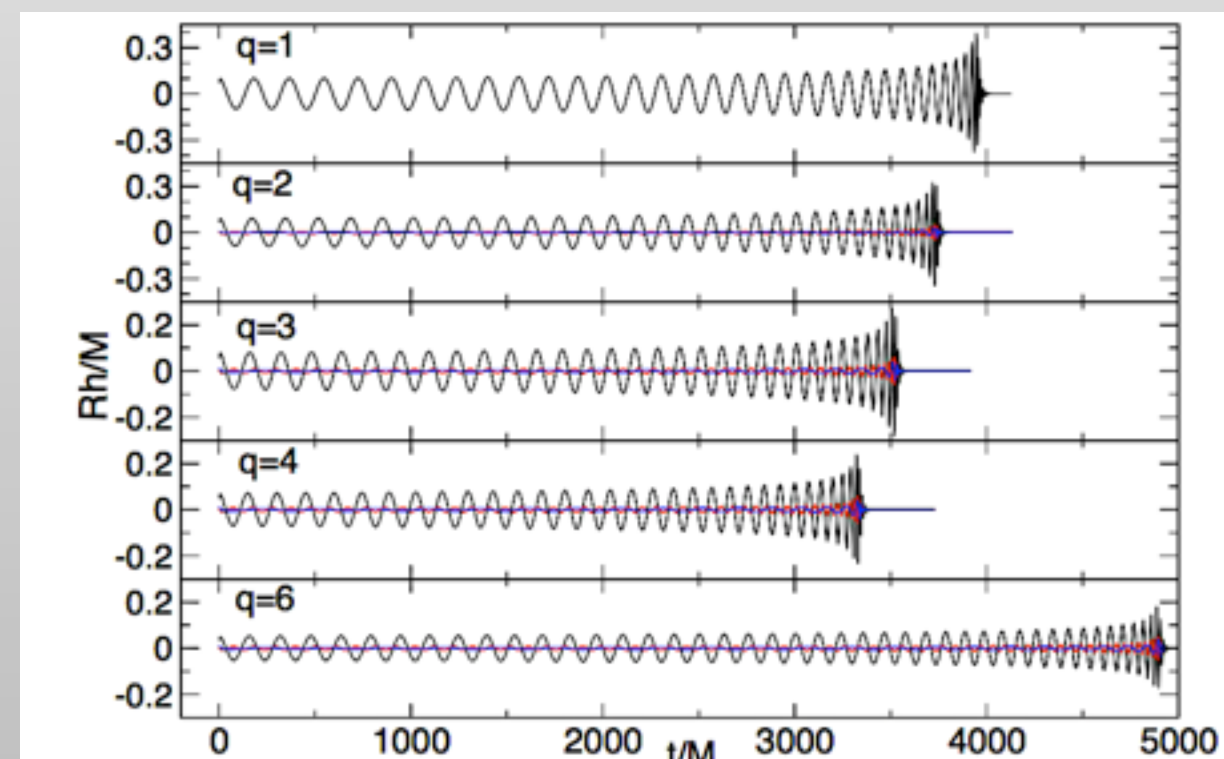
Multi-domain spectral methods
SpEC (Cornell-Caltech-CITA-Wash U-Fullerton)

Conventional wisdom:
- -- Less robust, "difficult"
- -- Few long simulations
- -- Higher accuracy, lower cost

Currently:
- -- about 20 orbits
- -- accuracy ok for param est'n



Schmidt ea 2011



Buchman ea (in prep)

NR capabilities (rough guide)

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	Easy	Moderate	Hard
Mass-ratio	<3	3-6	>6
Spin large BH	<0.5	0.5-0.9	>0.9
Spin small BH	0	<0.5 ish	>0.5 ish
# orbits	<8	8-15	>20

❖ Difficulty *multiplicative* (q=10, S/M^2=0.99, 30 orbits = hard^3)

❖ “Hard” generally involves novel research

- time-scale of simulation unpredictable
- Combining two “hard” categories *rarely* done
- Combining three “hard” categories has *not* been attempted so far

Current Waveform Models

Phenomenological, aligned spins

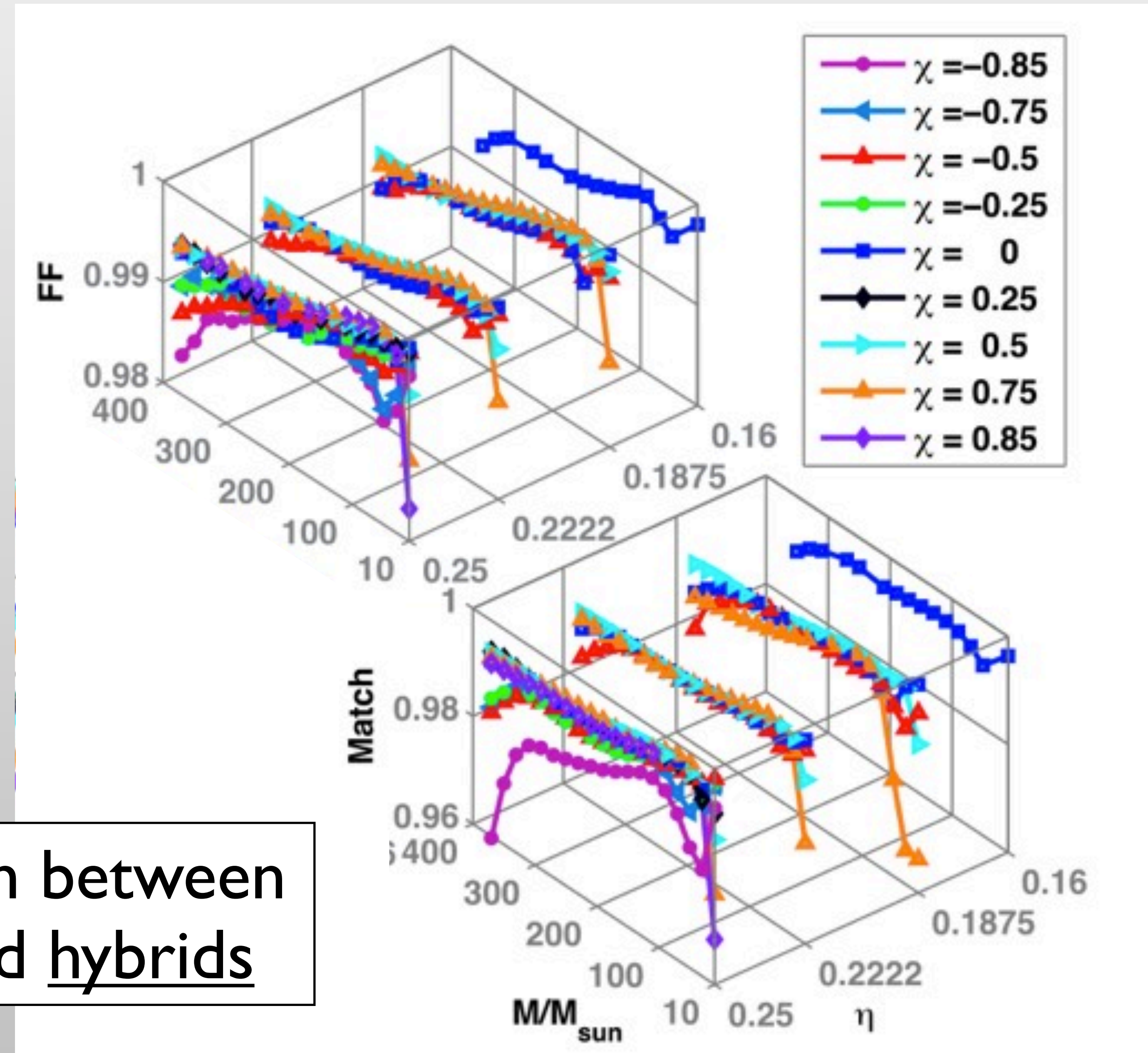
CITA-ICAT

- Unequal-mass, aligned spins (Ajith ea 2011) "IMRPhenomB"
  - 2-dim waveform family (mass-ratio, effective spin)
  - (2,2) mode calibrated against 24 sims (BAM, Ccatie, Llama)

- Two stages:
  1. construct TaylorT1+NR hybrids
  2. fit model to hybrids

$$A(f) \equiv C f_1^{-7/6} \begin{cases} f'^{-7/6}\left(1+\sum_{i=2}^{3} \alpha_i v^i\right) & \text{if } f < f_1 \\ w_m f'^{-2/3}\left(1+\sum_{i=1}^{2} \epsilon_i v^i\right) & \text{if } f_1 \leq f < f_2 \\ w_r \mathcal{L}(f, f_2, \sigma) & \text{if } f_2 \leq f < f_3, \end{cases}$$

$$\Psi(f) \equiv 2\pi f t_0 + \varphi_0 + \frac{3}{128 \eta v^5}\left(1 + \sum_{k=2}^{7} v^k \psi_k\right). \qquad (1)$$



FF & match between model and hybrids

EOB + NR

❖ Effective one body

- Buonanno, Damour 1999; many papers since

❖ Inspiral-Merger-Ringdown waveform model based on

- Effective Hamiltonian to capture conservative dynamics

$$H = \mu\sqrt{p_r^2 + A(r)\left[1 + \frac{p_r^2}{r^2} + 2(4-3\nu)\nu\frac{p_r^4}{r^2}\right]}, \qquad A(r) = \sum_{k=0}^{4}\frac{a_k(\nu)}{r^k} + \frac{a_5(\nu)}{r^5}$$

- Radiation reaction terms

$$\frac{dp_r}{dt} = -\frac{\partial H}{\partial p_r} + a^r_{\rm RR}\frac{\dot r}{r^2\Omega}\widehat{\mathcal{F}}_\phi$$

$$\frac{dp_\varphi}{dt} = = 0 - \frac{v_\Omega^3}{\nu V_\phi^6}F_4^4(V_\phi;\nu,v_{\rm pole}), \quad \text{using 4-PN term } \mathcal{F}_{8,\nu=0} + \nu A_8$$

- Attach ringdown modes

★ Fit parameters to NR simulations

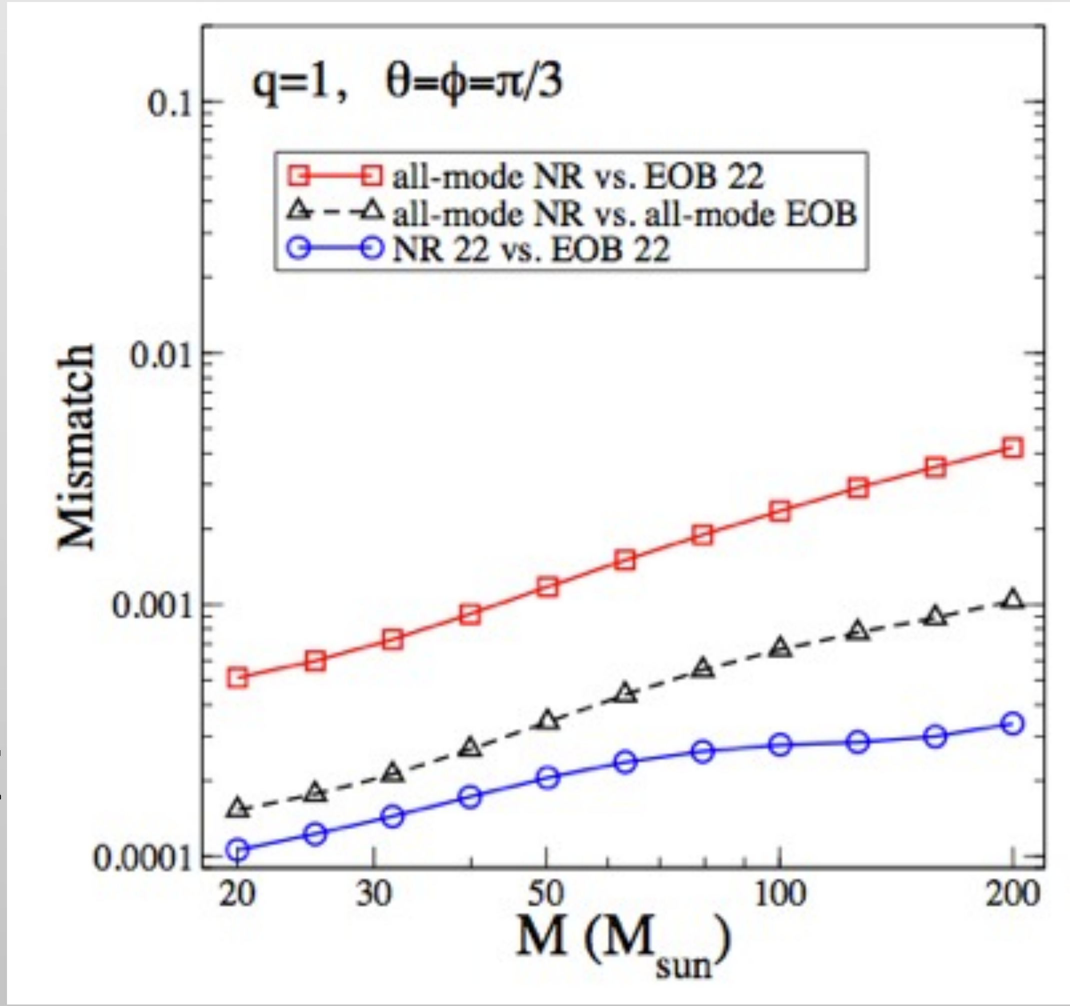
EOB for non-spinning BH-BH

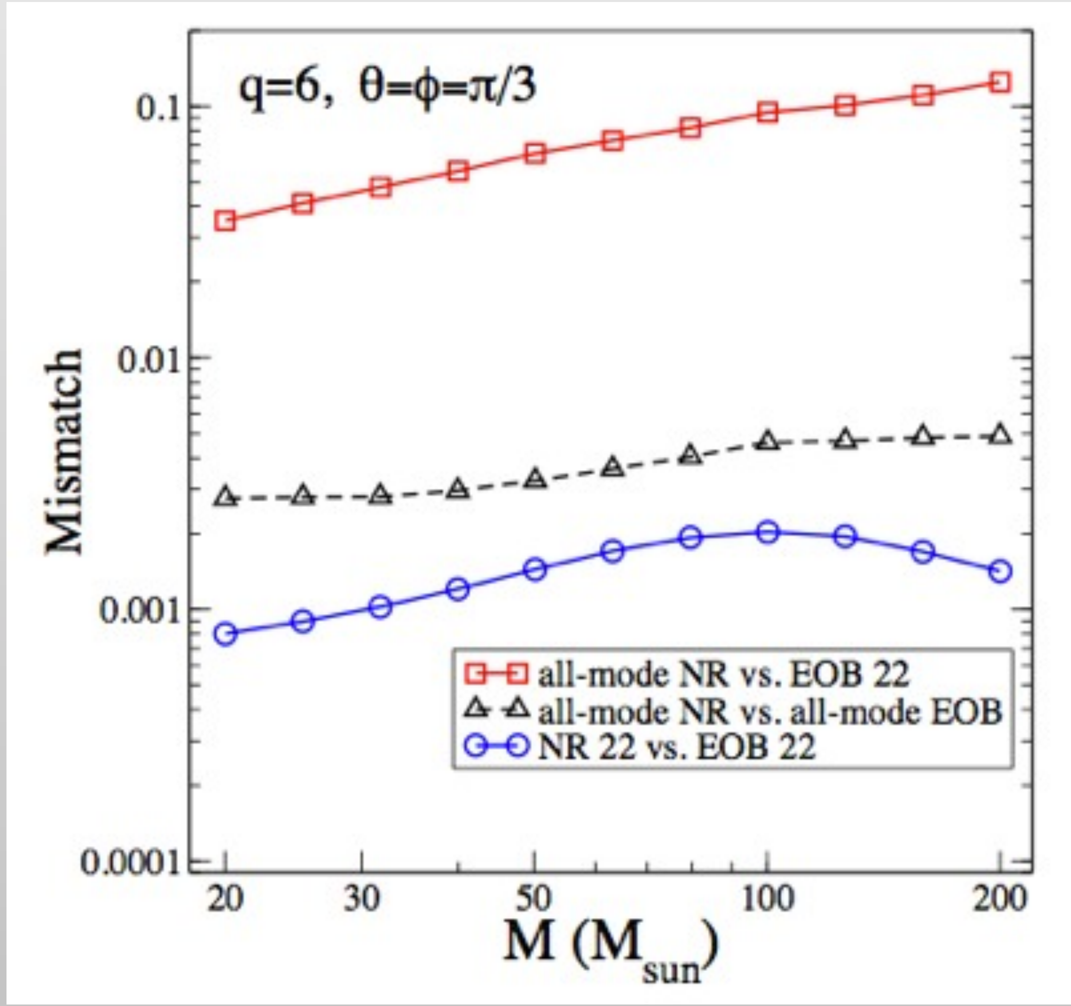
❖ Physical parameter mass-ratio q

❖ "EOBNRv2" Pan ea, 2011

- supersedes EOBNRv1 (Buonnano ea 2007)
- Five modes: (2,2), (2,1), (3,3), (4,4), (5,5)
- calibrated against SpEC q=1,2,3,4,6.

Mismatch with NR waveforms <u>used in fitting</u>





EOB for aligned spins

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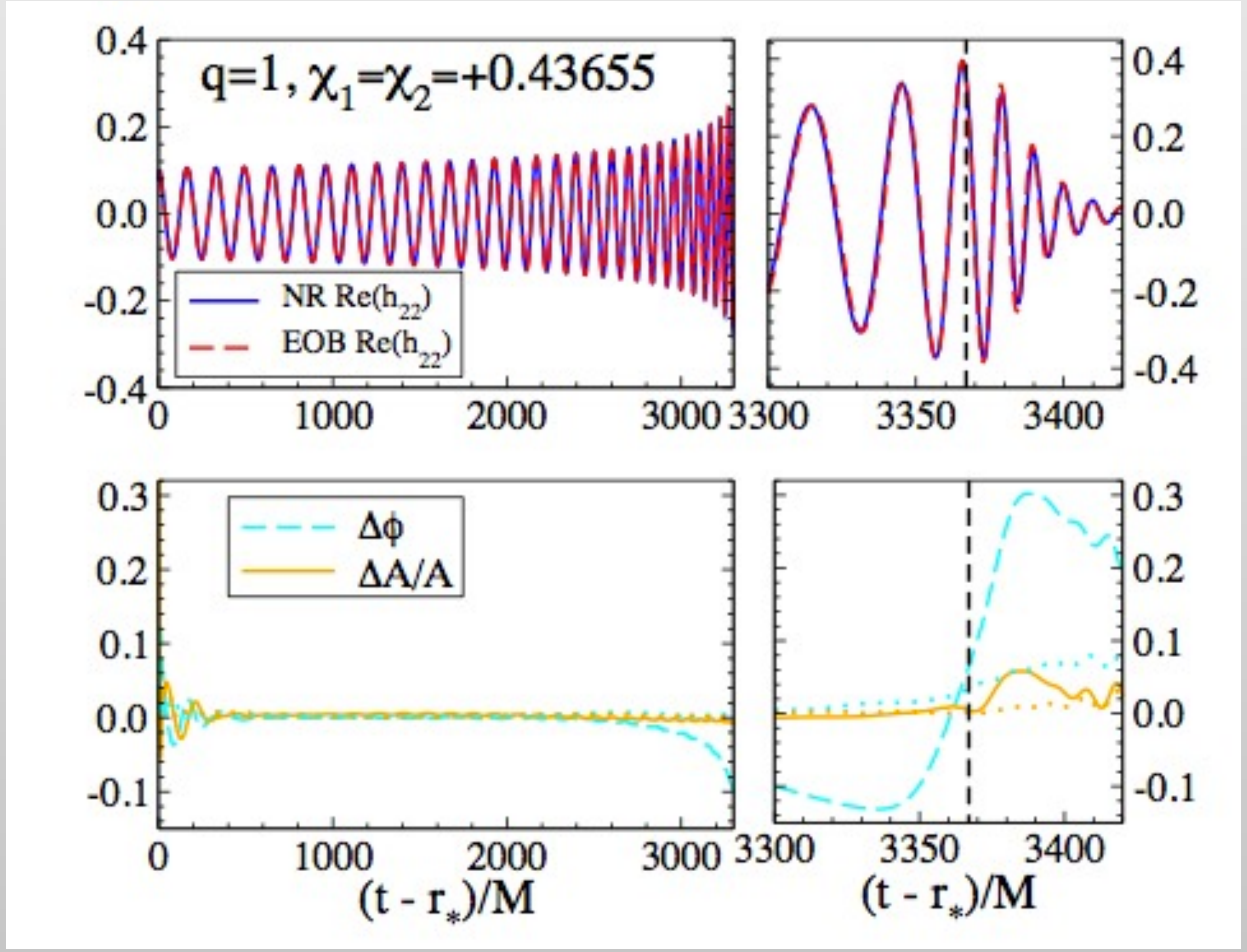
- EOB w/ aligned spins "SEOBNRv1"
  - Taracchini ea 2012
  - (2,2) mode calibrated against 7x SpEC & Teukolsky code
  - Prototype-model: Intended for re-calibration with more NR sims

