The dangers of systematic errors in parameter estimation
What waveforms do data analysts want?

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Why do we care about parameter estimation?

- If we want to do physics with GW detections -- astrophysics or studies of strong-field gravity -- we must know how to evaluate model fits and estimate parameters

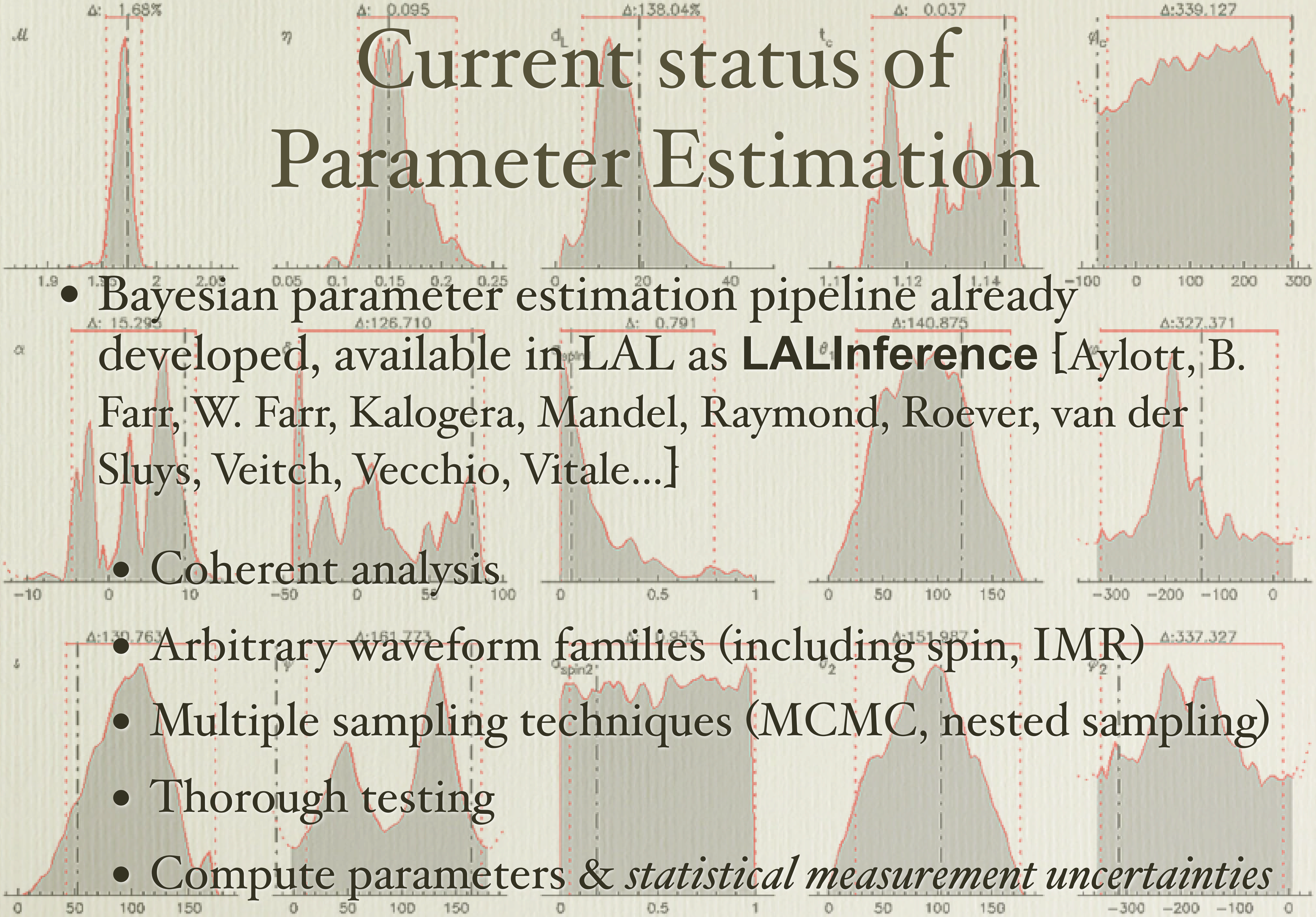
- See a number of PE posters at this meeting:
  - T. Sidery, Sky Localization
  - S. Vitale and R. Sturani, Spins
  - W. Vousden, Astrophysical Priors
  - R. Smith, IMRI waveforms
  - and talks by H. Tagoshi, T. Li, H. Pfeiffer, M. Vallisneri ...

Controversial Claims

- Parameter estimation problem is fundamentally solved when the model is known
- Systematic biases, particularly from imperfectly known waveforms, are the biggest challenge going forward
- We have been using inadequate techniques for estimating impact of bias [*A modest proposal for how to do this better*]
- Are BNSs clean systems from a PE perspective? [*Not quite*]
- What are the most urgent challenges for a data analyst? [*Personal view*]