- Caveats:
  - Calibrated in tiny region of param space:
    (a) zero spin q=1,2,3,4,6
    (b) q=1, equal spin ±0.44
  - Current EOB model fails for aligned spins >0.7

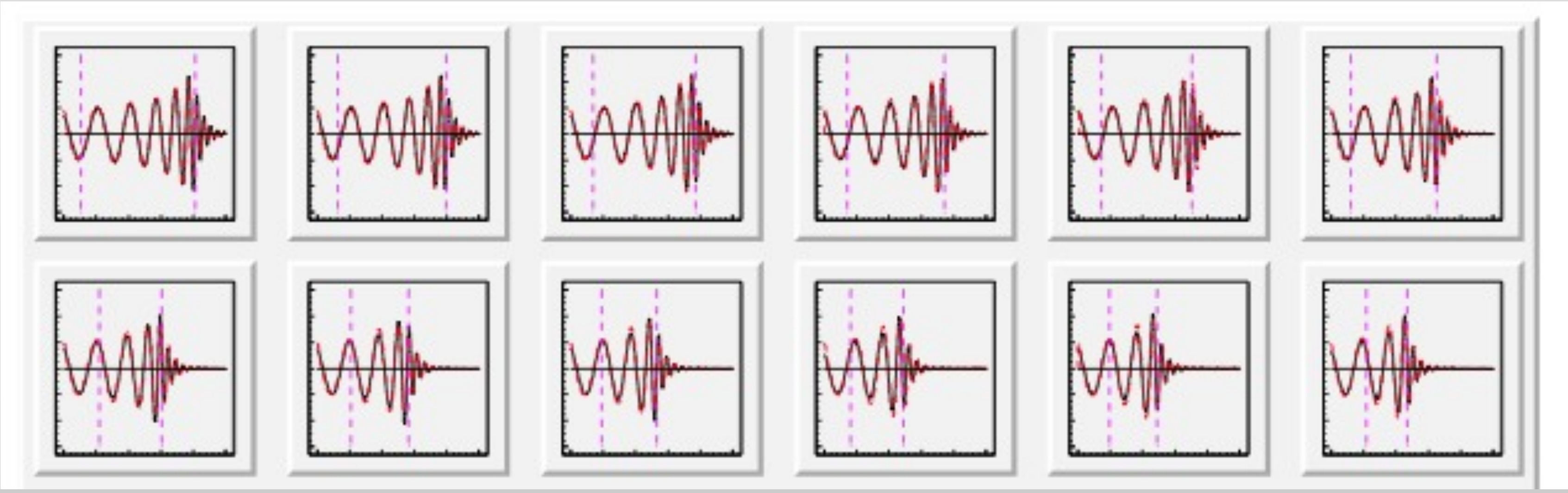


Taracchini ea 2012

Precessing BH-BH

❖ First generic spin model (Sturani ea 2010)

- Based on 24 MayaKranc sims
- TaylorT4 until very close to merger & phenomenological Ansatz



1/2 of the Sturani ea NR waveforms

Reasons for Pessimism

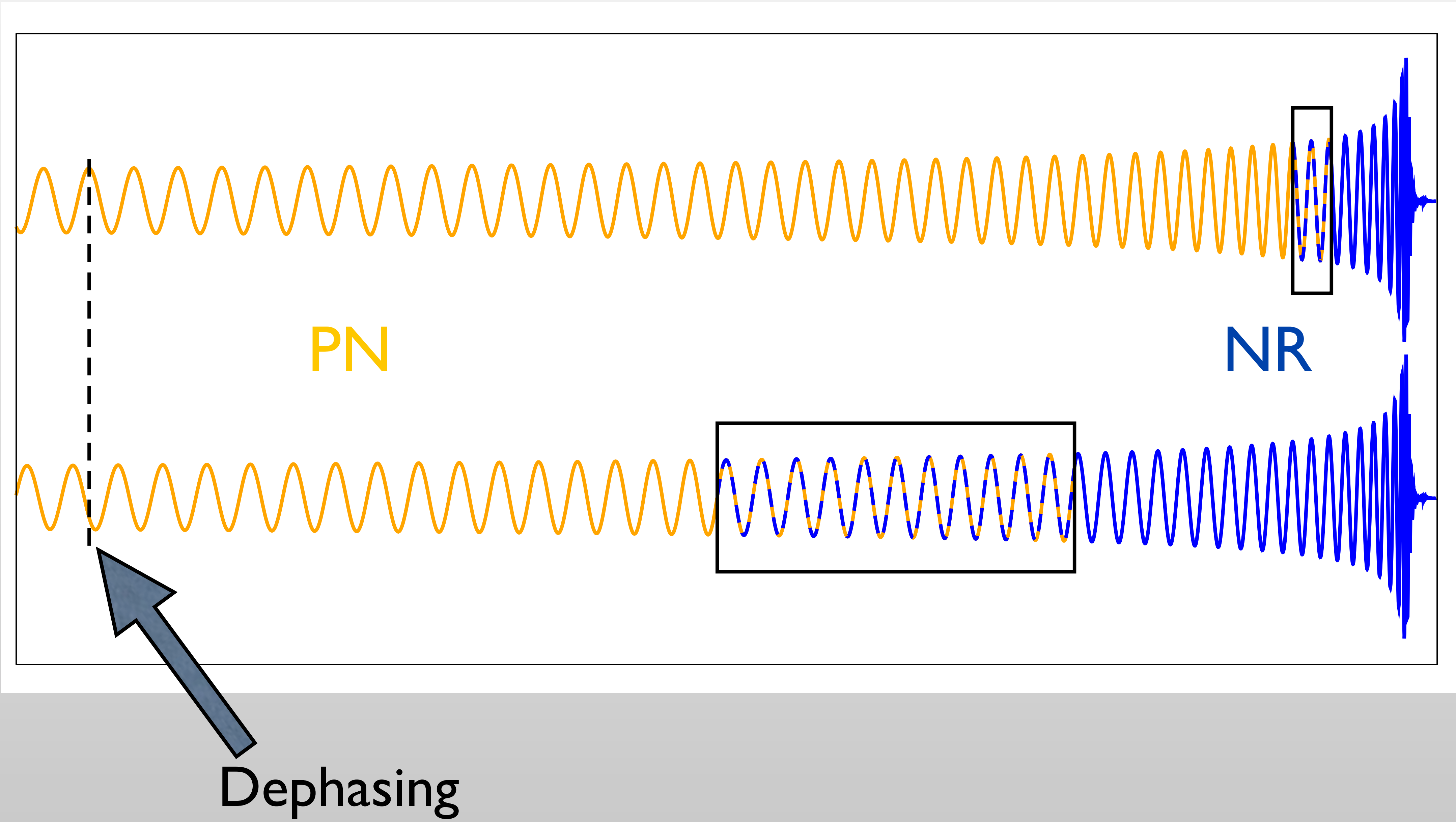
Quality

❖ Important properties of NR waveforms

- Accuracy of NR
- Length of NR
- # of NR waveforms / Parameter space coverage

Length requirements for NR

❖ Must switch to NR early enough to avoid large PN errors

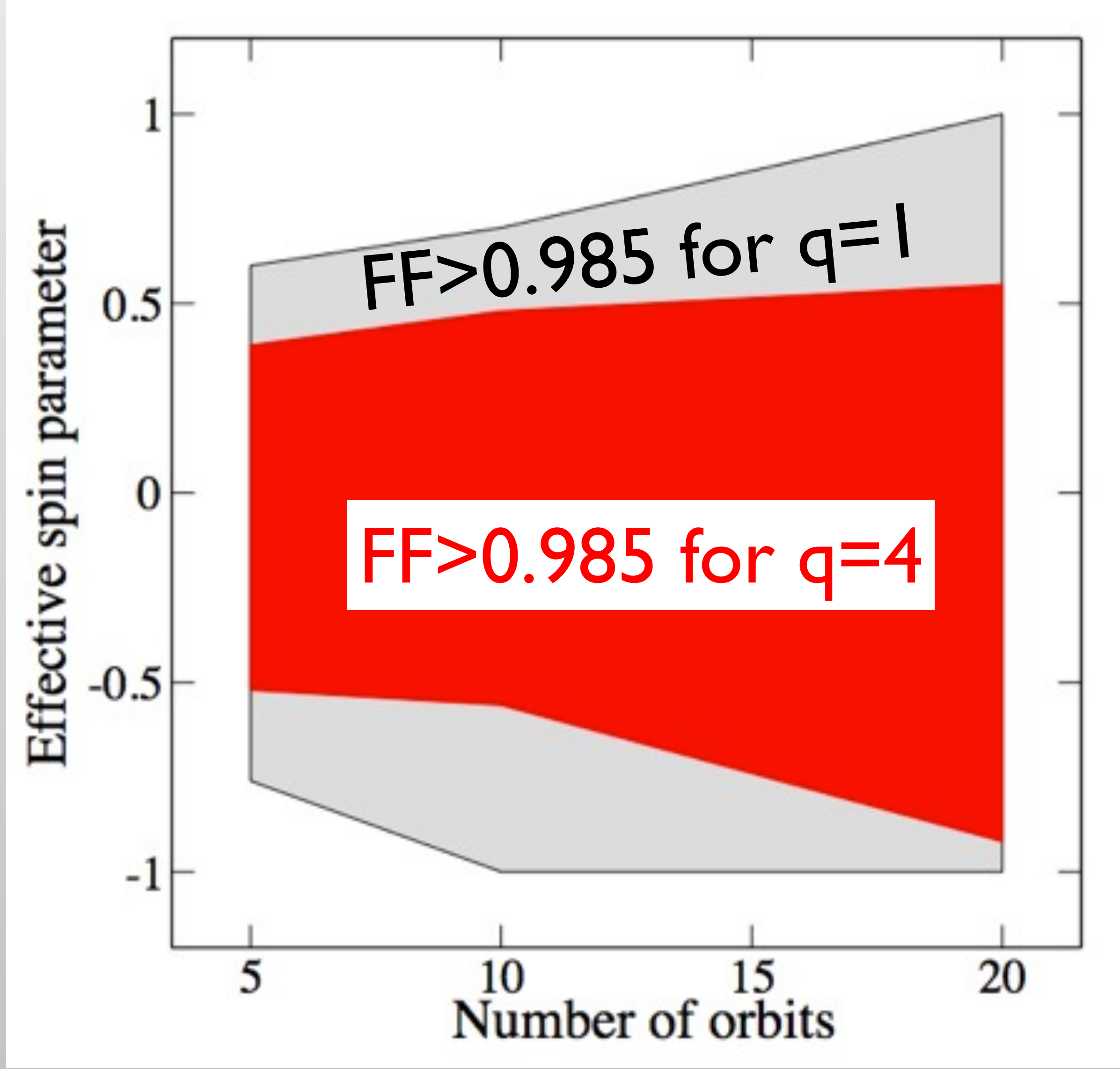


Length: GW-detection

- Maximize overlap over physical parameters $\vec{\lambda}$

$$\mathrm{FF}(\vec{\lambda}_0) \equiv \max_{\vec{\lambda}} \frac{\langle h_{\mathrm{correct}}(\vec{\lambda}_0), h_{\mathrm{PN+NR}}(\vec{\lambda})\rangle}{|h_{\mathrm{correct}}(\vec{\lambda}_0)|\,|h_{\mathrm{PN+NR}}(\vec{\lambda})|}$$

  - Less stringent requirements
  - More difficult to analyze (need continuous waveform models to perform maximization)



- Hannam ea '10, Ohme ea '11
  - ~10 NR orbits sufficient for large parts of non-precessing parameter space

Length: Parameter estimation

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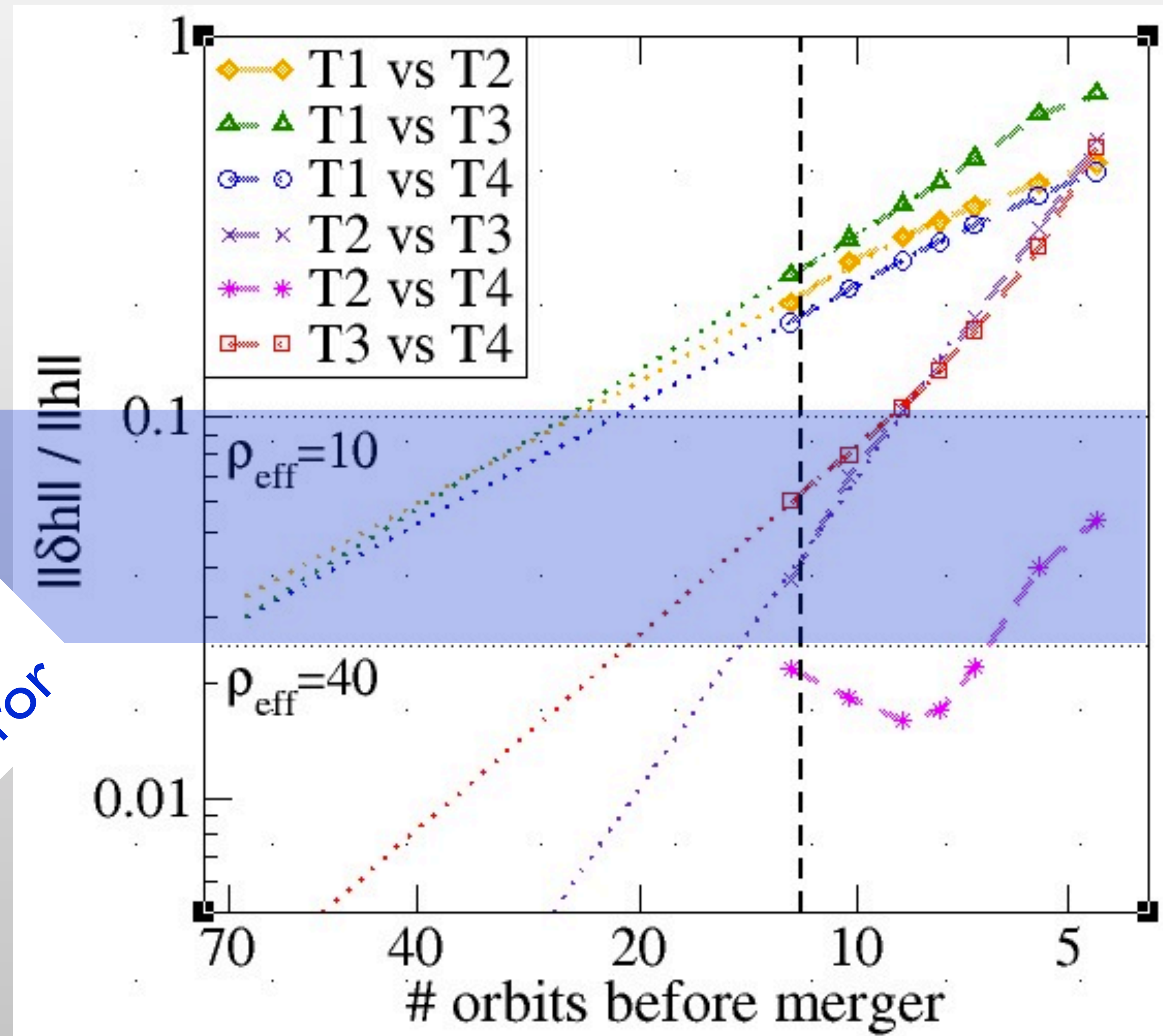
- ❖ Start NR so early that different PN approximants cannot be distinguished by LIGO
- ❖ need *much* longer NR waveforms
  - Hannam ea 2010
  - Ohme ea 2011
  - Boyle 2011
  - MacDonald ea 2011
  - Damour ea 2011

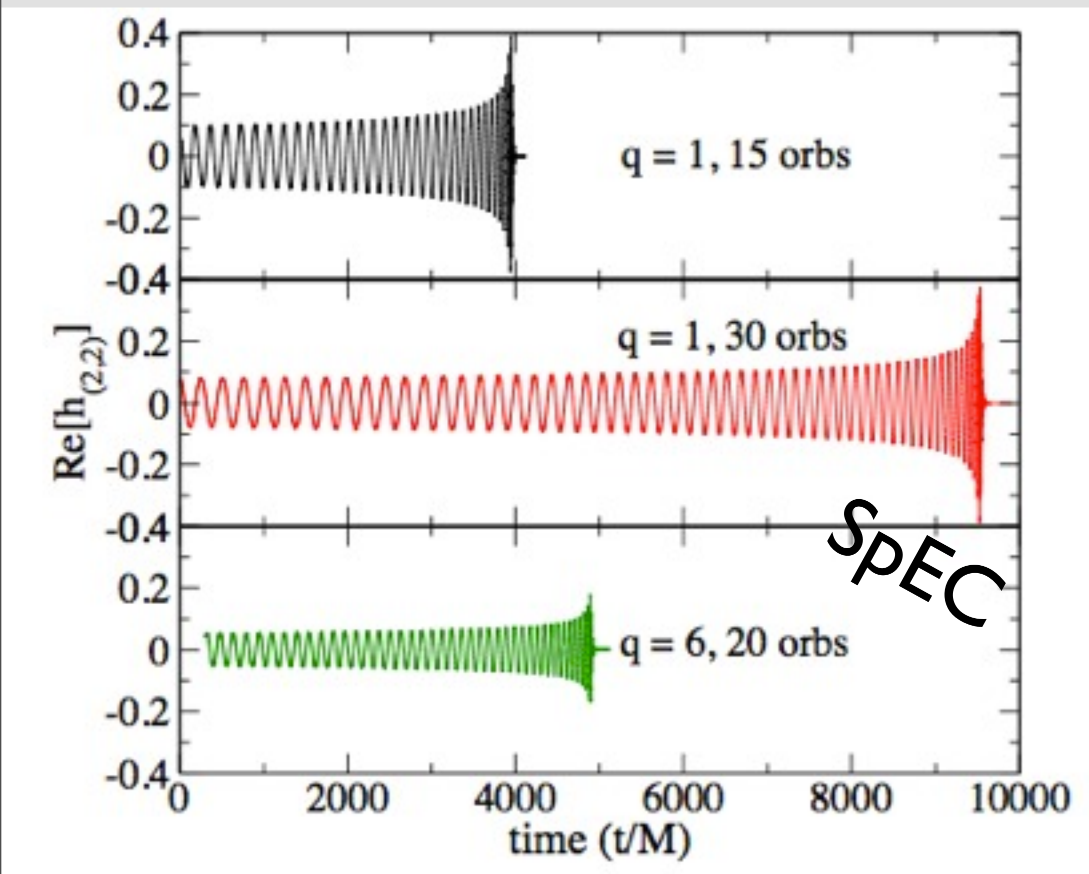


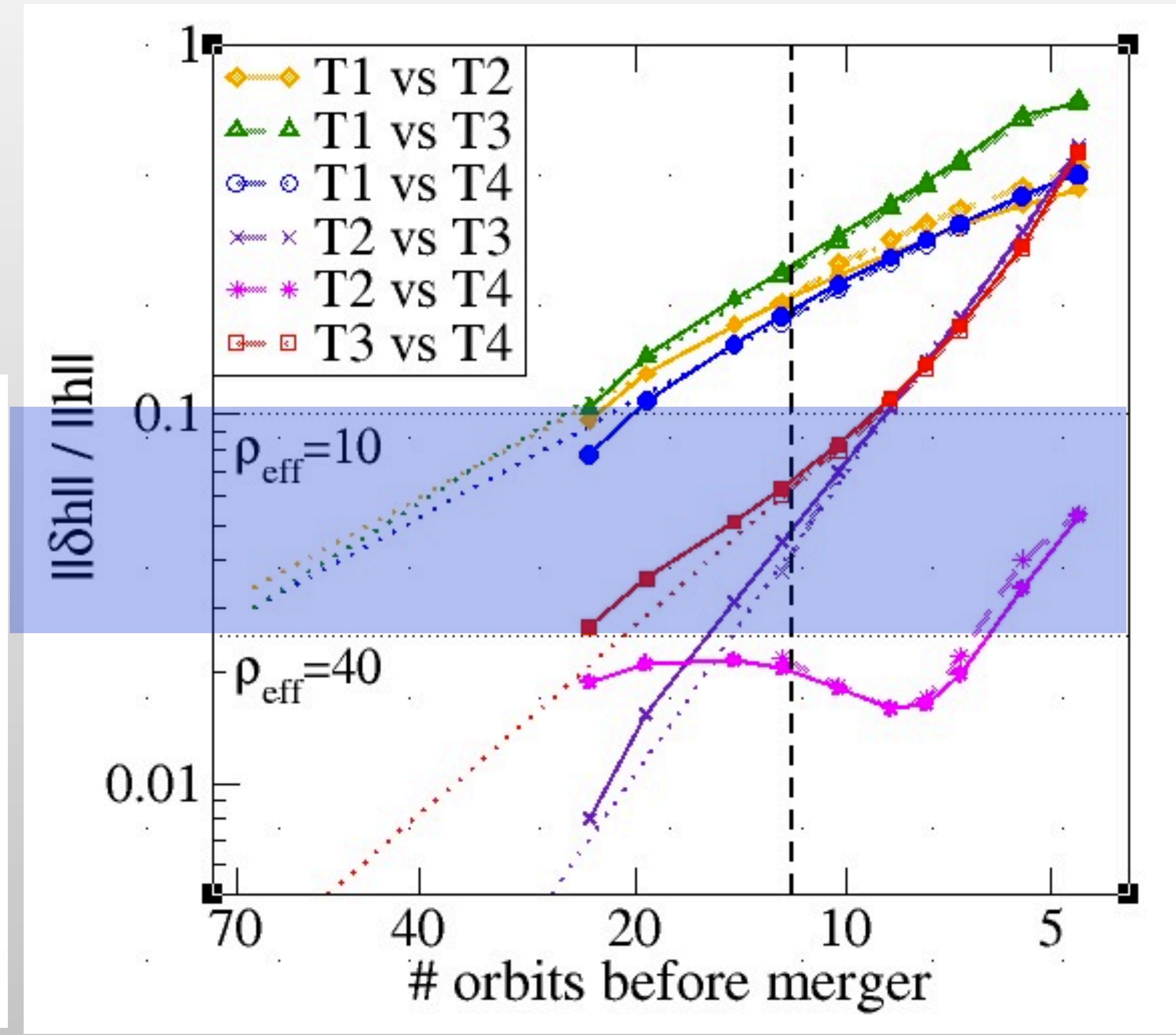
Systematic error ≲statistical error

MacDonald, Nissanke, HP 2011

Length: Parameter estimation

- New 30 orbit equal-mass, zero spin simulation
  - Confirm previous results
  - Long enough for *one* choice of parameters