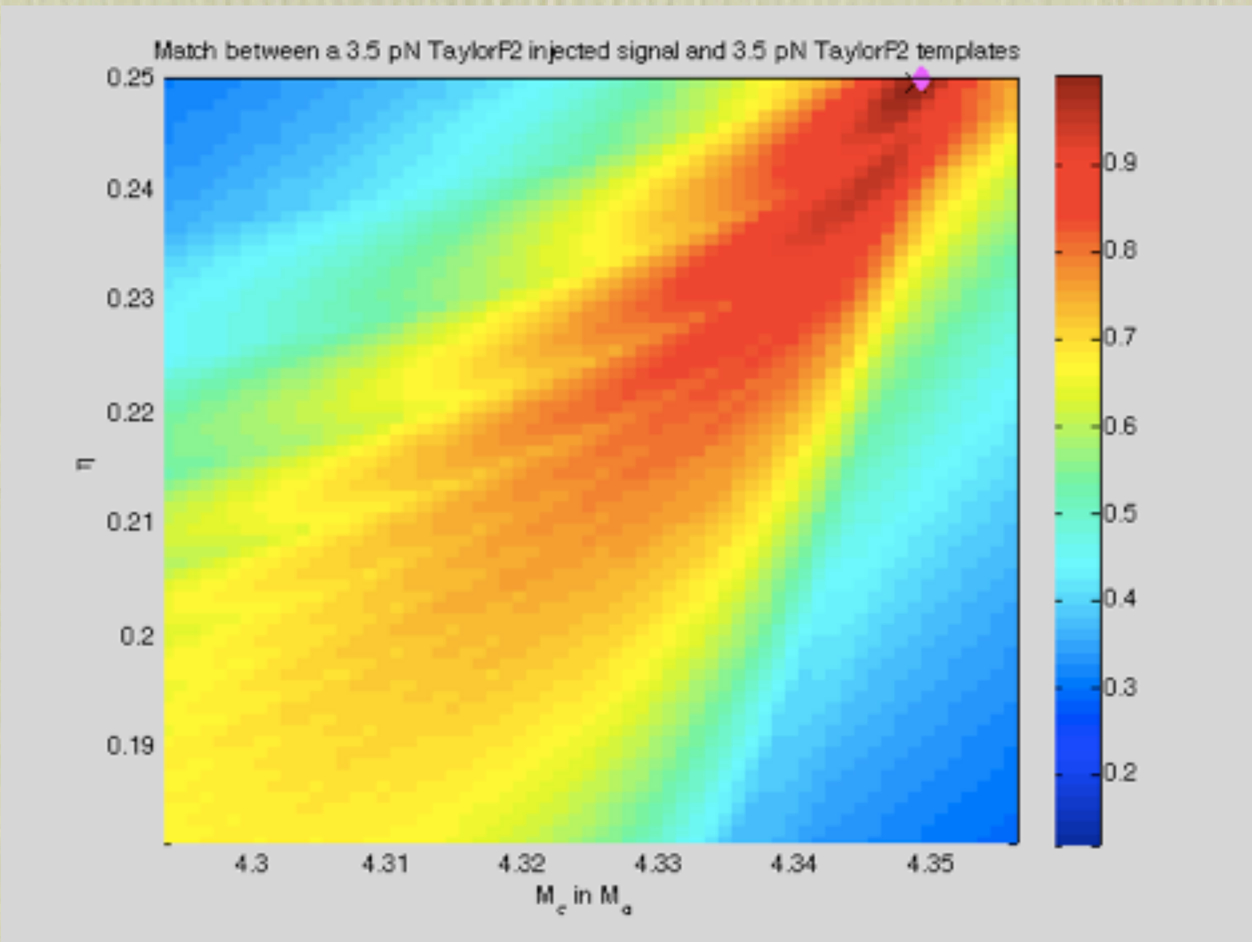
Current status of Parameter Estimation

- Bayesian parameter estimation pipeline already developed, available in LAL as **LALInference** [Aylott, B. Farr, W. Farr, Kalogera, Mandel, Raymond, Roever, van der Sluys, Veitch, Vecchio, Vitale...]
  - Coherent analysis
  - Arbitrary waveform families (including spin, IMR)
  - Multiple sampling techniques (MCMC, nested sampling)
  - Thorough testing
  - Compute parameters & *statistical measurement uncertainties*



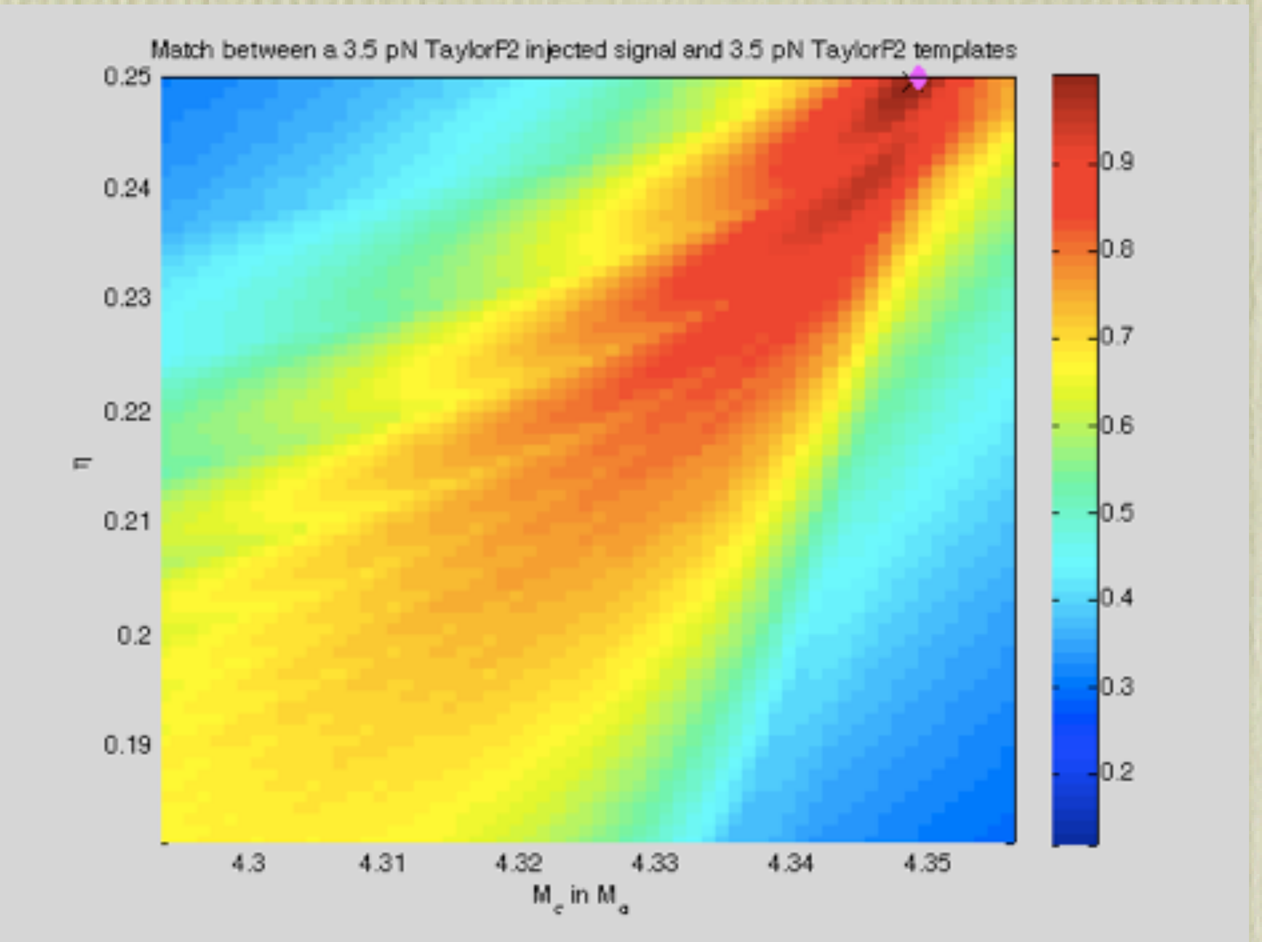
[van der Sluys, Mandel, Raymond, et al., 2009]

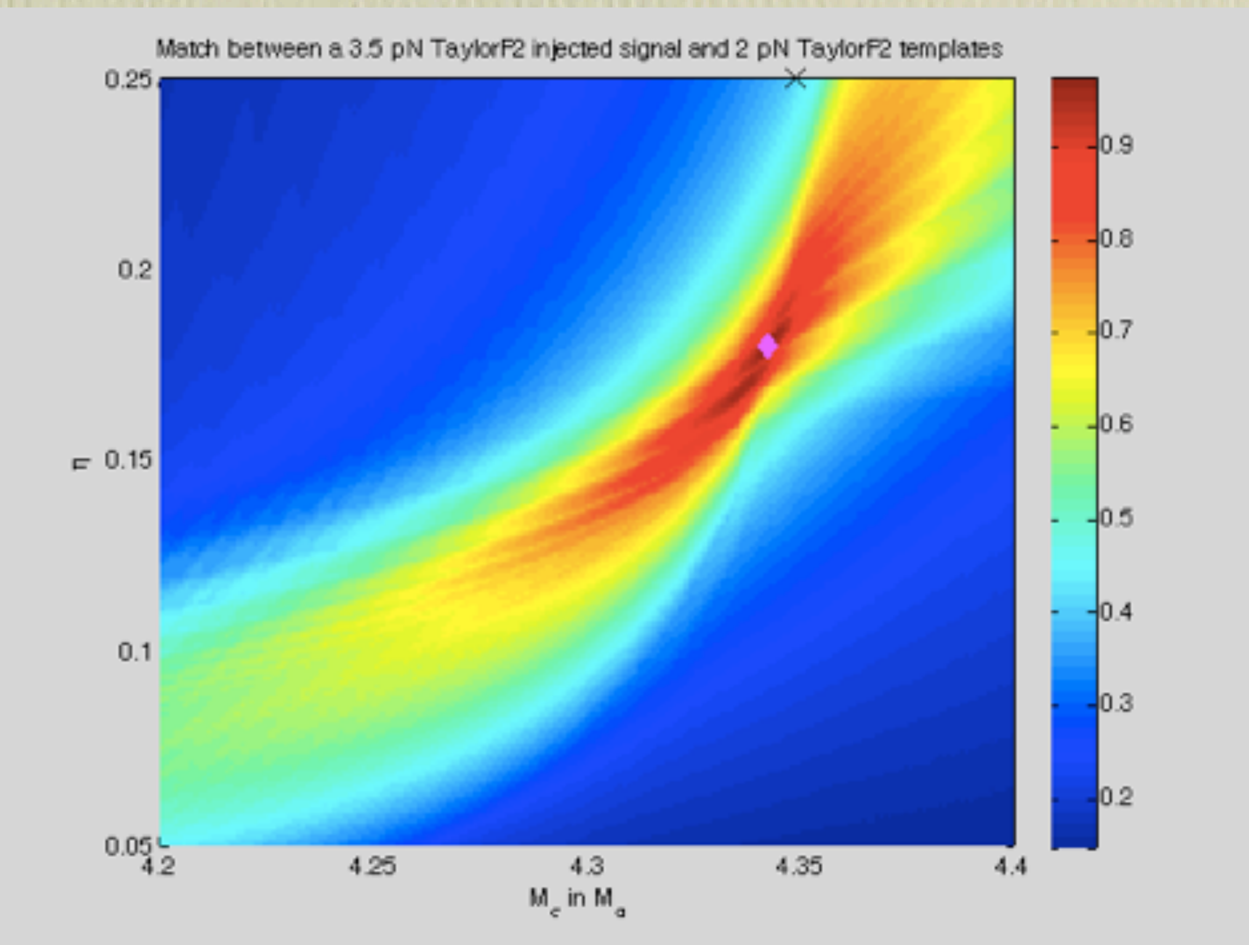
Statistical uncertainty



- 3.5pN injection and recovery
- 4.8+5.2 solar-mass BH system, overhead a single AdvLIGO detector
- Match of 0.97 contour corresponds to ~2-sigma confidence interval on masses (or does it?)

Systematic biases





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- 3.5pN injection, 2pN recovery
- Best fit (magenta) is 9.2+2.8 system -- NS-BH???
- [Match with injection is above 0.97]

Waveform models

- Parameter estimation, not detection, is the threshold -- so "good to a few percent" is not enough.
- We need more confident waveform families that incorporate all of the relevant effects:
  - **Inspiral, merger, ringdown**
  - **Spinning, precessing waveforms**
  - Higher harmonics
  - Intermediate mass ratios
  - Matter effects
  - Eccentric binaries
  - Deviations from Kerr / GR?

How accurate do waveforms *really* need to be?