MacDonald, Mroue, HP in prep

Length–Statements depend on λ

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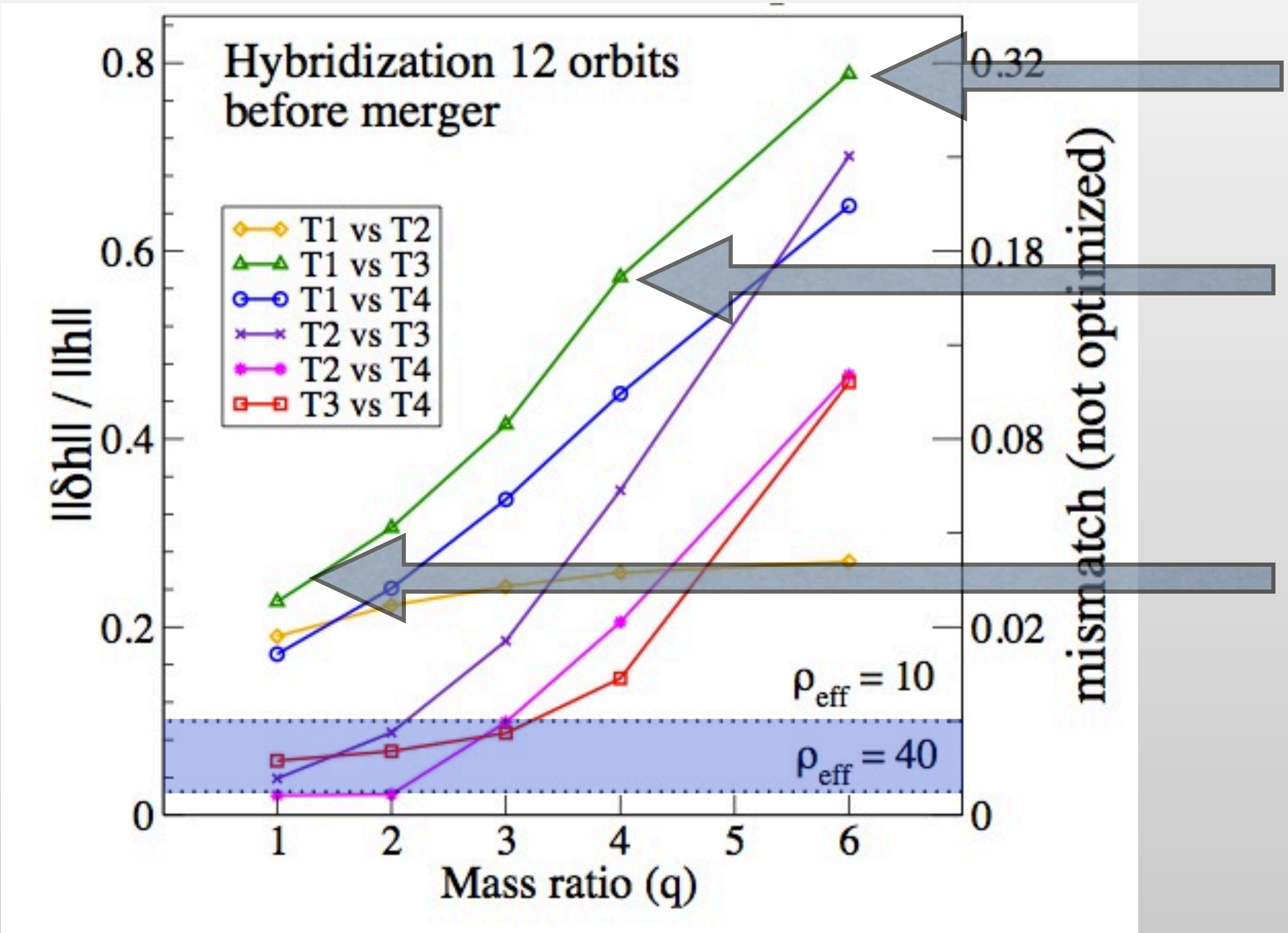
❖ Non-spinning, unequal masses



q=6, MM=0.3

q=4, MM=0.2

q=1, MM=0.03

MacDonald, Mroue, HP in prep
(similar results in Ohme ea, 2011)

Longer NR-waveforms: Alternatives

CITA–ICAT

- ~~Option~~ 1: Longer NR?
  - Can **not** perform long enough sims

$$\frac{T}{M} \approx 5\nu^{3/5}(2\pi N)^{8/5}$$

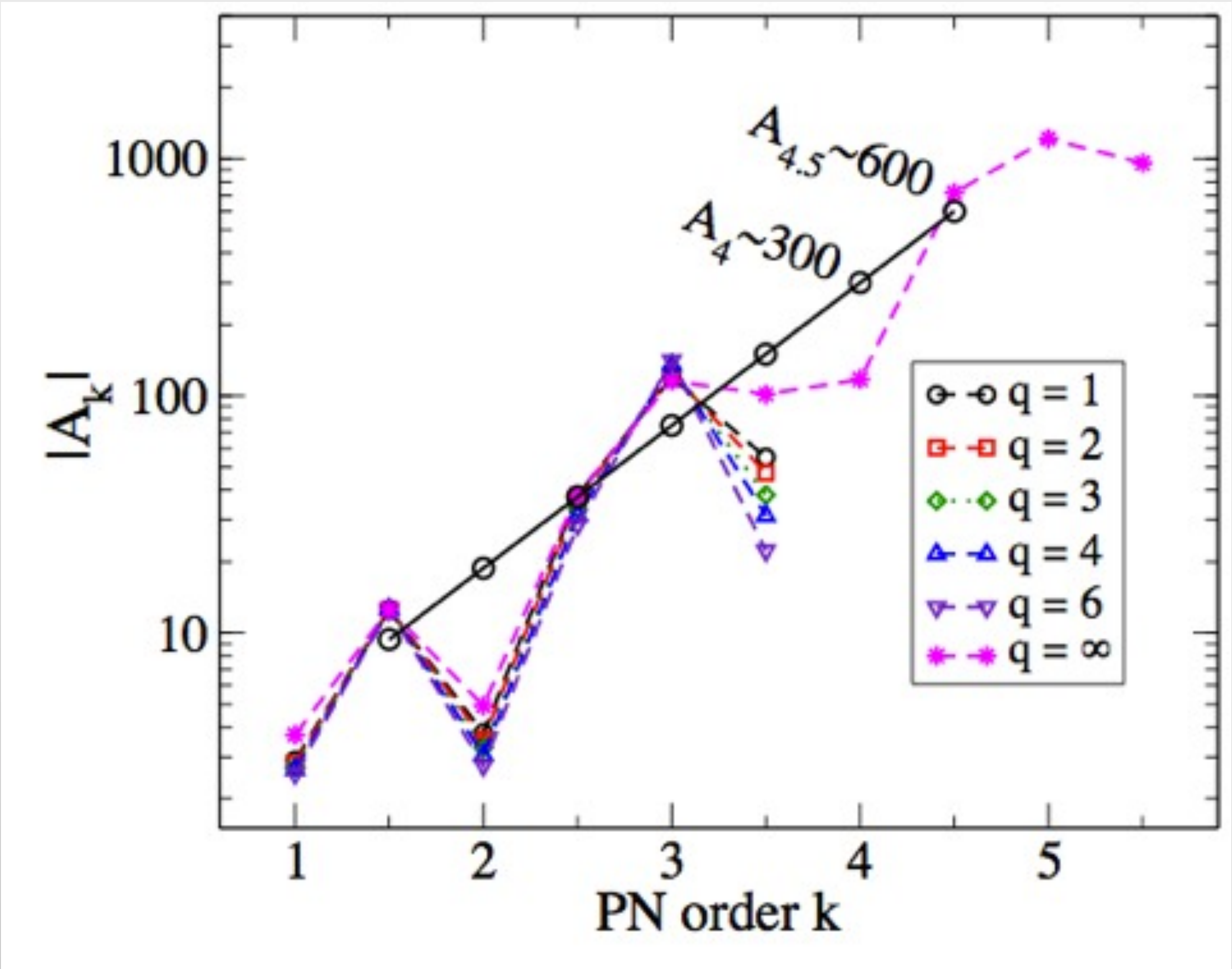
- Option 2: Live with it
  - Ohme ea 2011: Systematic errors δM/M~0.1%, δ(S/M^2)~ 0.1
- Option 3: Wait for 4PN
  - Buys us a factor of 2
- Option 4: Relax rigor
  - Only δh *tangential* to signal-manifold causes systematic errors (→Ilya Mandel's talk). *Give up on testing GR with orthogonal δh*
  - Fit PN or EOB to improve agreement with NR

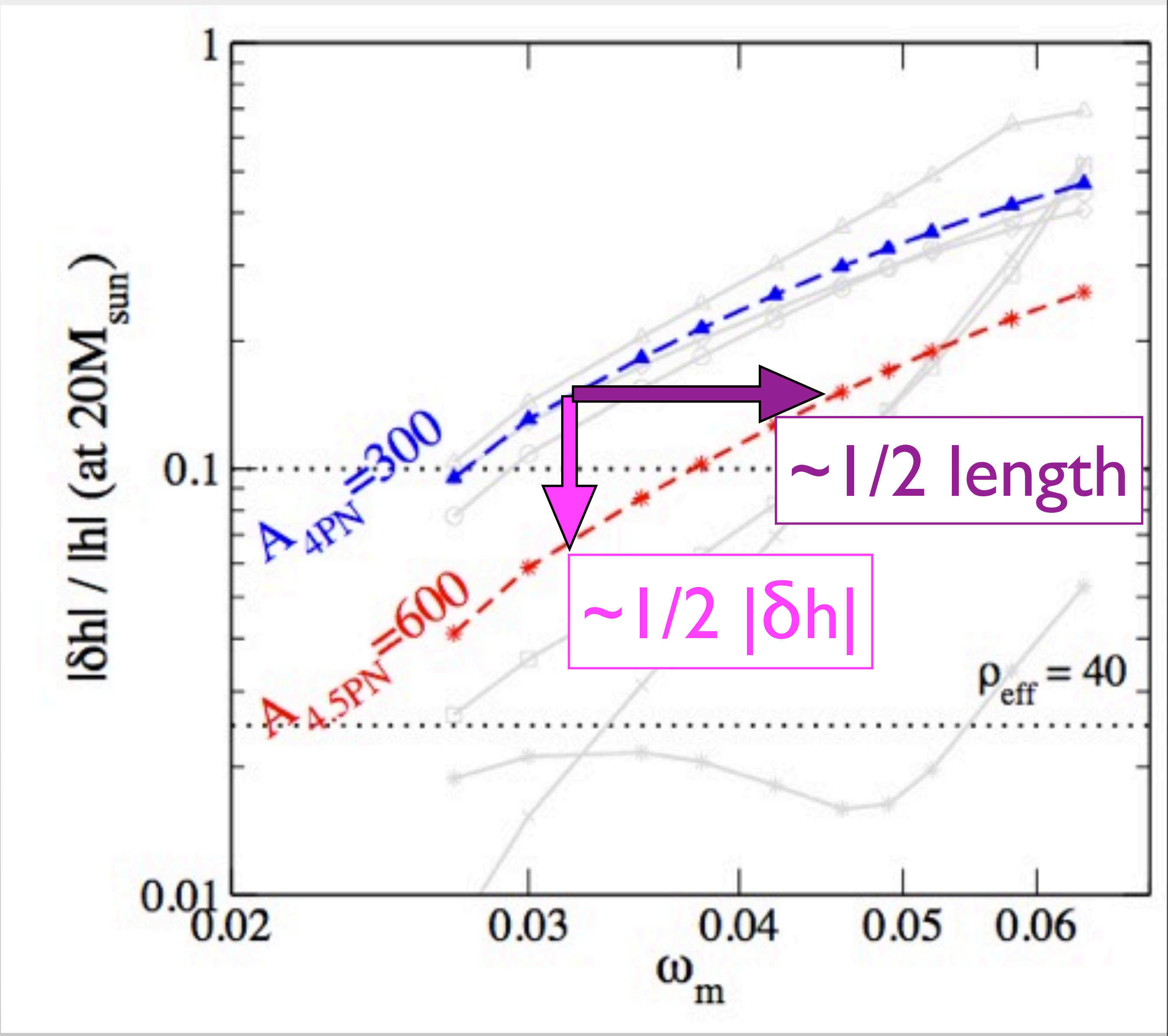
    Introduces dependence between NR and analytical waveforms, which may bias accuracy estimate of model.

Estimated impact of 4-PN

❖ TaylorT4 phase-evolution

$$\frac{dx}{dt} = \frac{64c^3\nu}{5GM}x^5\left(1+\sum_k A_k x^k\right)$$

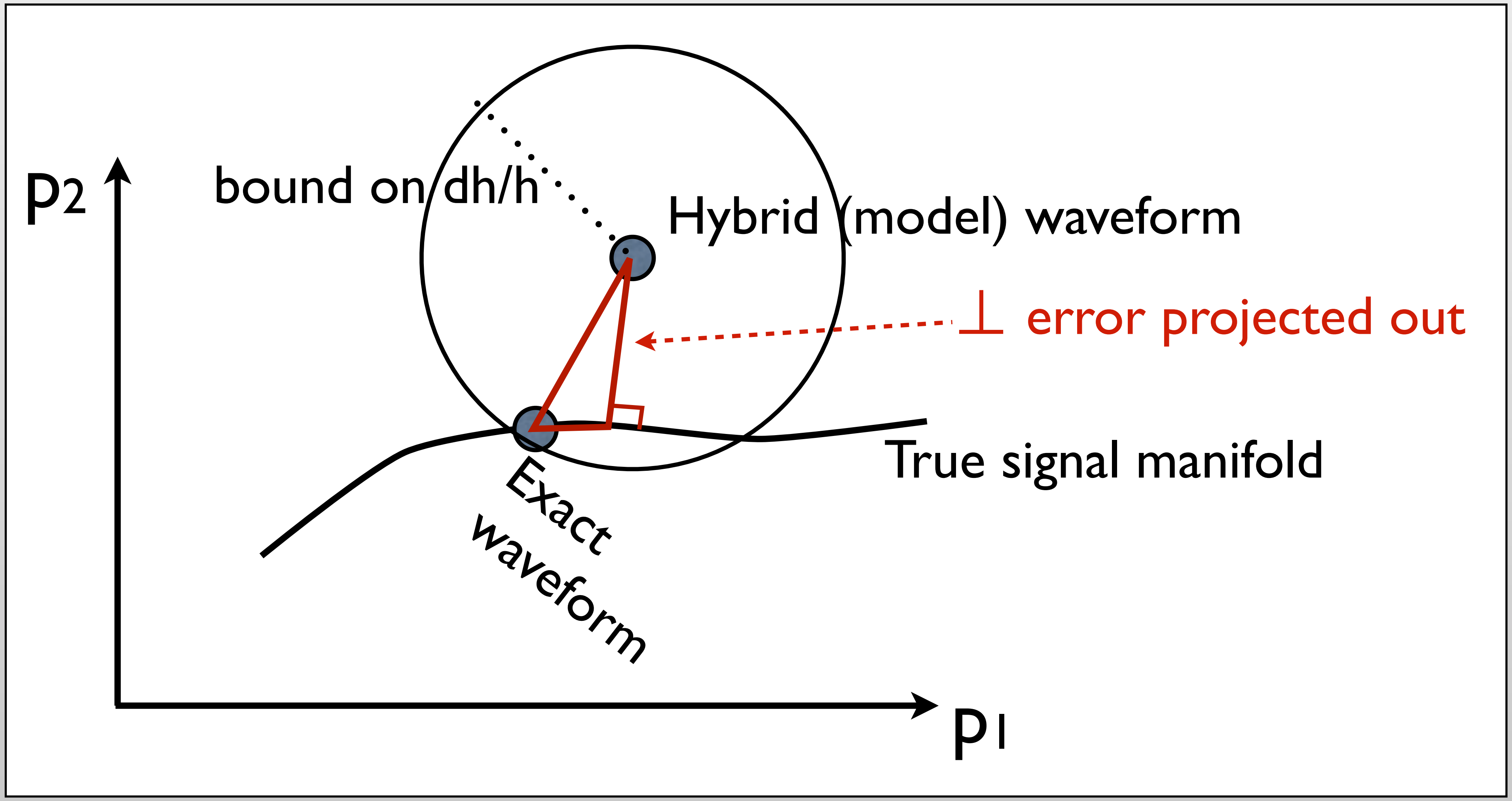




MacDonald, Mroue, HP (in prep)