- Standard criterion: $\langle \delta h | \delta h \rangle \leq 1$ [Lindblom, Owen, Brown; 2008]

$$
\begin{aligned}
h(\lambda, f) &= (1-\lambda) h_e(f) + \lambda h_m(f), \\
&\equiv h_e(f) + \lambda \delta h(f),
\end{aligned}
$$

$$
\sigma_\lambda^{-2} = \left\langle \frac{\partial h}{\partial \lambda} \middle| \frac{\partial h}{\partial \lambda} \right\rangle = \langle \delta h | \delta h \rangle
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- Detectability of deviation between two alternative models with all other parameters fixed

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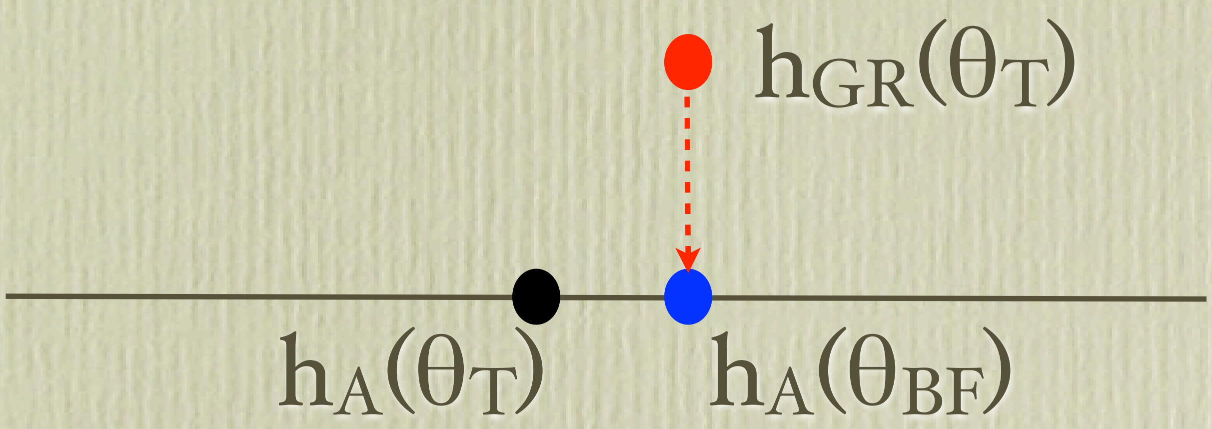
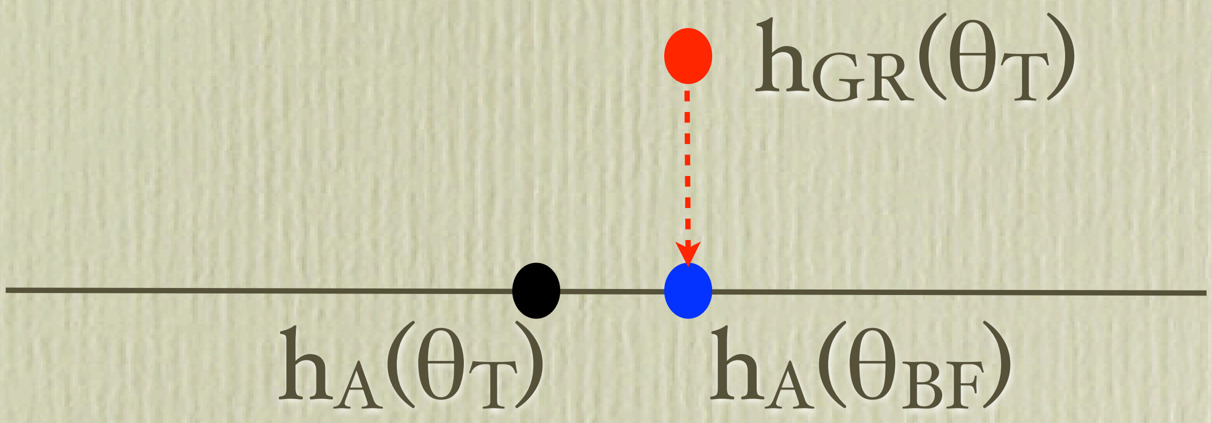


- Standard criterion: $\langle \delta h | \delta h \rangle \leq 1$ [Lindblom, Owen, Brown; 2008], $\delta h = h_{GR}(\theta_T) - h_A(\theta_T)$: Can we tell red and black apart?
- But the real question is, do we care about the difference between black and blue?
  - That difference can be much smaller because of projection; might also manifest in irrelevant parameters

How accurate do waveforms *really* need to be?



- Standard criterion: $\langle \delta h | \delta h \rangle \leq 1$ [Lindblom, Owen, Brown; 2008], $\delta h = h_{GR}(\theta_T) - h_A(\theta_T)$: Can we tell red and black apart?
- But the real question is, do we care about the difference between black and blue, for *parameters of interest*?
- Standard criterion sufficient, but not necessary -- can be far too demanding

How to estimate impact of systematic bias?

- Separately compute statistical uncertainty and systematic bias [e.g., Cutler & Vallisneri, 2007]
- Systematic bias: best-fit interesting parameters (*mass)* from grid-based search maximizing over uninteresting parameters (*time & phase)*
- Statistical uncertainty: ~~poor~~ computationally-limited man's version of Bayesian PE

Statistical uncertainty, I

- Likelihood: $L(h, \vec{\theta}) = p(s|\vec{\theta}) \propto \exp\left(-\frac{\langle s - h(\vec{\theta}) \mid s - h(\vec{\theta})\rangle}{2}\right)$

- Zero-noise realization:

$$L(\delta\vec{\theta}) \propto \exp\left(-\frac{\langle h(\vec{\theta_{\rm inj}}) - h(\vec{\theta_{\rm inj}} + \delta\vec{\theta}) \mid h(\vec{\theta_{\rm inj}}) - h(\vec{\theta_{\rm inj}} + \delta\vec{\theta})\rangle}{2}\right) \equiv \exp\left(-\frac{\langle \delta h \mid \delta h\rangle}{2}\right)$$