Sufficient vs. necessary

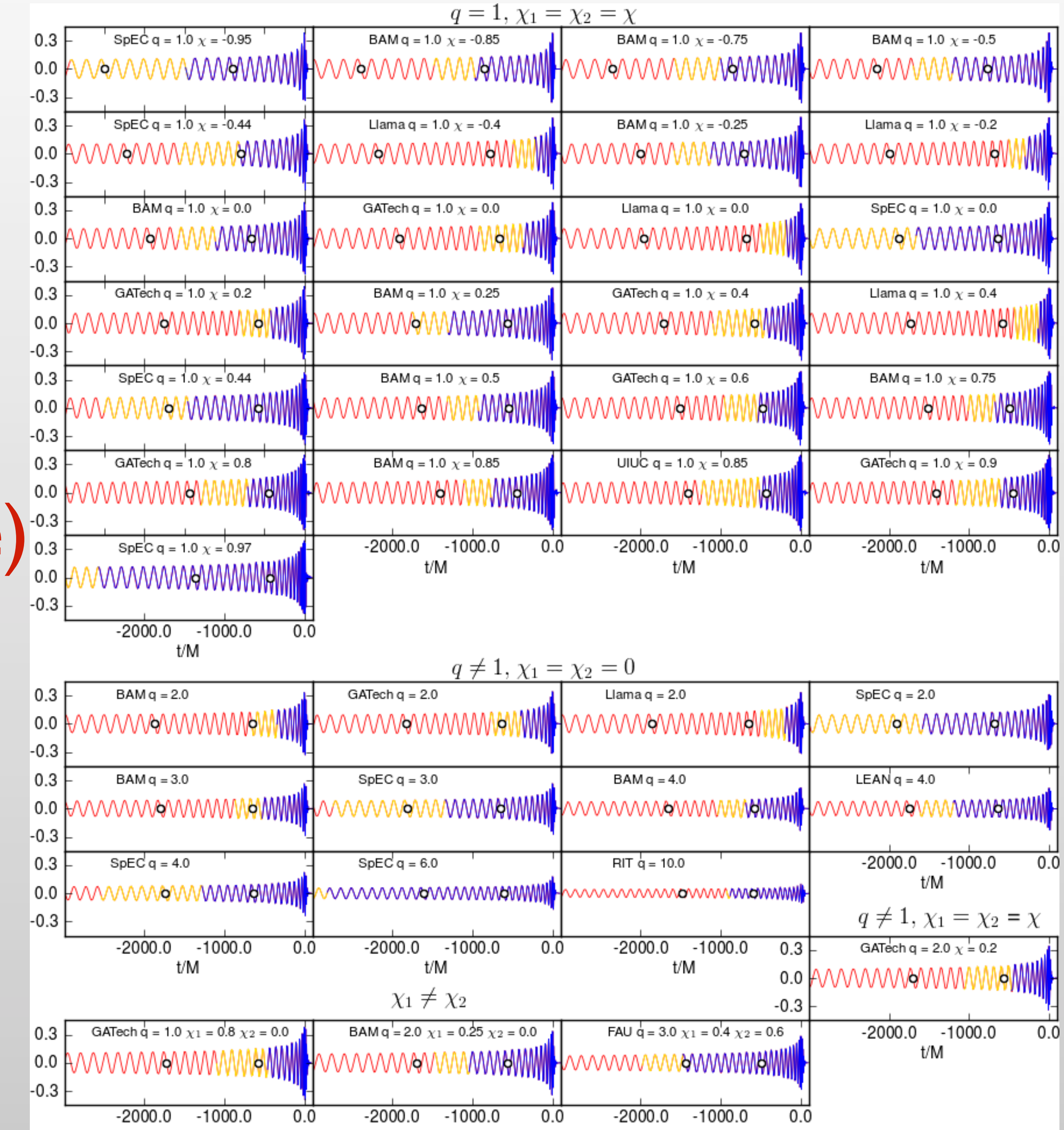
- |dh|/|h|<1/SNR is ***sufficient*** criterion, but may not necessary
  - Detector calibration might dominate error budget
  - Error orthogonal to signal manifold does not impact parameter estimation
- More work needed



Parameter space coverage

Ninja 2

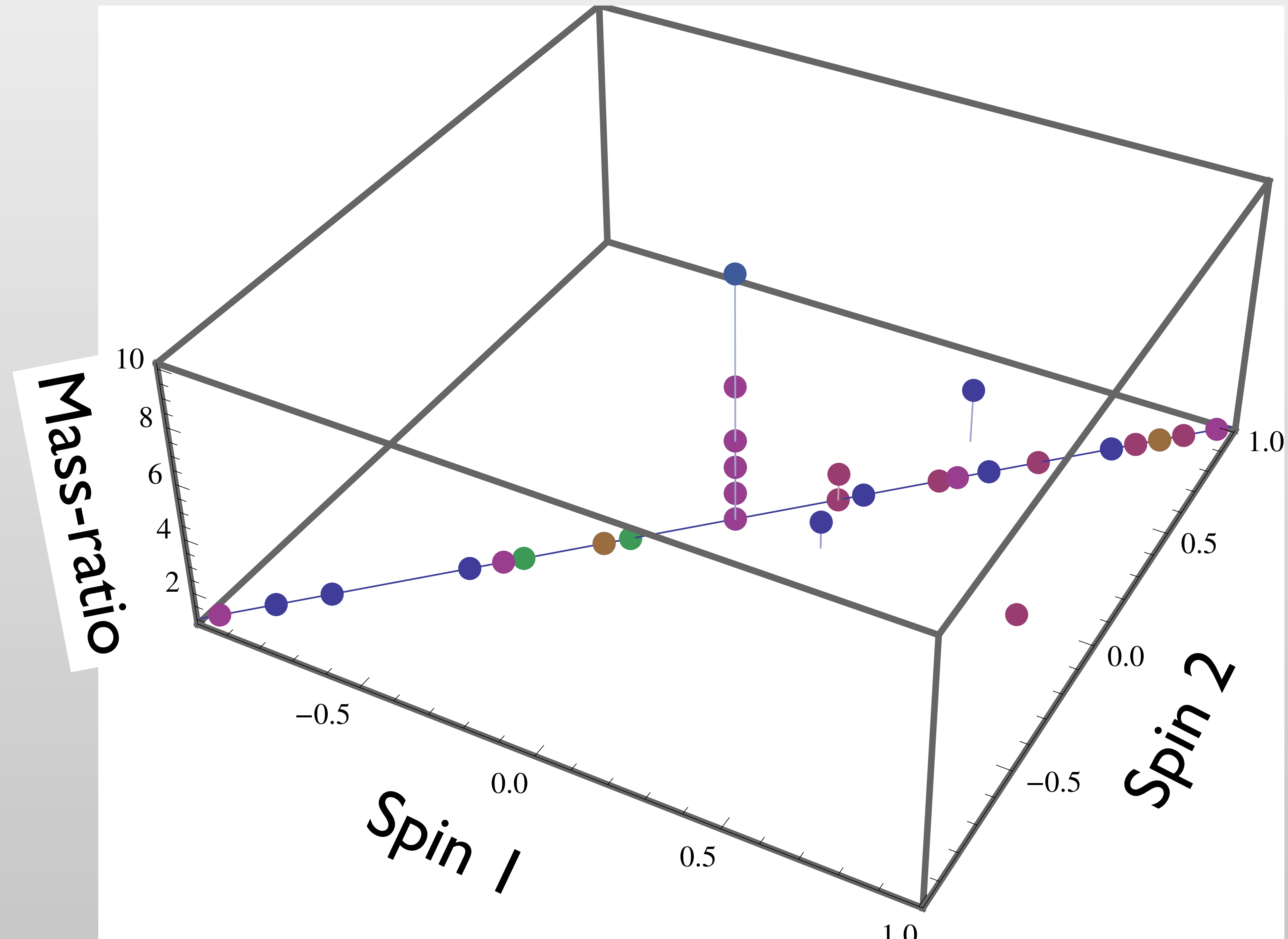
- ❖ 8 NR groups participate
- ❖ Required NR length & accuracy roughly in line with event detection needs (on lenient side)
  - Quick results, not perfect ones
- ❖ Begin
  - Summer 2009
- ❖ Waveforms complete
  - Spring 2012



41 NR waveforms   Ajith ea 2012

Ninja 2 parameter space coverage

- Goal: Sample aligned-spin parameter space
- Outcome: Two 1-dim subspaces sampled:
  - Equal-mass & equal-spin
  - Non-equal mass & zero spin
- Sprinkling of NR runs away from these subspaces.



Ajith ea, 2012

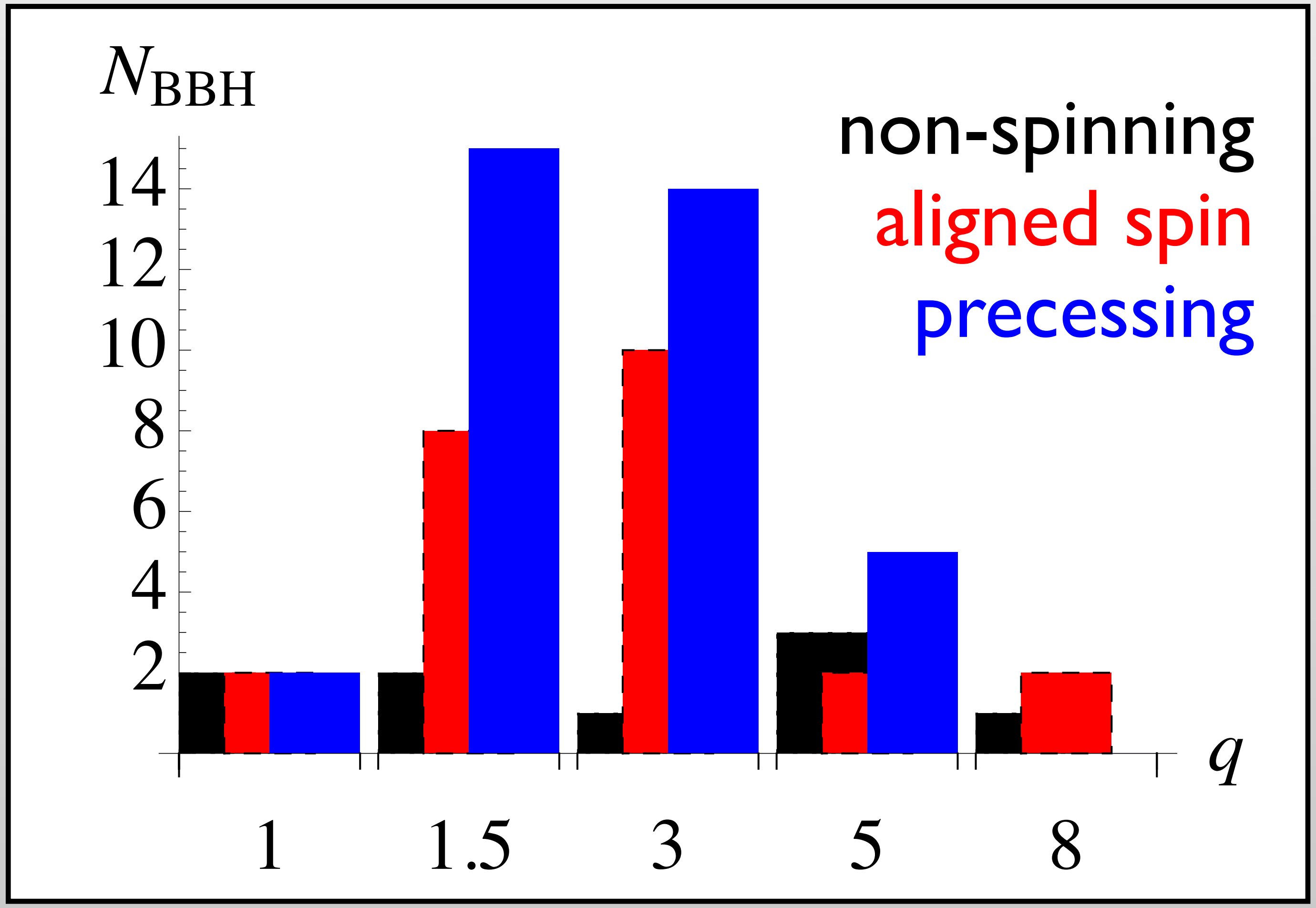
NR–AR collaboration

CITA–ICAT

- **N**umerical **R**elativity -- **A**nalytical **R**elativity
  - Perform high-quality NR simulations, use waveforms to construct waveform models
  - 9 NR groups participate
  - 11 Mio CPU-hours from NSF + indidvidual group's resources
- NR-length and -accuracy more demanding than Ninja 2
  - 10 orbits, 0.25rad phase error
    (still insufficient for indistinguishability criteria)
- Start: Late 2009
- Expected completion of Waveform catalog: Summer 2012
  - ~30 waveforms
  - Extended coverage of aligned spin systems

SpEC Parameter Survey

- Led by Abdul Mroue+HP at CITA, in collaboration with Caltech, Cornell & Fullerton
  - 30 Mio CPU-hours on Compute Canada systems
  - 100 NR runs (+600 further configurations circularized)
- Started Sep 2009
- Runs mostly complete
  - Improvements of mergers in progress
  - Extrapolation of precessing waveforms in progress



Why does this take so long?

CITA–ICAT

- ❖ high CPU-cost
  - Runs take months, use 100,000's of CPU-hours each.
  - This compounds supercomputer problems
- ❖ Genuine novel research is done along the way
  - Eccentricity removal, hybridization, error estimates
  - Boldly go where no code has gone before
    - {longer, more accurate, precessing, mergers} require many trials
- ❖ Validation, data-interfaces
  - Validate PN codes, hybridization codes.
  - Fix problems in NR waveforms. Standardize data-formats
- ❖ Finite man-power, tedious and repetitive
  - Easily distracted with more fun, shorter time-scale research
  - Big Dog

Precessing BH–BH

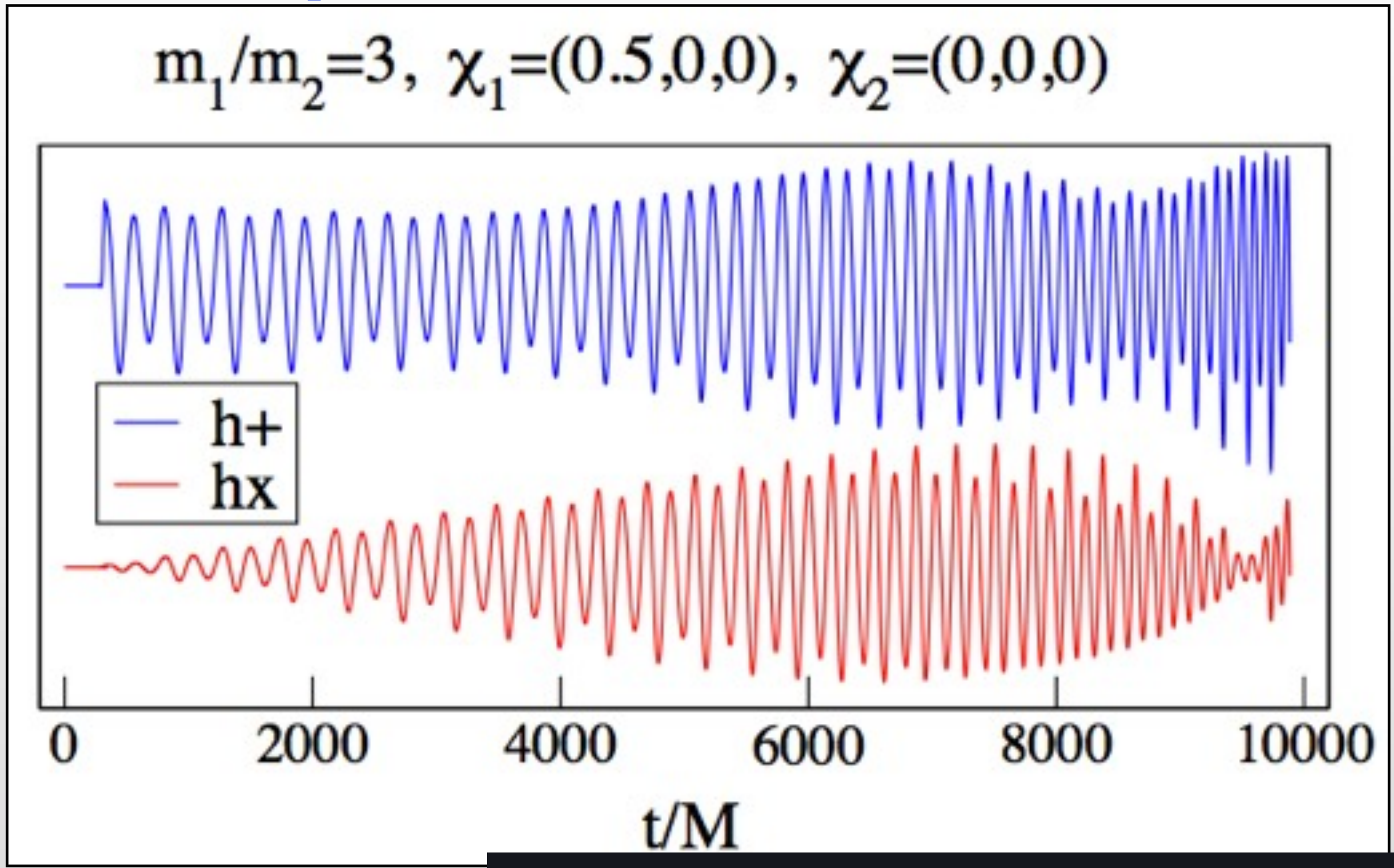
The Frontier: Precessing systems

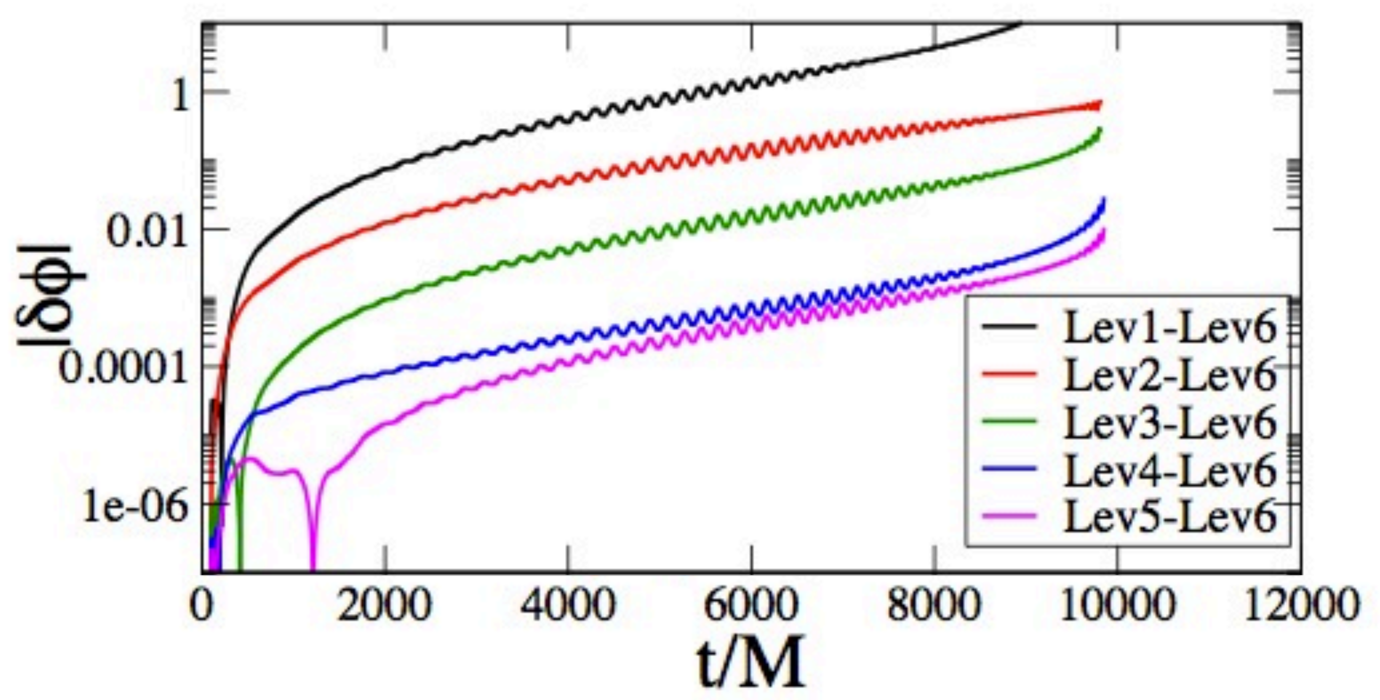
- ❖ Vast parameter space
  - Precessing 7-dim vs. non-precessing 3-dim
  - Eccentricity adds 2-dim in both cases
- ❖ Little concerted effort so far
  - Only short sims, or individual longish simulations
- ❖ Efforts ramping up:
  - NR-AR collab: ~dozen waveforms in progress
  - SpEC parameter survey
    - SXS collab. (Cornell, Caltech, CITA, U Wash, Fullerton)
    - ~50 waveforms in progress

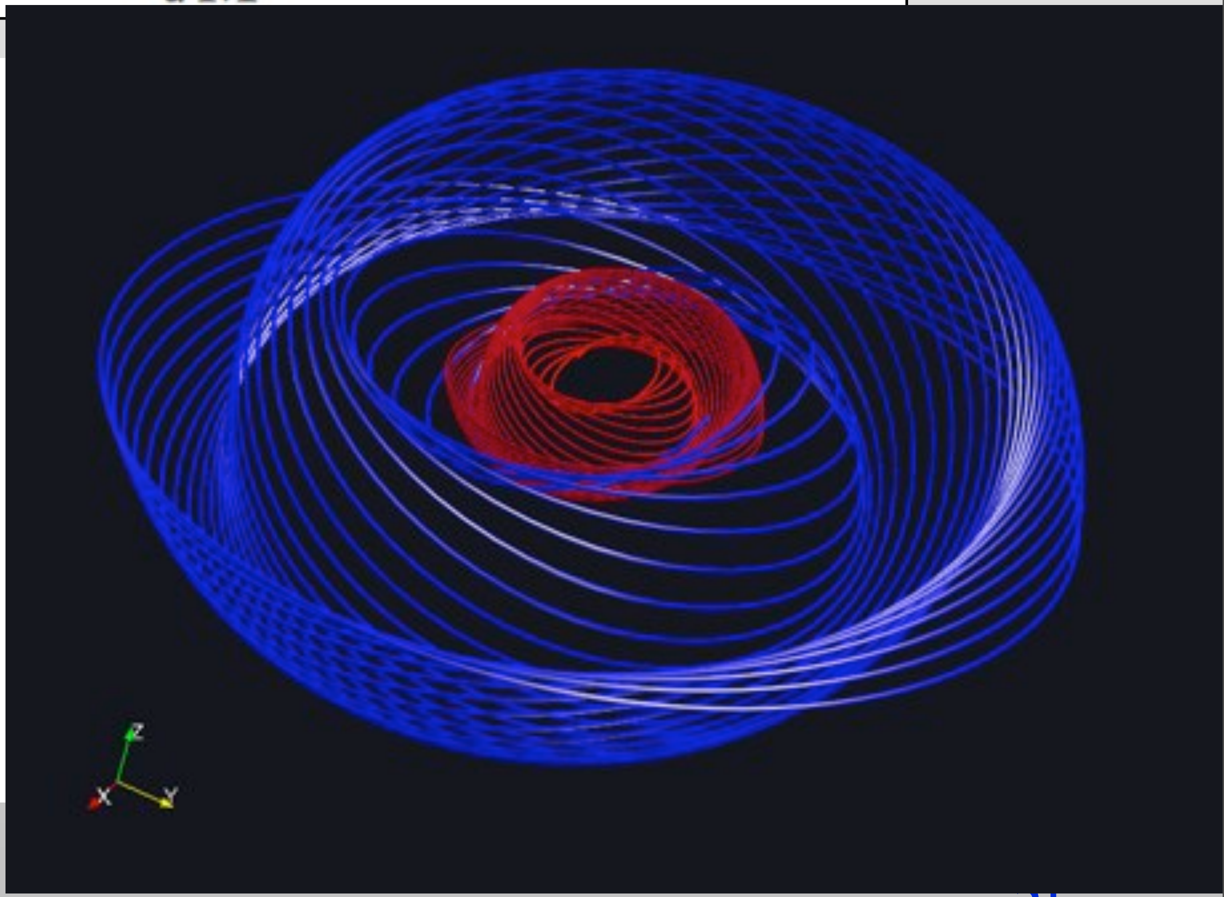
Pushing the envelope

- q=3, χ_A=0.5, χ_B=0
  33 orbits

Mroue, HP + SXS





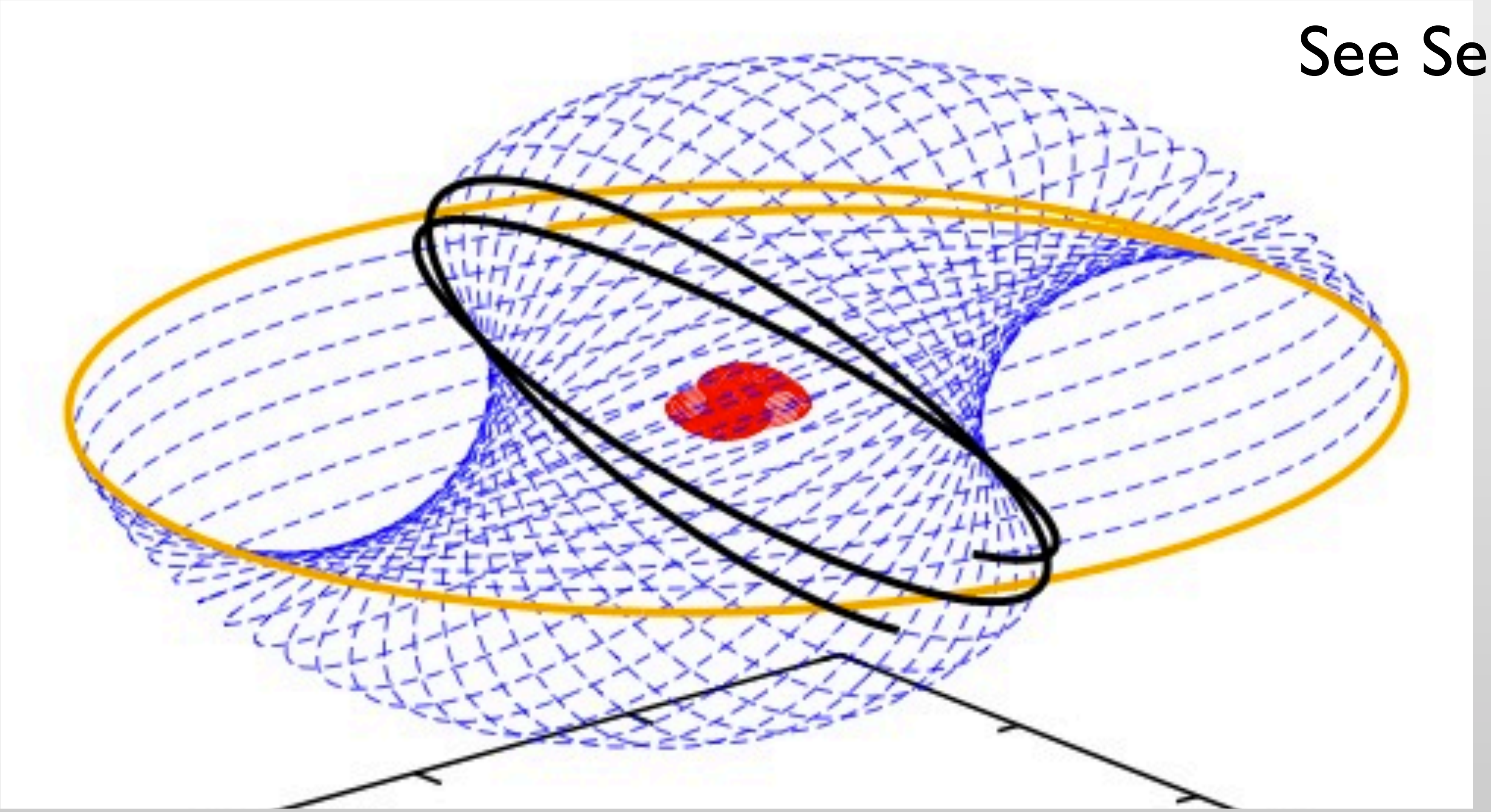


Pushing the envelope

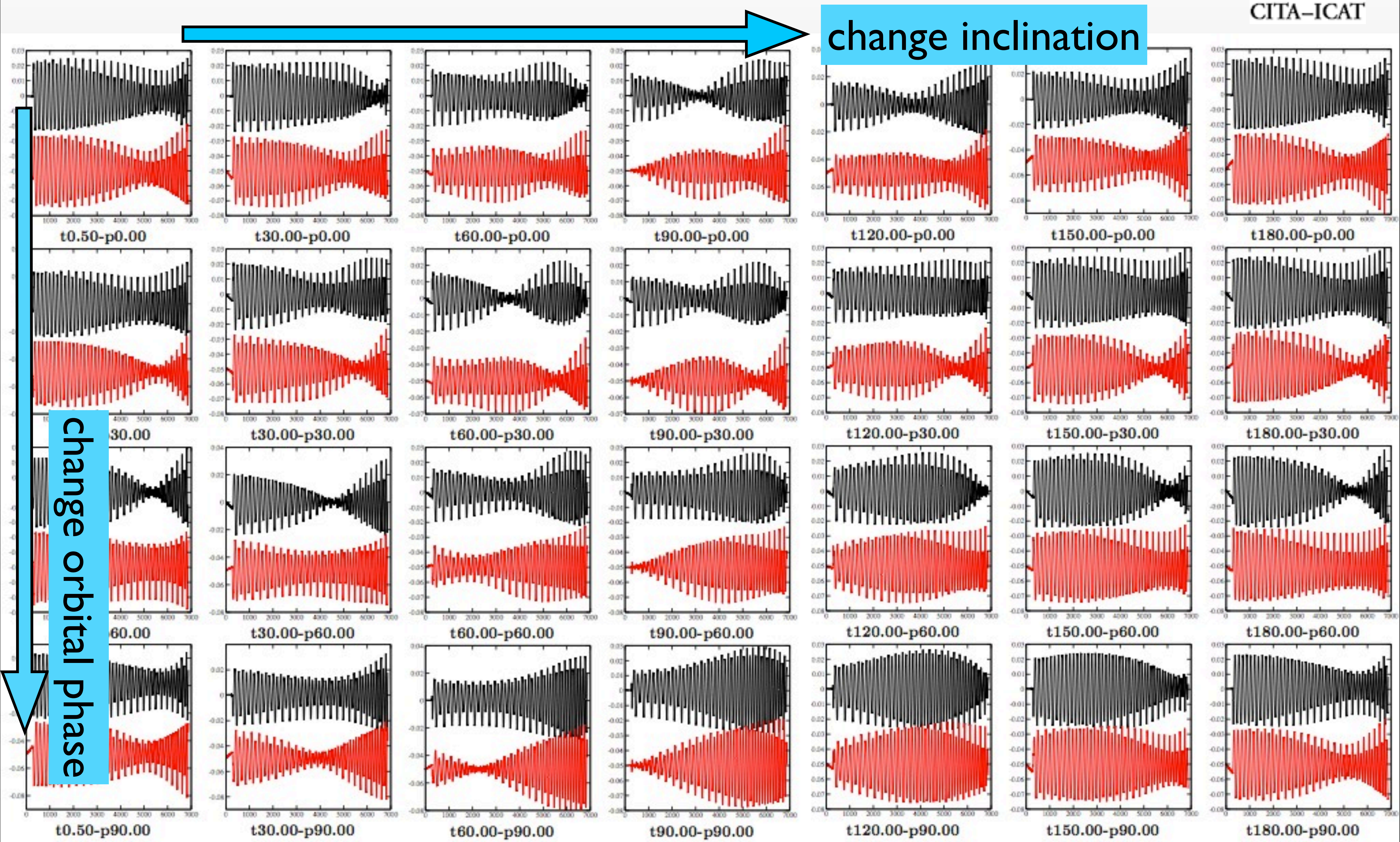
- q=9.5, χ_A=0.5, χ_B=0
- Orbital plane flips over

Serguei Ossokine, HP + SXS

See Serguei's poster!



Waveforms in different directions

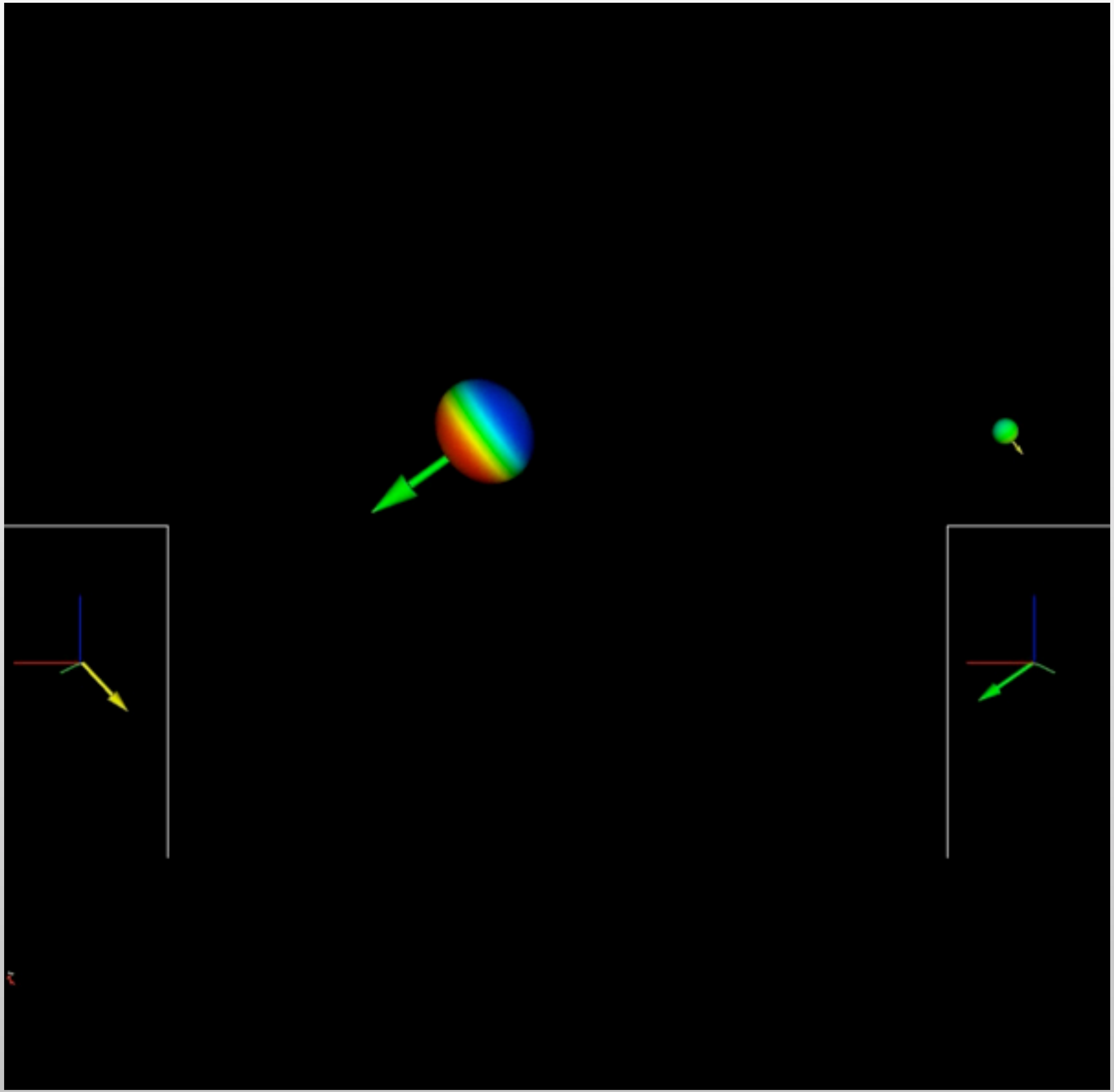


S. Ossokine, HP + SXS

Pushing the envelope

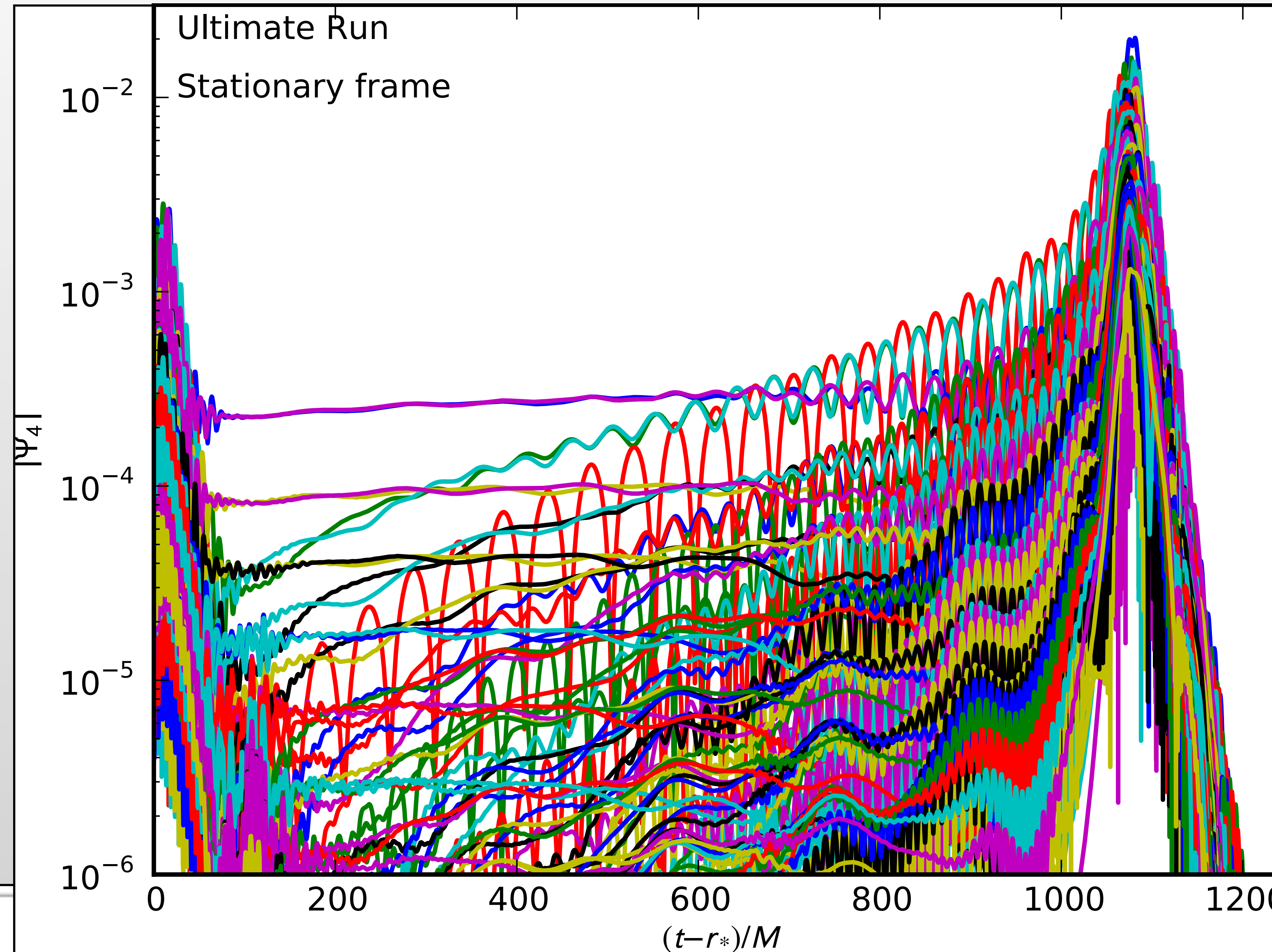
- q=6, χ_A=0.9, χ_B=0.3
  8 orbits

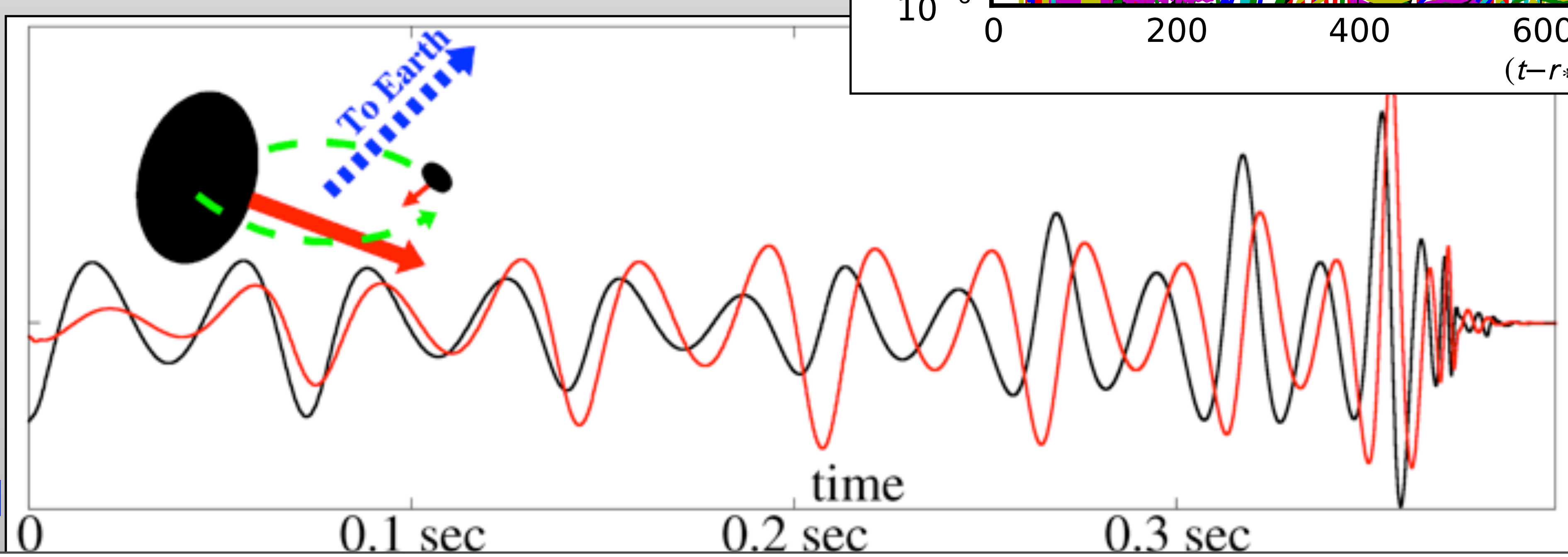
Larry Kidder + SXS



Ylm decomposition of GW

- q=6, χ_A=0.9, χ_B=0.3, 8 orbits
- As orbital plane precesses, all modes contribute





A cynic's summary so far

CITA–ICAT

❖ We have analytical waveform models for aligned spin systems

- Based on a ~dozens of NR waveforms, that ...
  ... are so short that parameter estimation is compromised
  ... are restricted (mostly) to two 1-dim subspaces of the 3-dim param space

❖ Computing a few dozens of NR waveforms...