- Overlap / match:

$$M(s, h(\vec{\theta})) = \frac{\langle s \mid h(\vec{\theta})\rangle}{\sqrt{\langle s \mid s\rangle\langle h(\vec{\theta}) \mid h(\vec{\theta})\rangle}} \to \frac{\langle h \mid h + \delta h\rangle}{\sqrt{\langle h \mid h\rangle\langle h + \delta h \mid h + \delta h\rangle}}$$

where we maximize over “uninteresting” parameters

Statistical uncertainty, II

- If $\delta h << h$ and $\langle h|\delta h\rangle = 0$, then

$$\log L = -\frac{1}{2}\langle \delta h \mid \delta h\rangle \sim -(1-M)\langle h \mid h\rangle = -(1-M)\ \mathrm{SNR}^2$$

- Further, if $\langle \delta h \mid \delta h\rangle \approx \langle \partial h_{,\theta} \mid \partial h_{,\theta}\rangle (\delta\theta)^2$

$$\int_{-\Delta\theta}^{+\Delta\theta} L(\theta) d\theta = \int_{-\Delta\theta}^{+\Delta\theta} d\theta e^{-\frac{1}{2}\langle \delta h \mid \delta h\rangle} \sim erf\left(\frac{\Delta\theta}{\sqrt{2\langle \partial h_{,\theta} \mid \partial h_{,\theta}\rangle^{-1/2}}}\right)$$

- Under these assumptions, the boundary of the N-sigma confidence interval is given by $\sqrt{2(1-M)}\mathrm{SNR}$

- So, for example, for SNR=8, M=0.97 is 2-sigma boundary

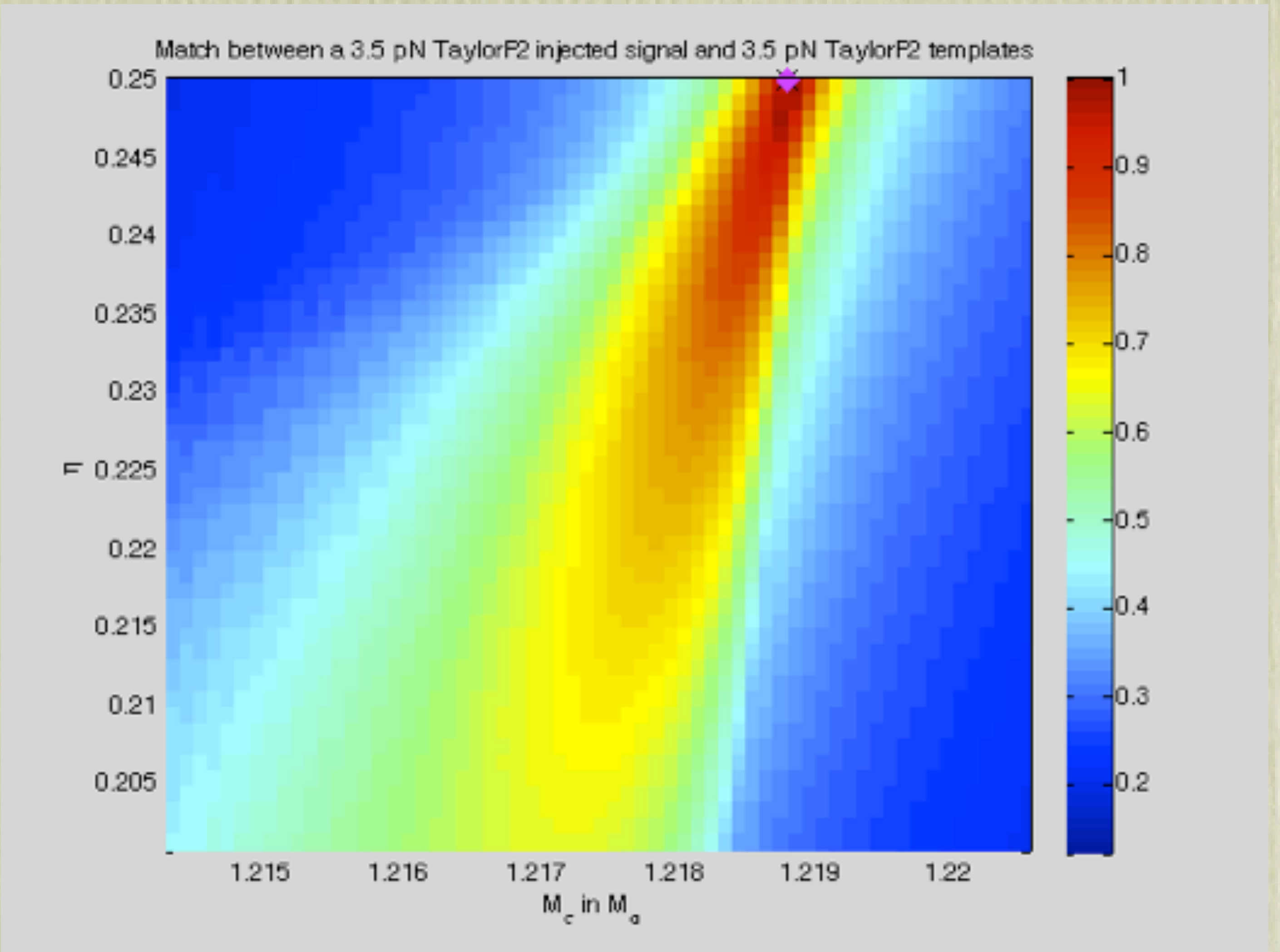
A Modest Proposal

- Can directly compute the desired confidence interval from a grid-based search:

$$\int_{-\Delta\theta}^{+\Delta\theta} L(\theta) d\theta = \int_{-\Delta\theta}^{+\Delta\theta} d\theta e^{-\frac{1}{2}\langle \delta h(\theta) \mid \delta h(\theta) \rangle}$$

- Inner product maximized over *uninteresting* parameters
- Minimal over-head relative to standard overlap calculation
- Faster than Bayesian techniques in small dimensions [at the expense of not getting the priors right on parameters that are maximized over]

Are binary neutron stars clean systems?