- ... takes three years

❖ The precessing waveform parameter-space is ...

- ... humungous
- ... interesting and complicated

4^7 / (dozens/year) = centuries

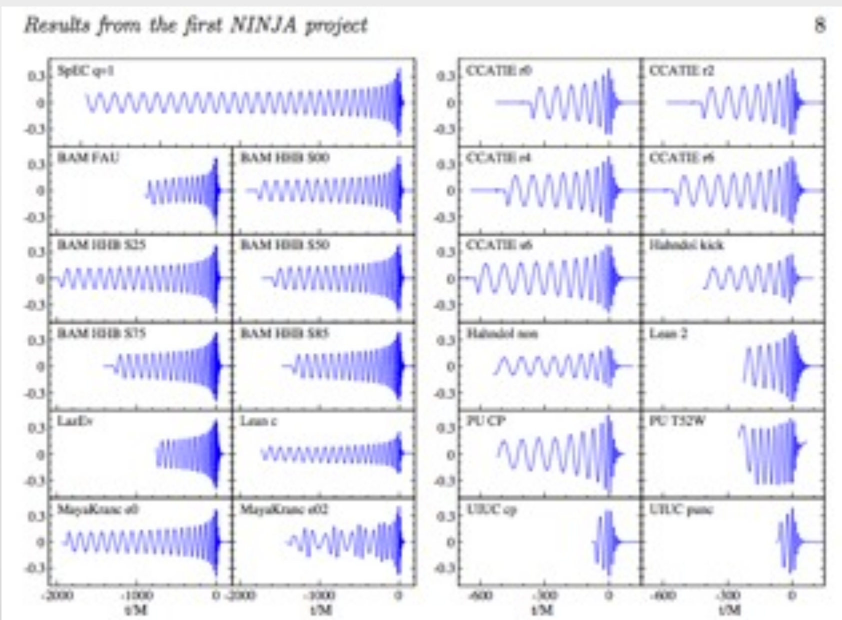
Reasons for Optimism

Optimism

- ❖ For event-detection, we're doing quite good, actually:
  - Two independent waveform models (cross-checks!)
  - Reasonably accurate (sensitive to most non-precessing systems)
  - Fitting analytical models to NR has been easier than producing NR waveforms. With new NR simulations, I'll expect the analytical waveform models to quickly adopt and further improve.
- ❖ NR has learned a lot during the last round of 3-year efforts. Many tools were developed.
  - robustness, automation, behavior of codes, error requirements, hybridization, extrapolation, PN codes, ...

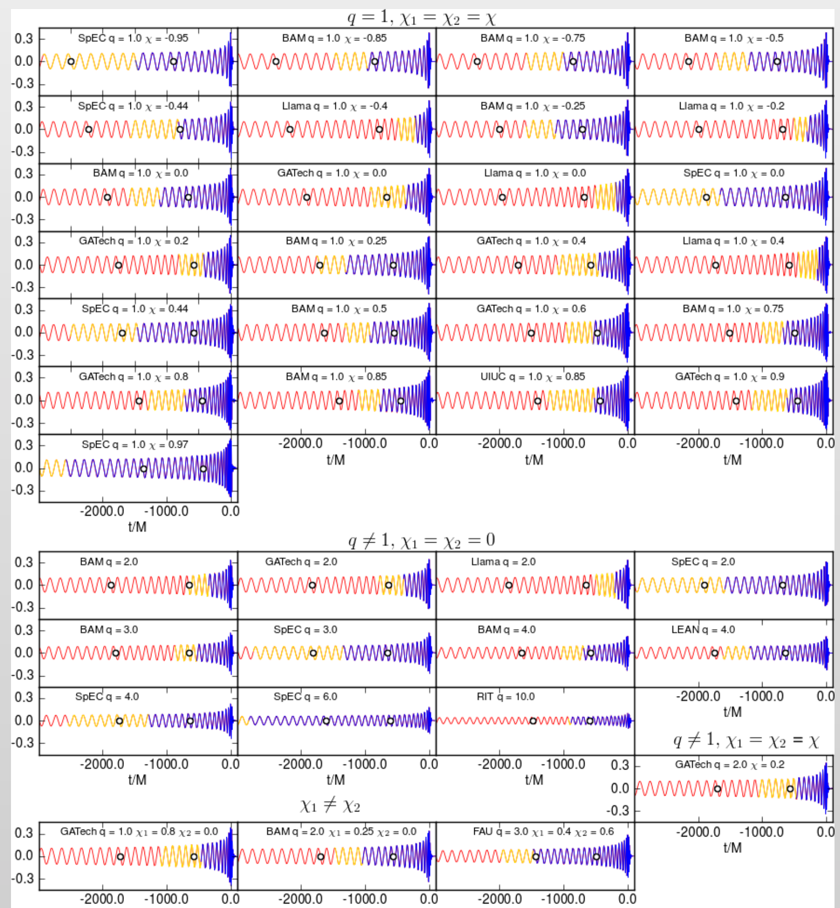
NR is advancing

PAST



Ninja1

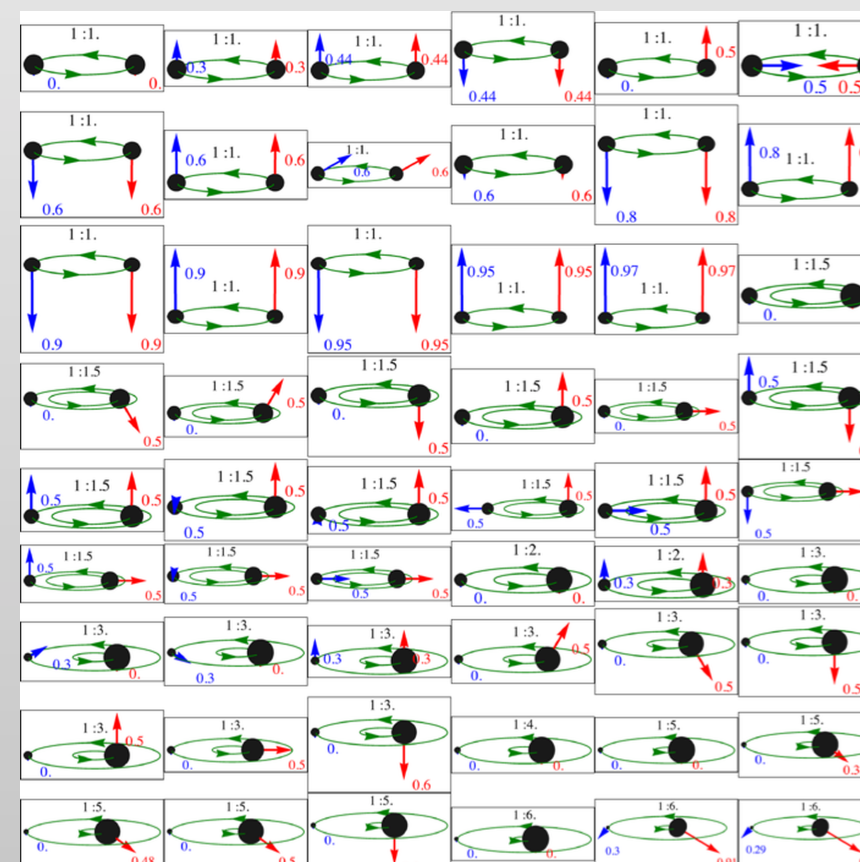
NOW



Ninja2

SOON

NR-AR
(no img)

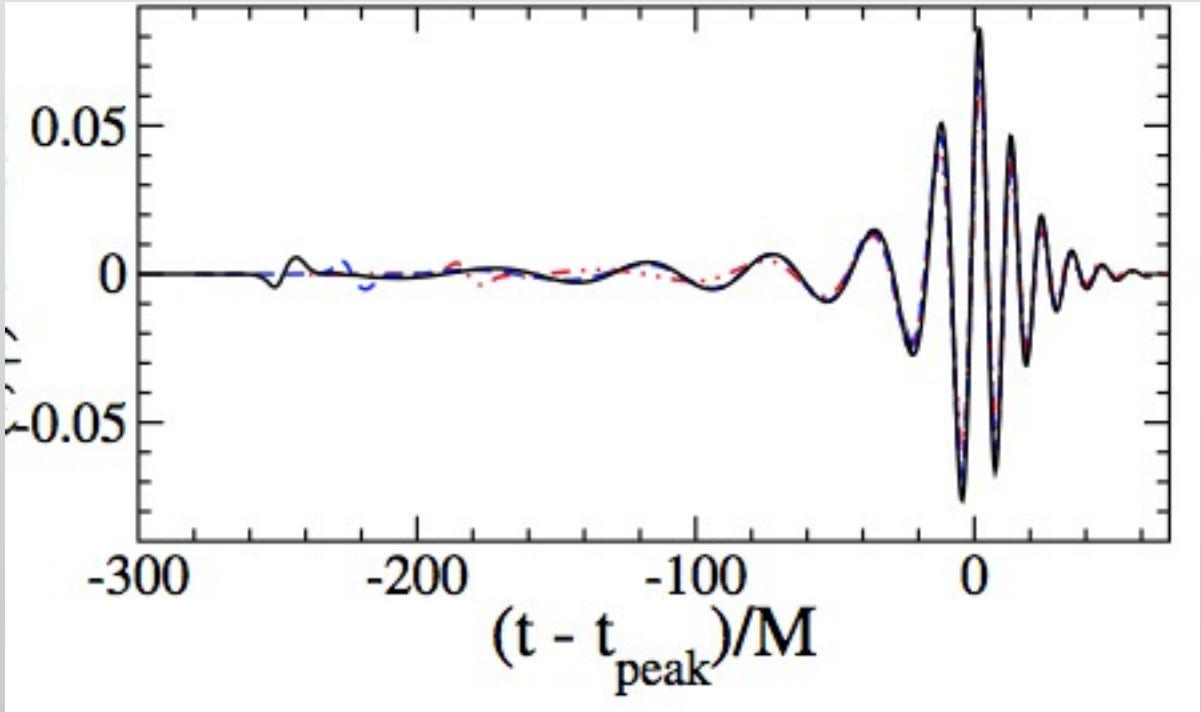


SpEC configurations

Simplicity

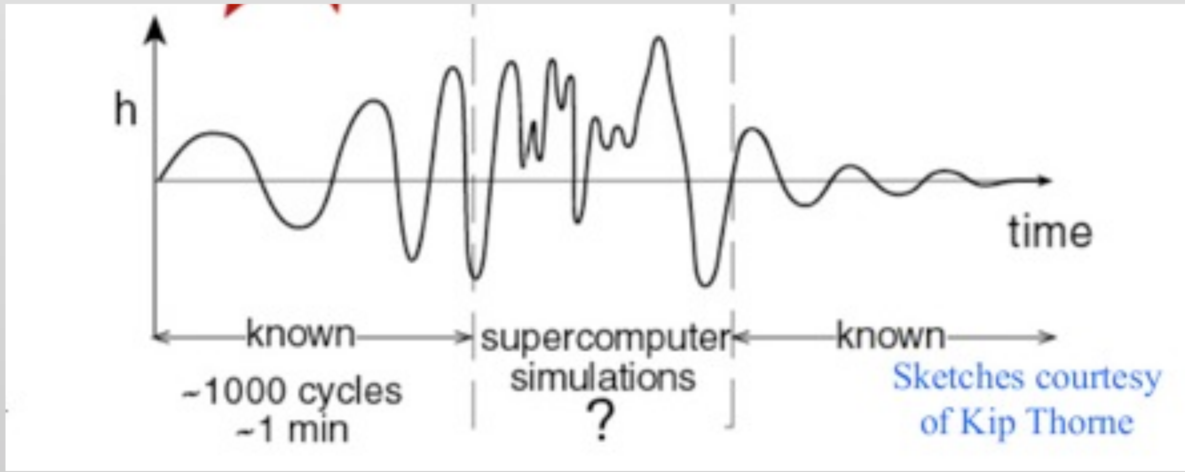
❖ Perhaps **precessing BH-BH** waveforms continue the trend of non-precessing waveforms and **are simple:**

- BH-BH waveforms are "nice" chirps with little structure (Shoemaker, Laguna, 200x)



Buonnano, Cook, Pretorius, '07

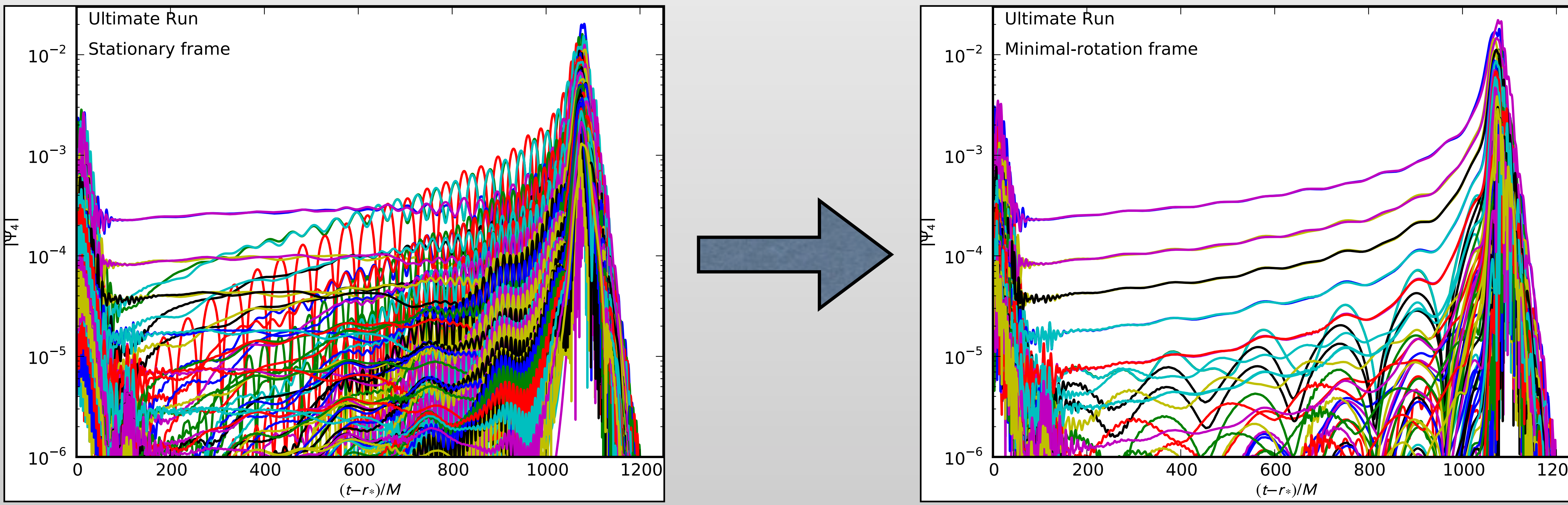
≠



Radiation-aligned minimally-rotating frame

CITA–ICAT

- ❖ Decompose radiation in a good frame, not an inertial frame
- ❖ Schmidt ea 2011, O'Shaugnessy ea 2011:
  - Polar axis of Ylm-decomposition along dominant emission direction



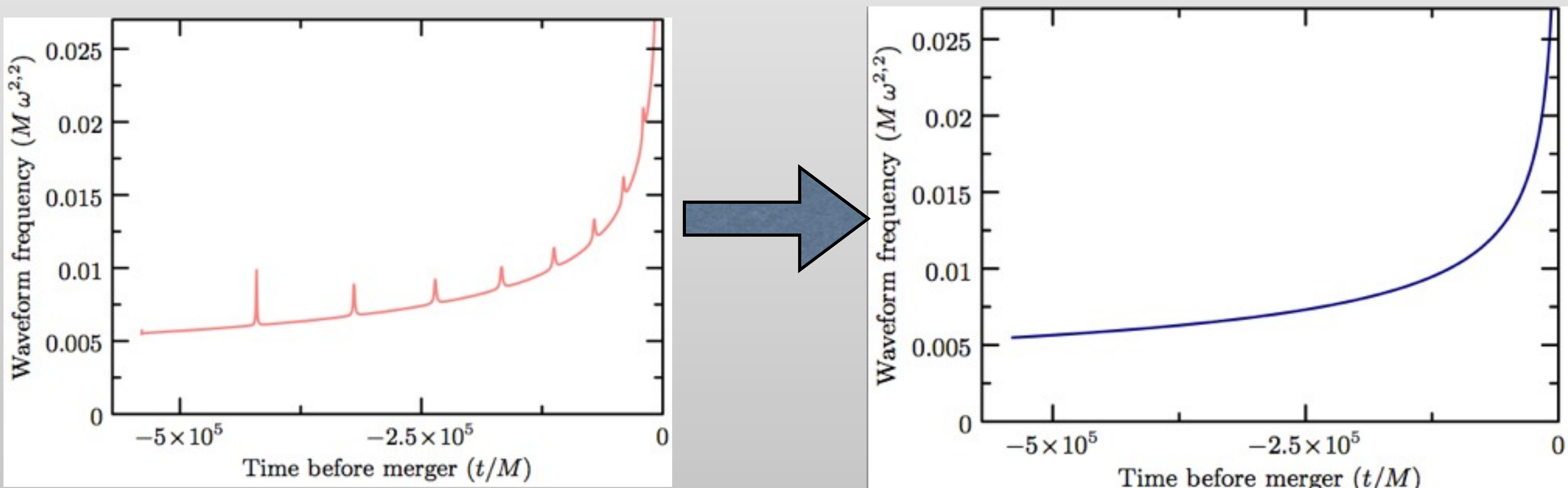
q=6, χA=0.9, χB=0.3, 8 orbits

Figures courtesy Mike Boyle & Larry Kidder

Radiation-aligned minimally-rotating frame

CITA–ICAT

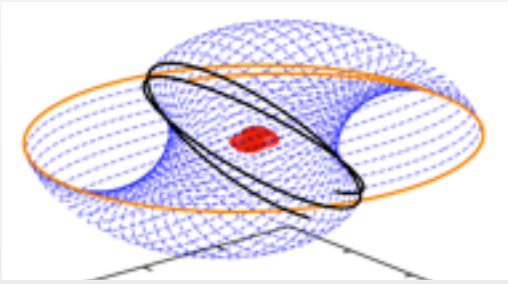
- Decompose radiation in a good frame, not an inertial frame
- Schmidt ea 2011, O'Shaugnessy ea 2011:
  - Polar axis of Ylm-decomposition along dominant emission direction
- Boyle, Owen, HP 2011
  - Unique preferred rotation *__about__* emission direction



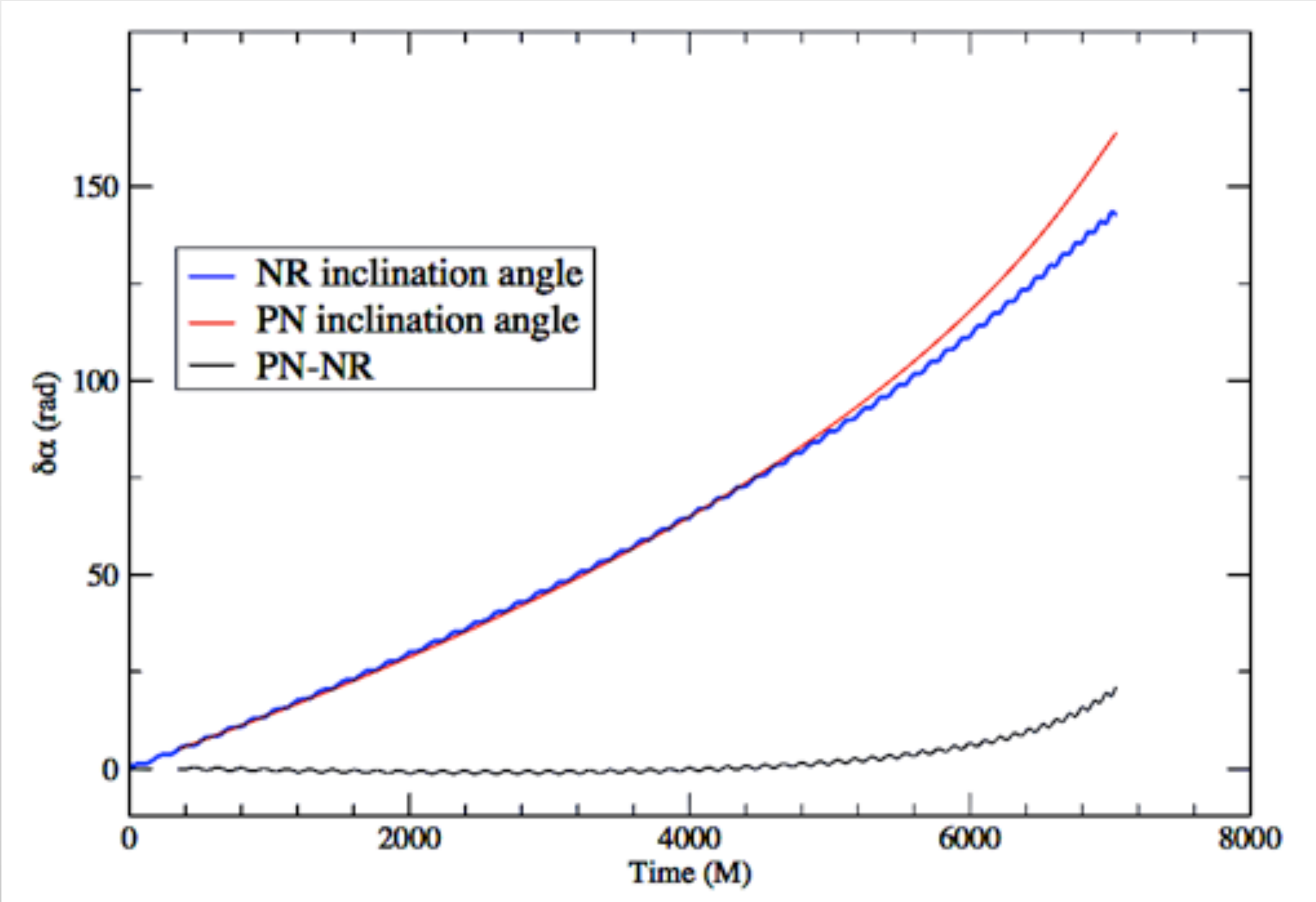
Boyle, Owen, HP 2011

Analytical Results are powerful

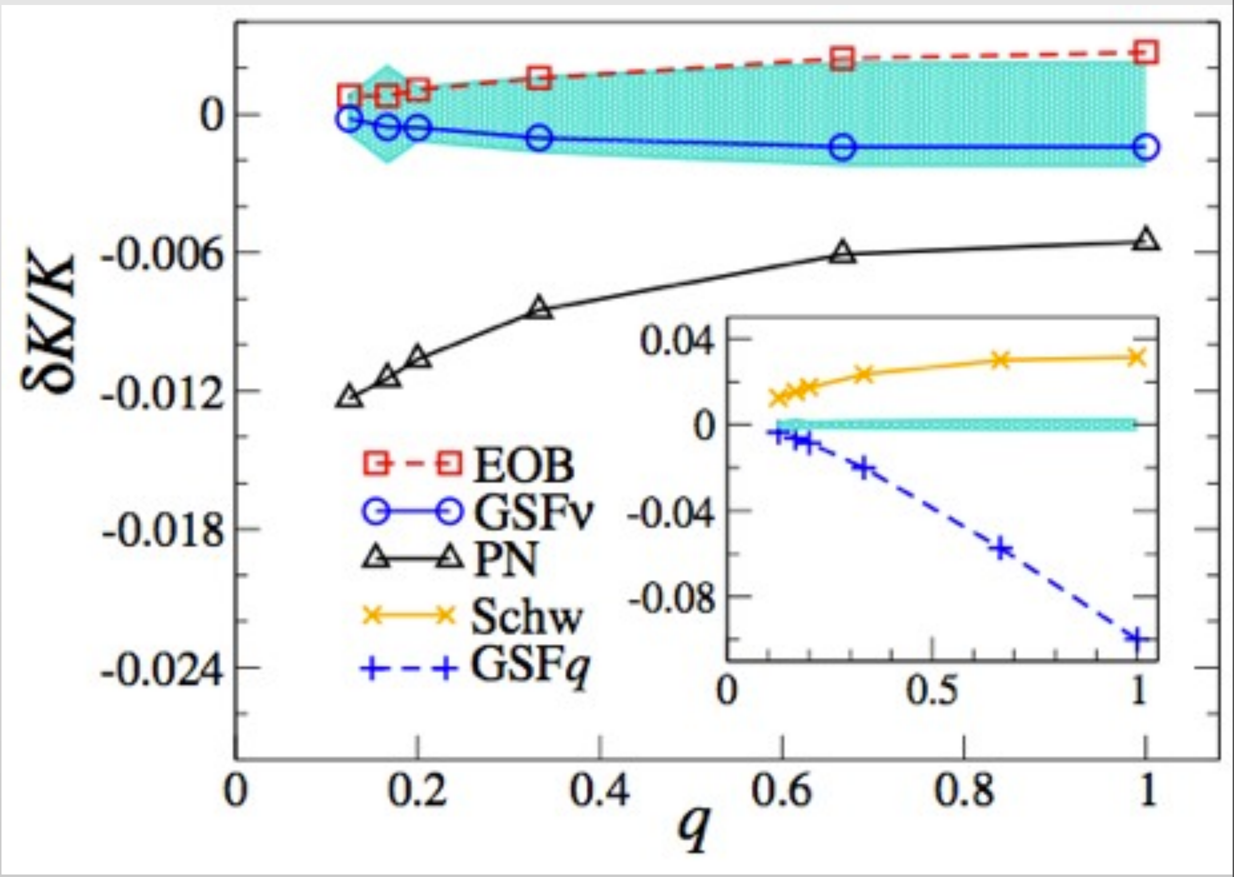
❖ Inclination angle for "flipping" BH-BH run well predicted by PN



❖ Perturbation theory and EOB predict periastron-advance for BH-BH at *all* mass-ratios



S. Ossokine, A. Mroue

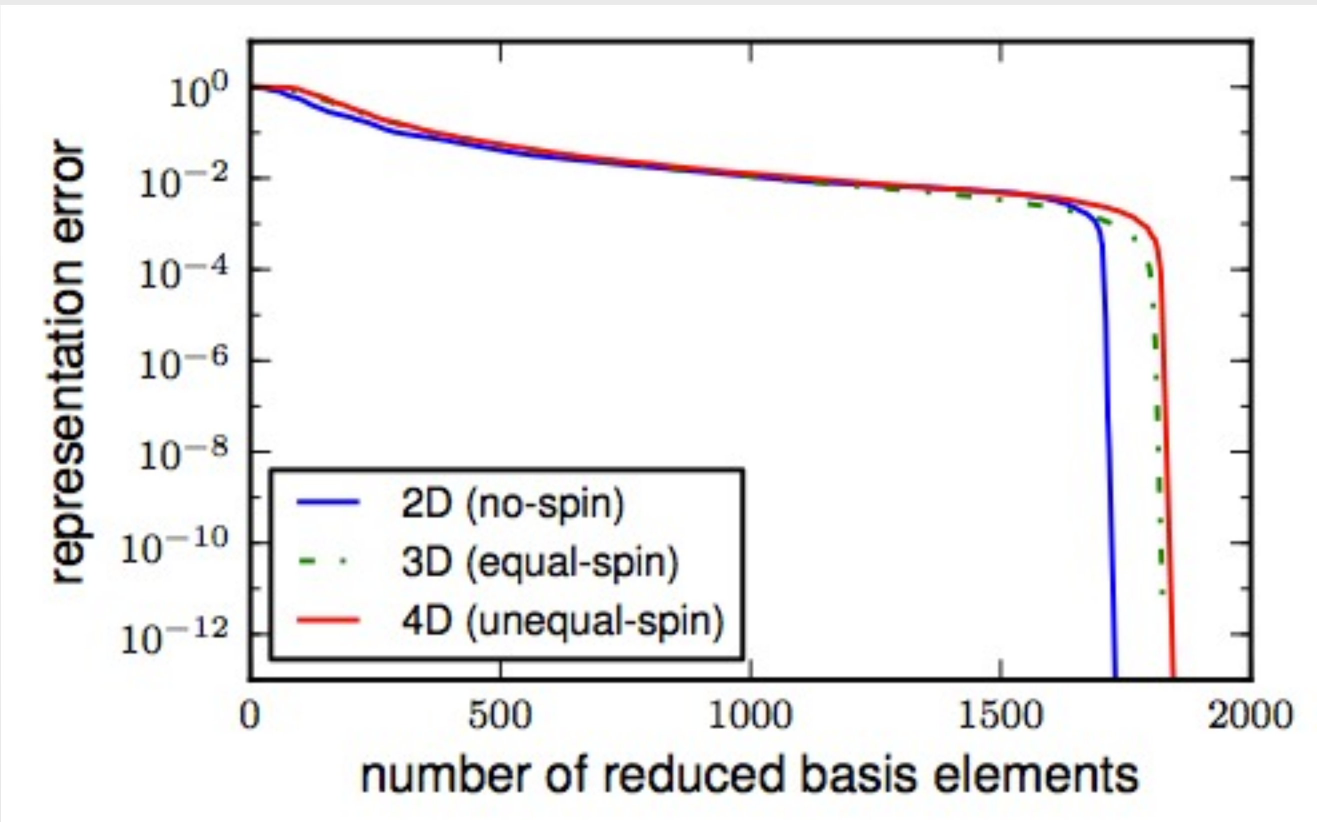


Le Tiec, Mroue, ea, 2011

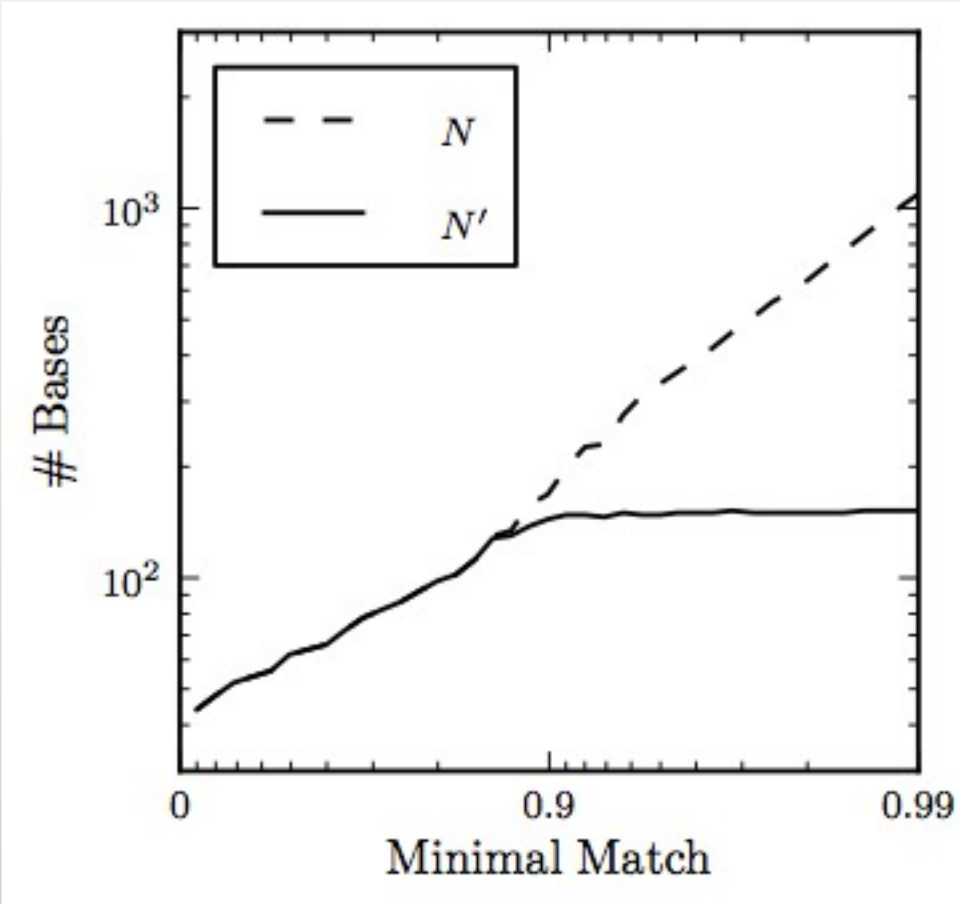
Dimensionality of waveform space

CITA–ICAT

❖ CBC waveforms may span only a low-dimensional space
- Reduced basis-methods
- SVD decomposition of waveforms



Hermann ea, arXiv:1205.6009



Cannon, Hanna, Keppel arXiv:1101.4939

CAVEAT:

BH–NS, NS–NS

BH–NS, NS–NS waveform modeling

- NR simulations harder
  - Hydro
  - micro-physics
  - larger param space (total mass, EOS)
  - Accuracy lower than for vacuum BH-BH sims
- Fewer simulations
- Simulations cover fewer inspiral cycles

NS–NS

❖ Comparison NR with TaylorT4 (point-particle and w/ tidal terms)

- Bernuzzi, Thierfelder, Bruegmann 2012

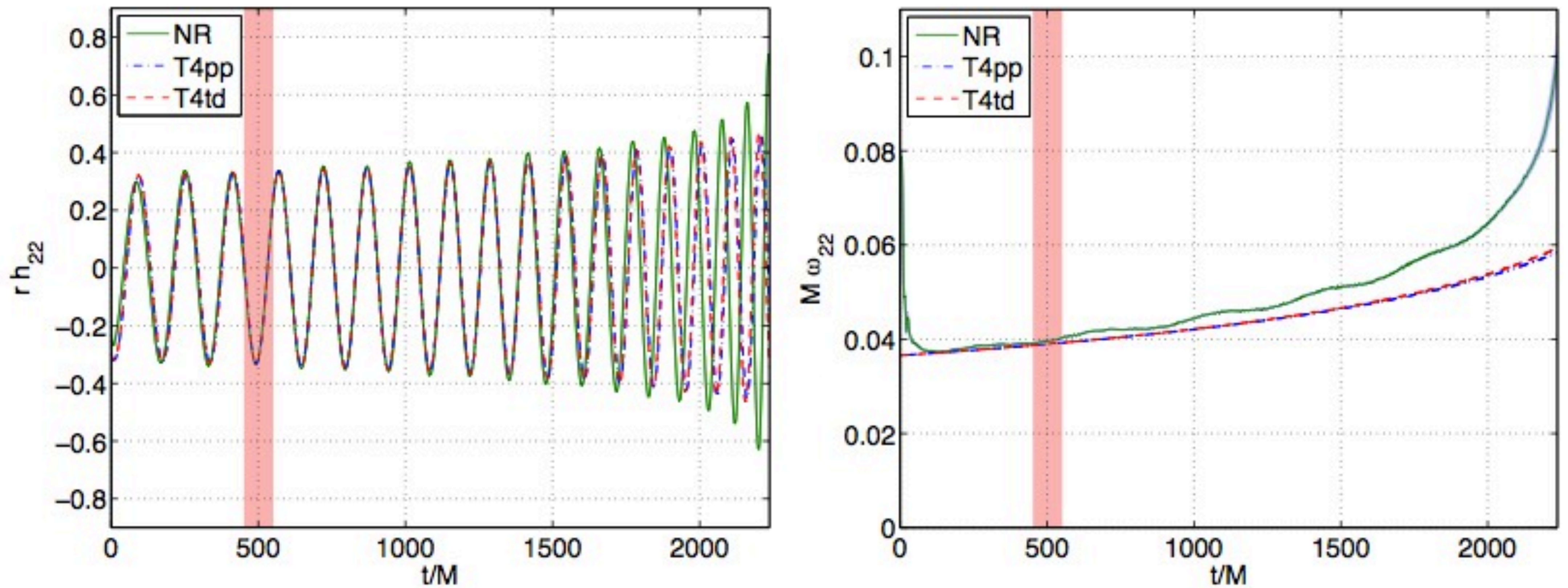
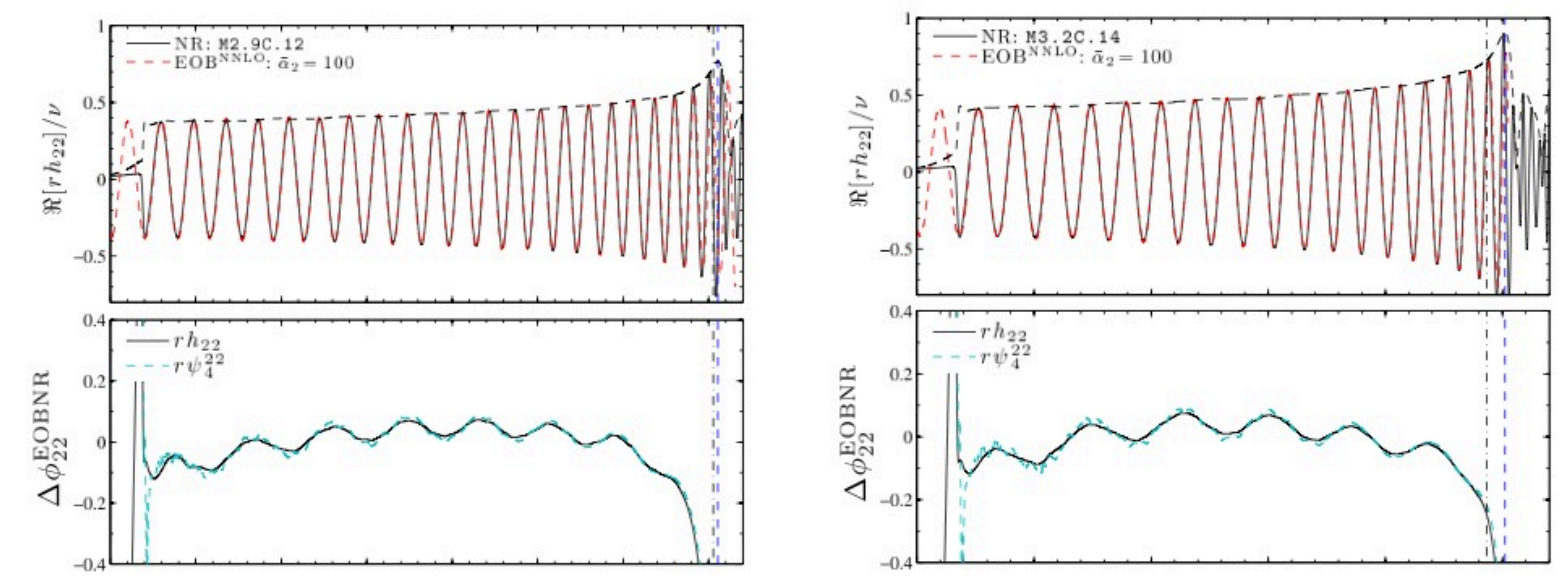


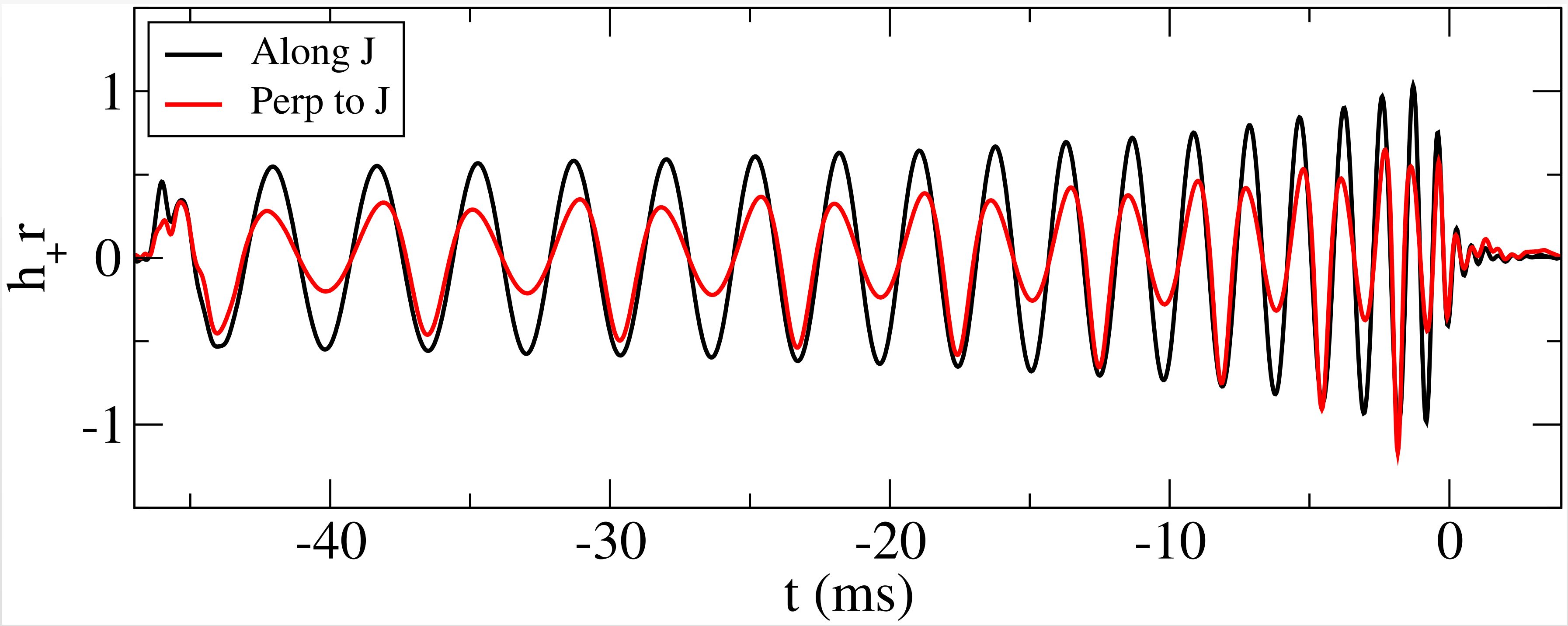
FIG. 10: Comparison T4pp/T4td–NR, $r\,h_{22}$ (right) and $M\,\omega_{22}$ (left). The shaded red area at early time is the alignment region. A very thin shaded blue area (barely distinguishable on this scale) shows the uncertainty of NR data.