- 3.5pN injection and recovery
- 1.4+1.4 solar-mass BH system, overhead a single AdvLIGO detector
- Match of 0.97 contour corresponds to ~2-sigma confidence interval on masses
- What about pN uncertainty? Matter effects? Spins?

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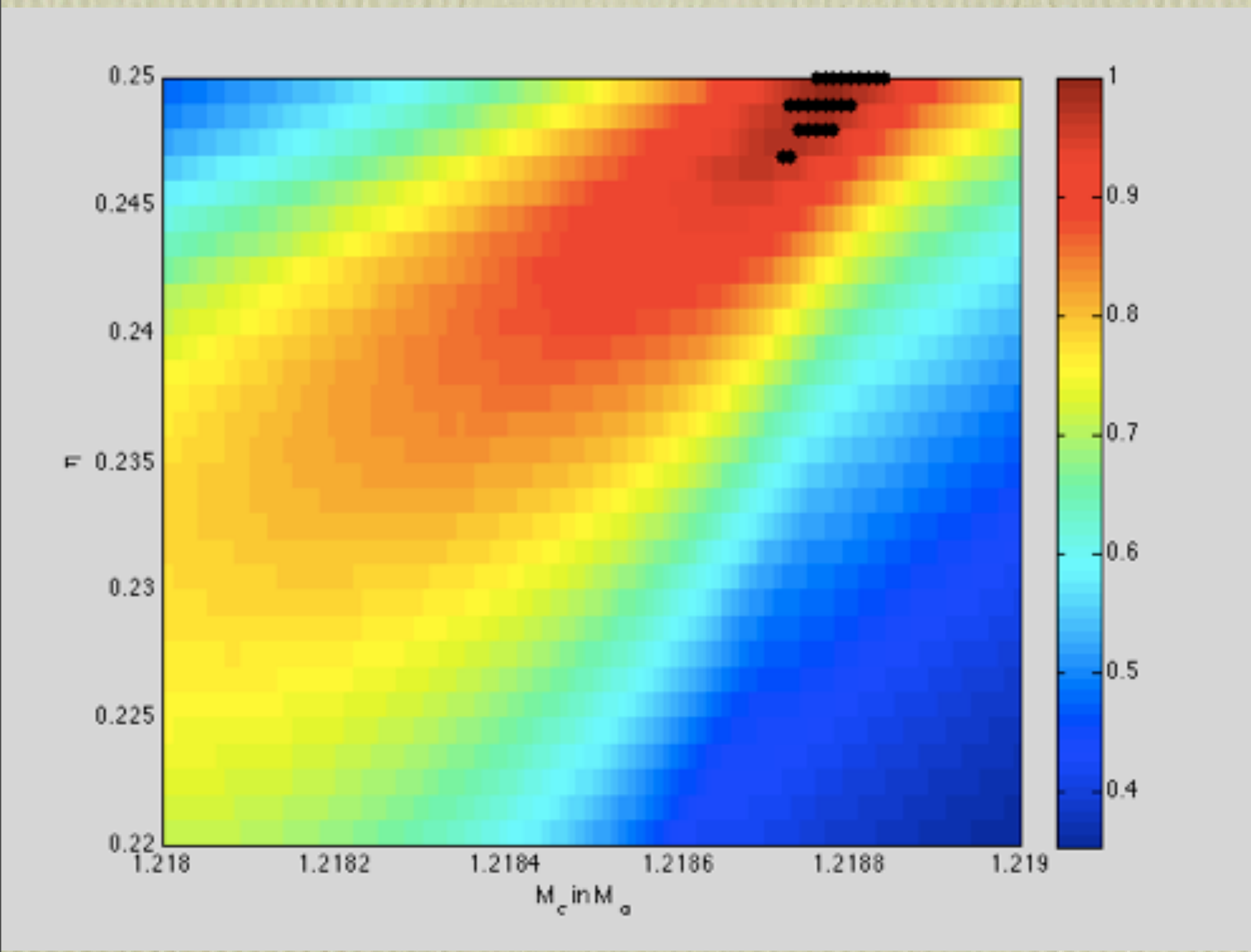
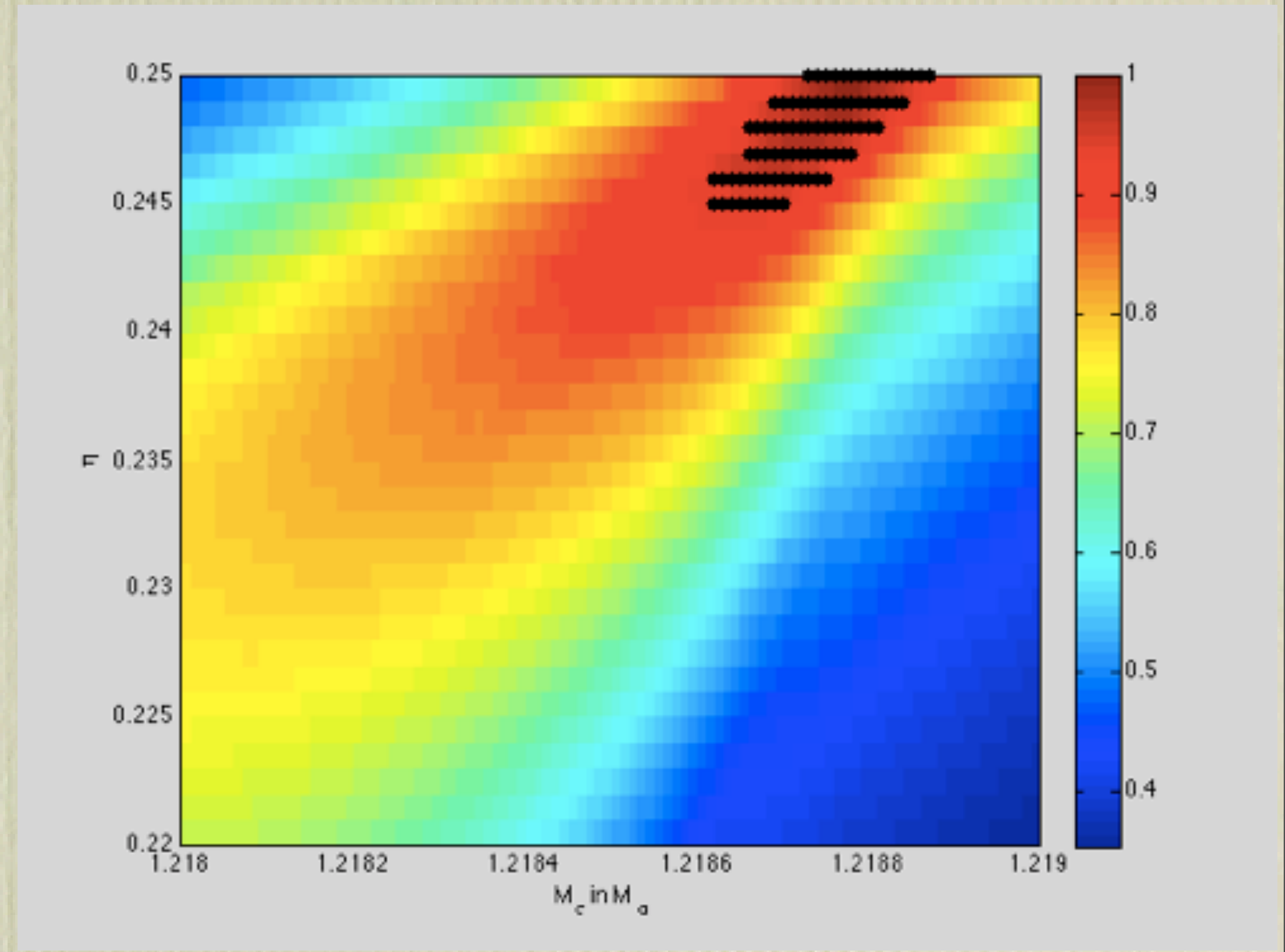
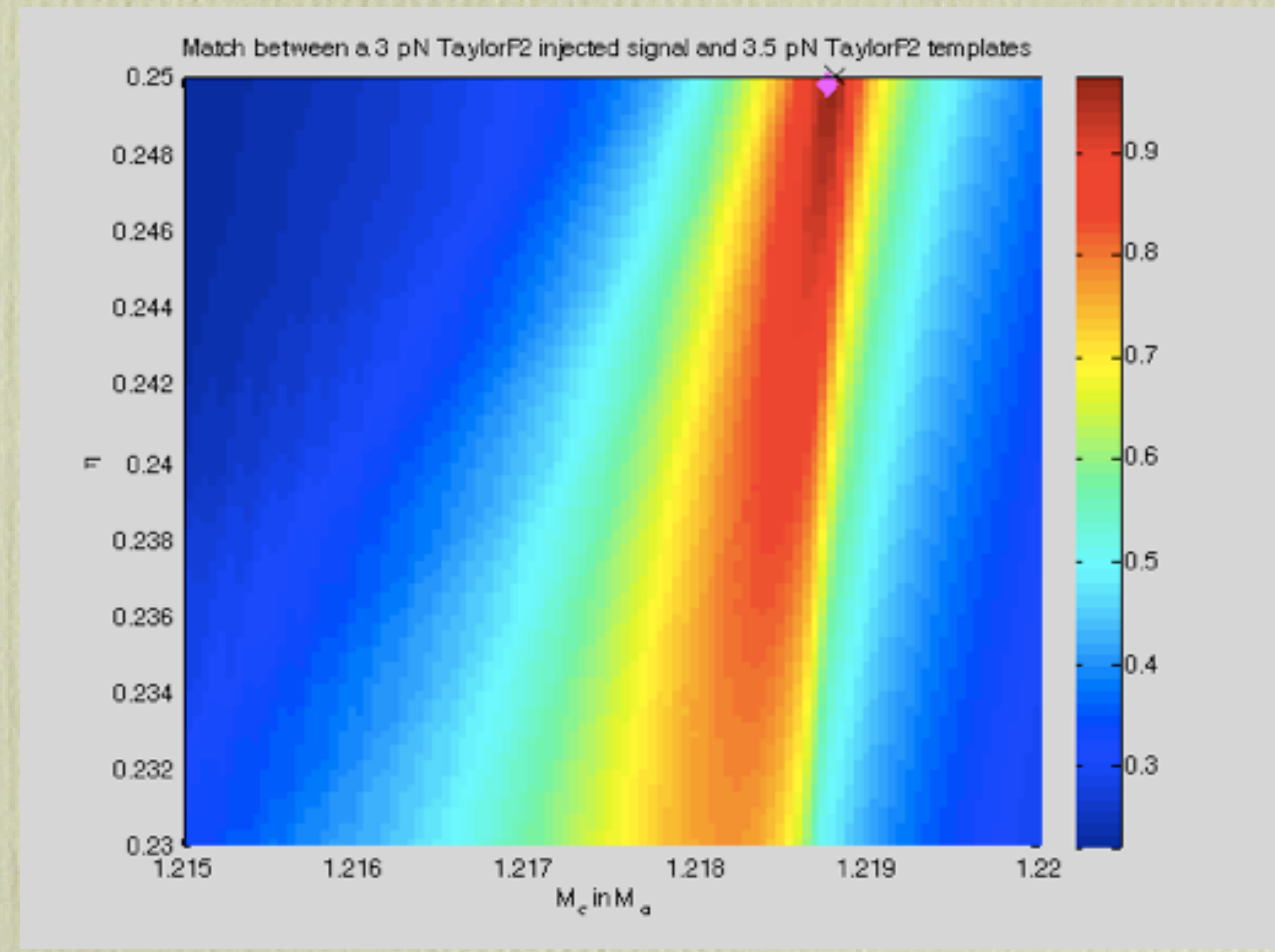