NS–NS

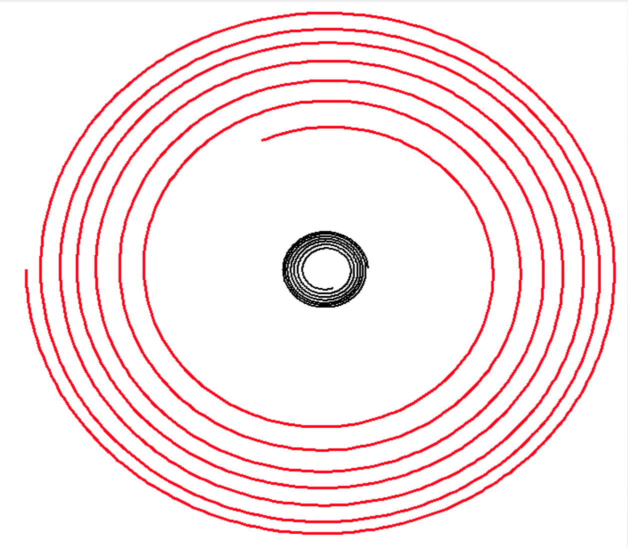
- Fit EOB w/ tidal terms to equal mass NS-NS
  - Baiotti ea 2011



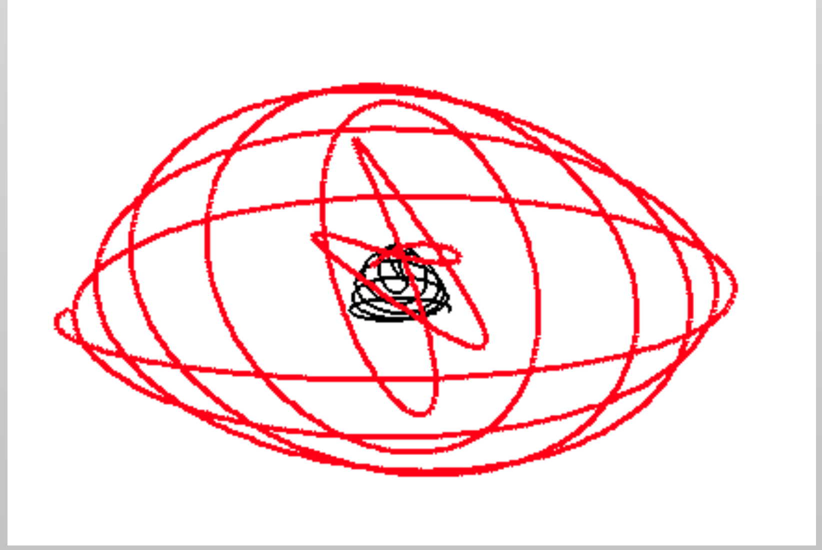
BH–NS at mass–ratio 7

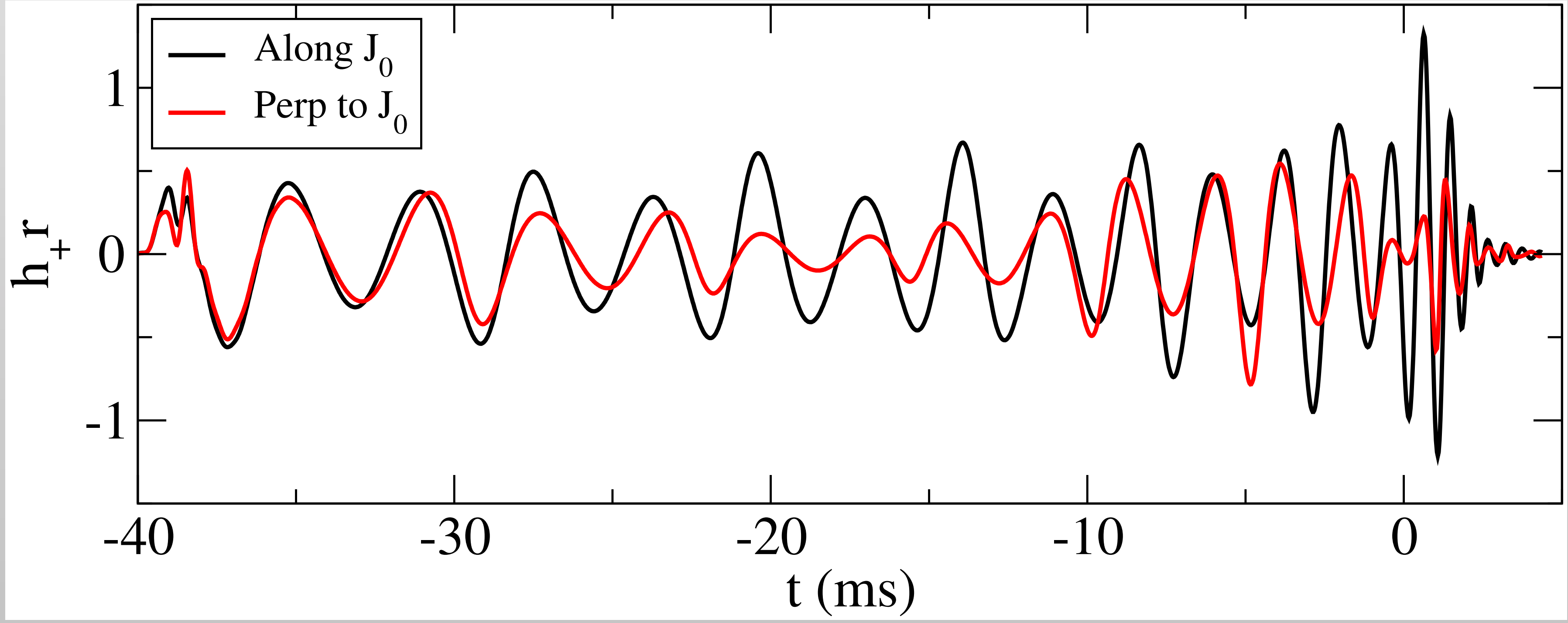


aligned-spin
(Foucart ea 2012)



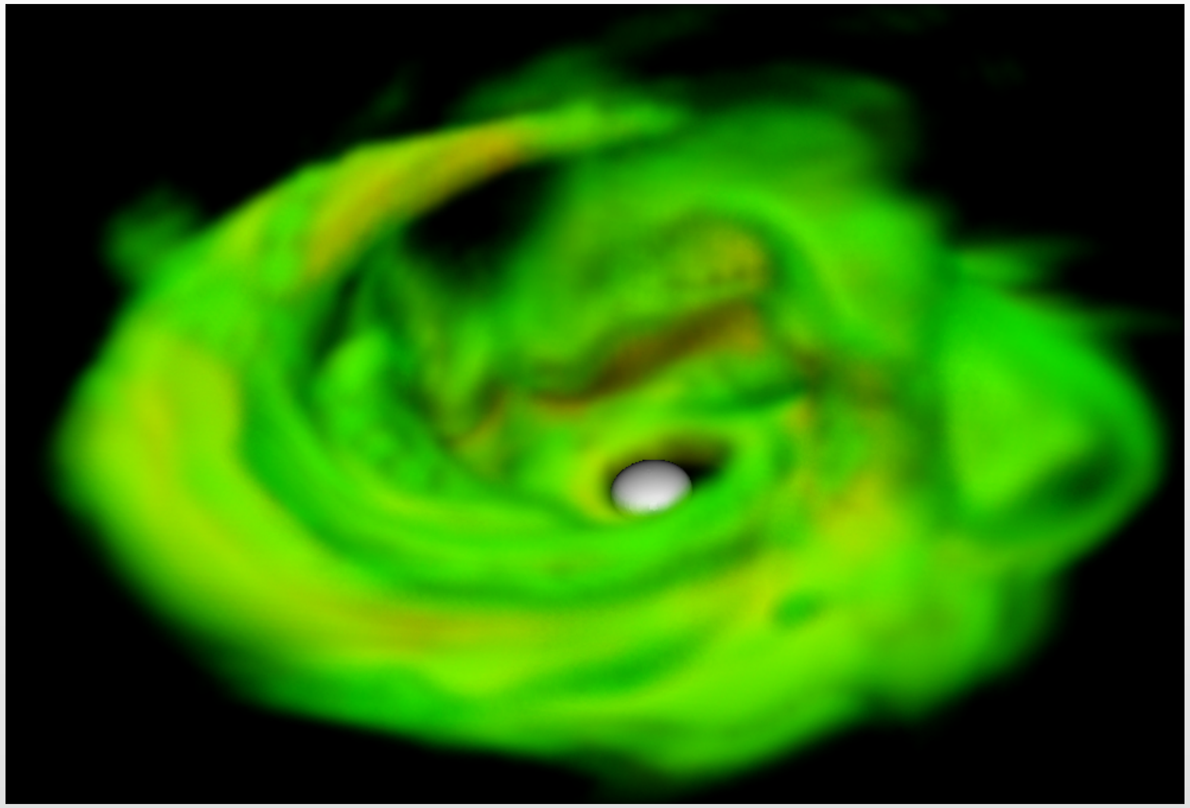
precessing
(Foucart ea, in prep)



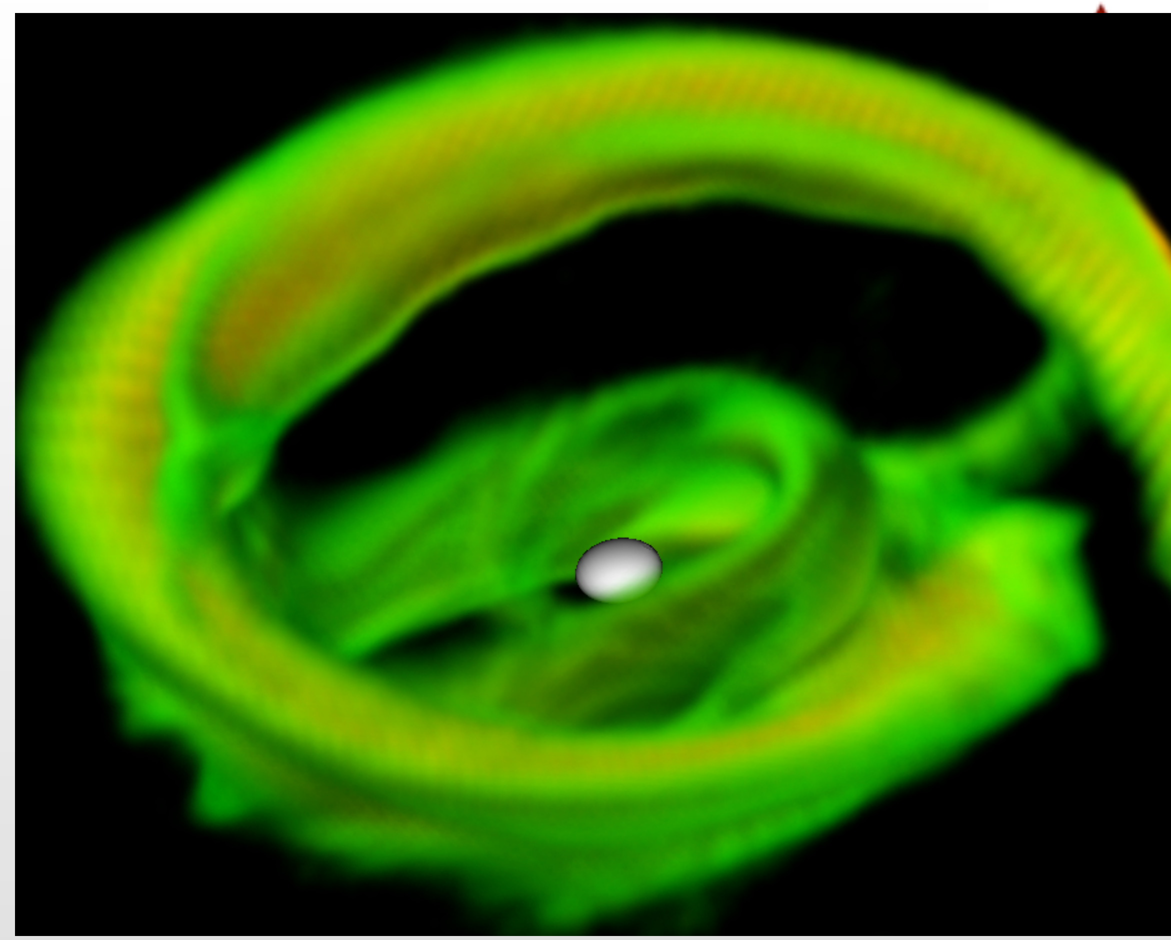


BH-NS q=7

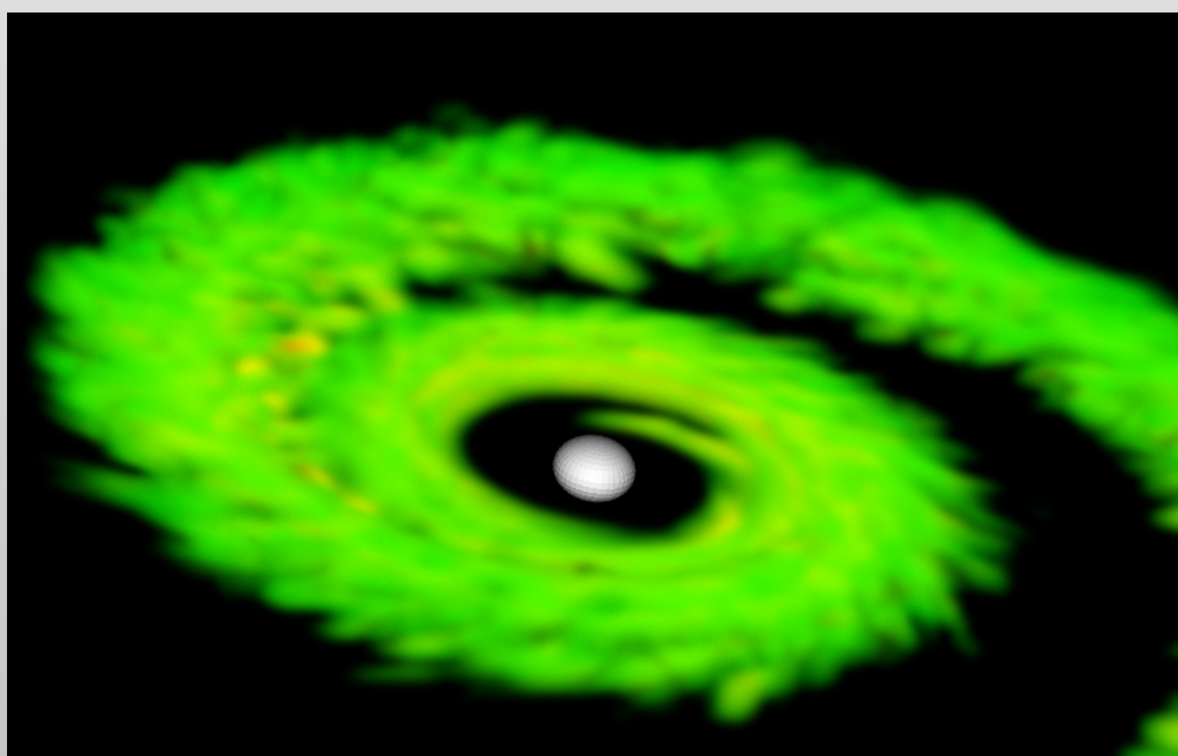
Images courtesy F. Foucart



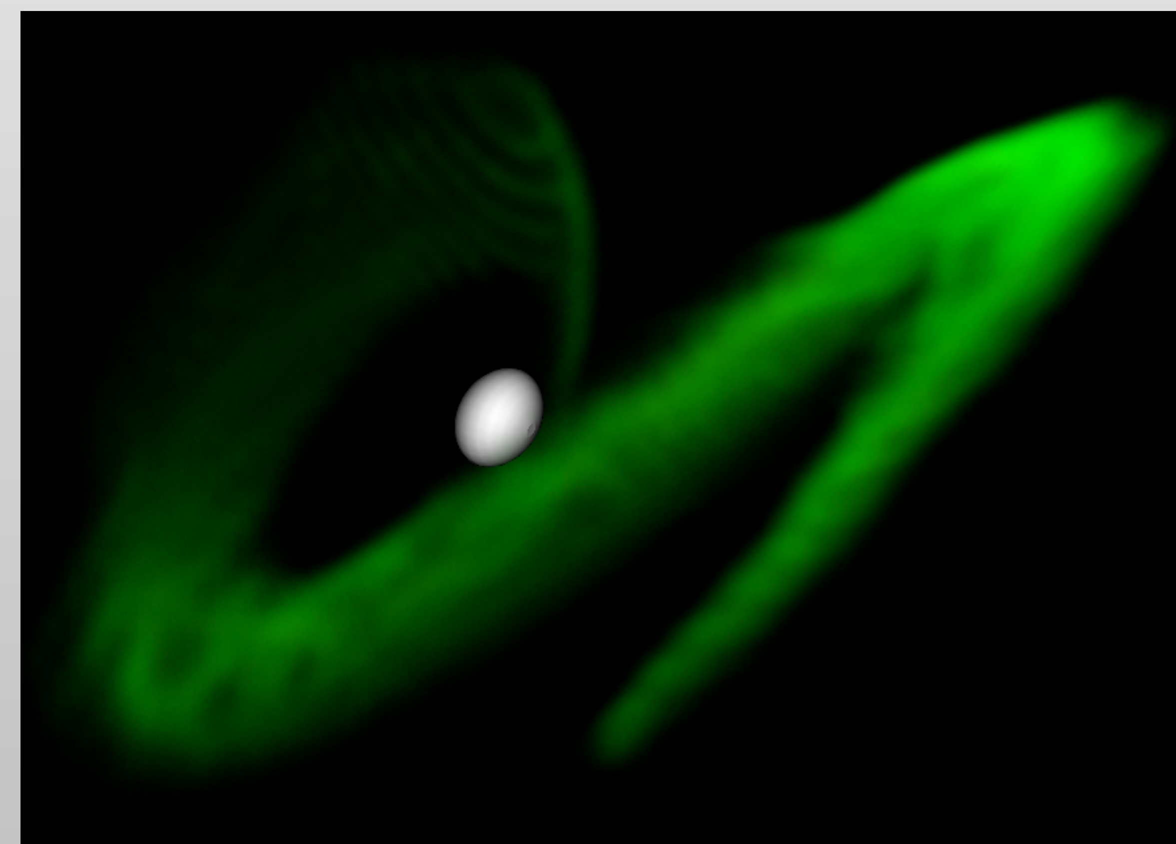
aBH=0.9 aligned



aBH=0.9, misaligned



aBH=0.7 aligned



aBH=0.9, strongly misaligned

Summary

- BH-BH waveform models for *aligned spins* well developed
  - Further work needed to shore up confidence
  - Next years will see improvements with new NR sims
- *Precessing* waveform models are ambitious, but likely doable
  - Lots of work remains! It's not a "solved problem"
- Modeling-effort increases *steeply* with desired accuracy
  - Collaborate with data-analysis to find good compromise
- *Redundancy* is essential
  - Phenom **and** EOB, finite-difference codes **and** SpEC
  - Allows consistence checks and avoids single point of failure
  - Data-analysis should insist on redundancy and perform independent cross-checks to validate and to guide development.