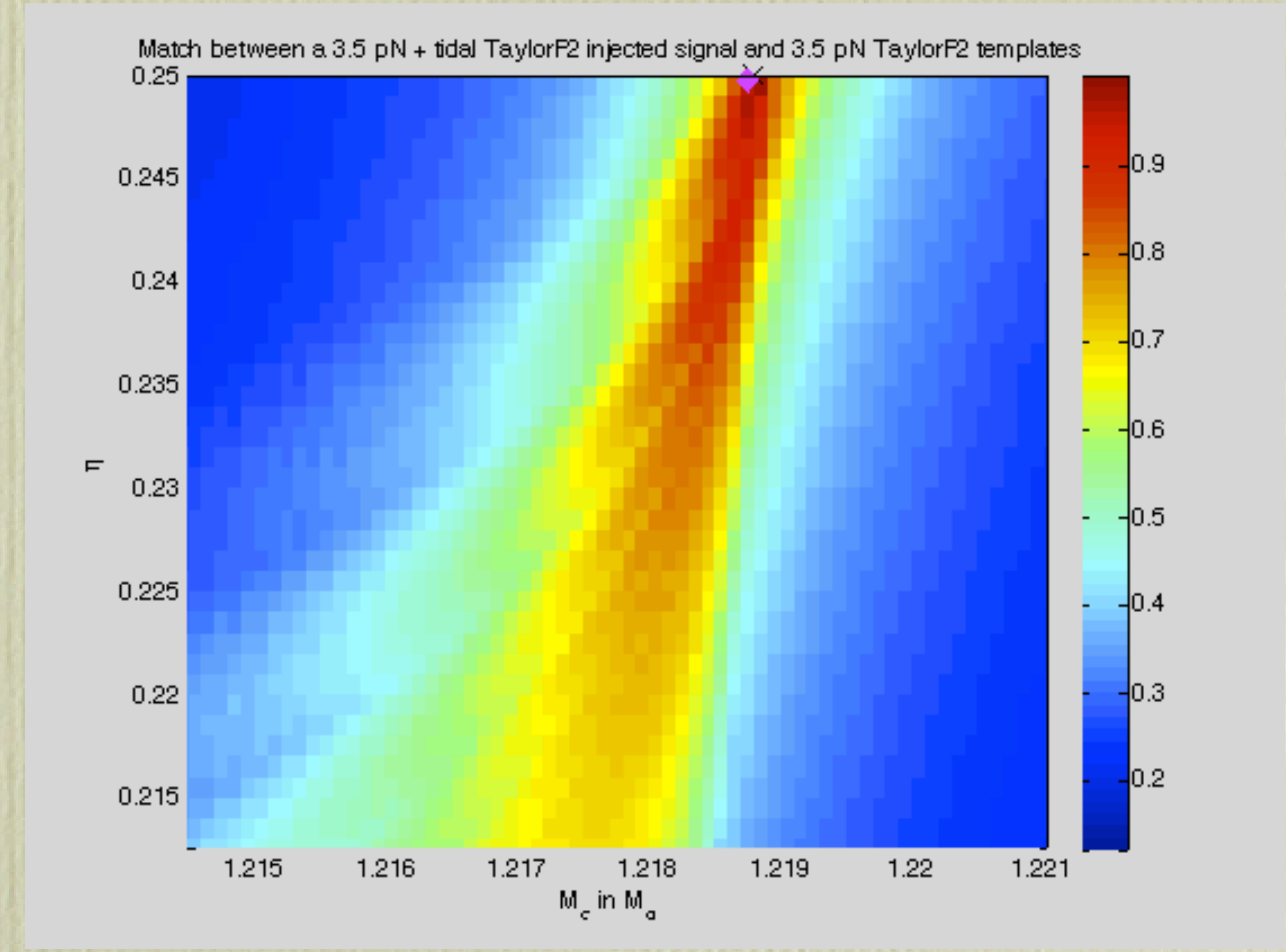
- 3.5pN injection and recovery, 1.4+1.4 solar-mass BH system, overhead a single AdvLIGO detector
- 95% confidence interval on masses includes matches ≳ 0.94 at SNR=8, $\langle \delta h | \delta h \rangle \lesssim 8$
- 68% confidence interval on masses includes matches ≳ 0.976 at SNR=8, $\langle \delta h | \delta h \rangle \lesssim 3$
- What about pN uncertainty? Matter effects? Spins?

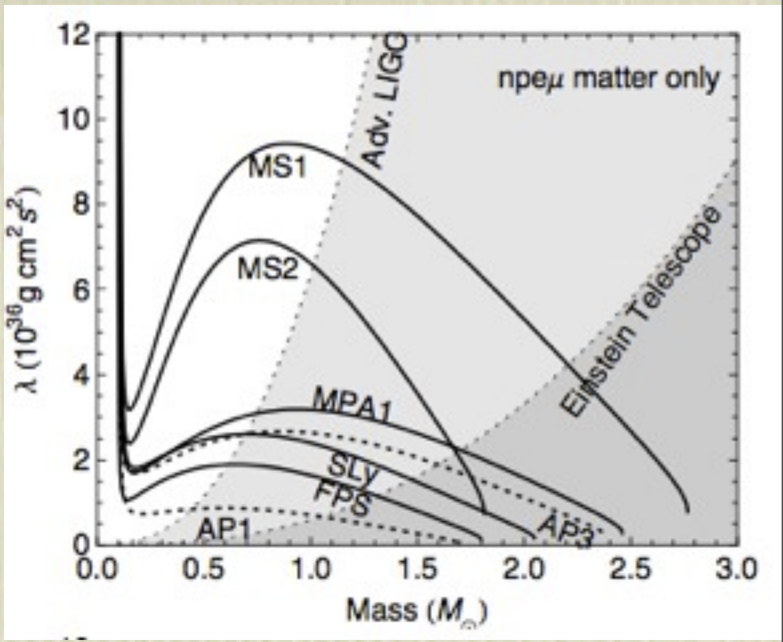
Effect of higher-order pN terms



- 3pN injection, 3.5 pN recovery
- Best-fit masses are 1.44+1.36

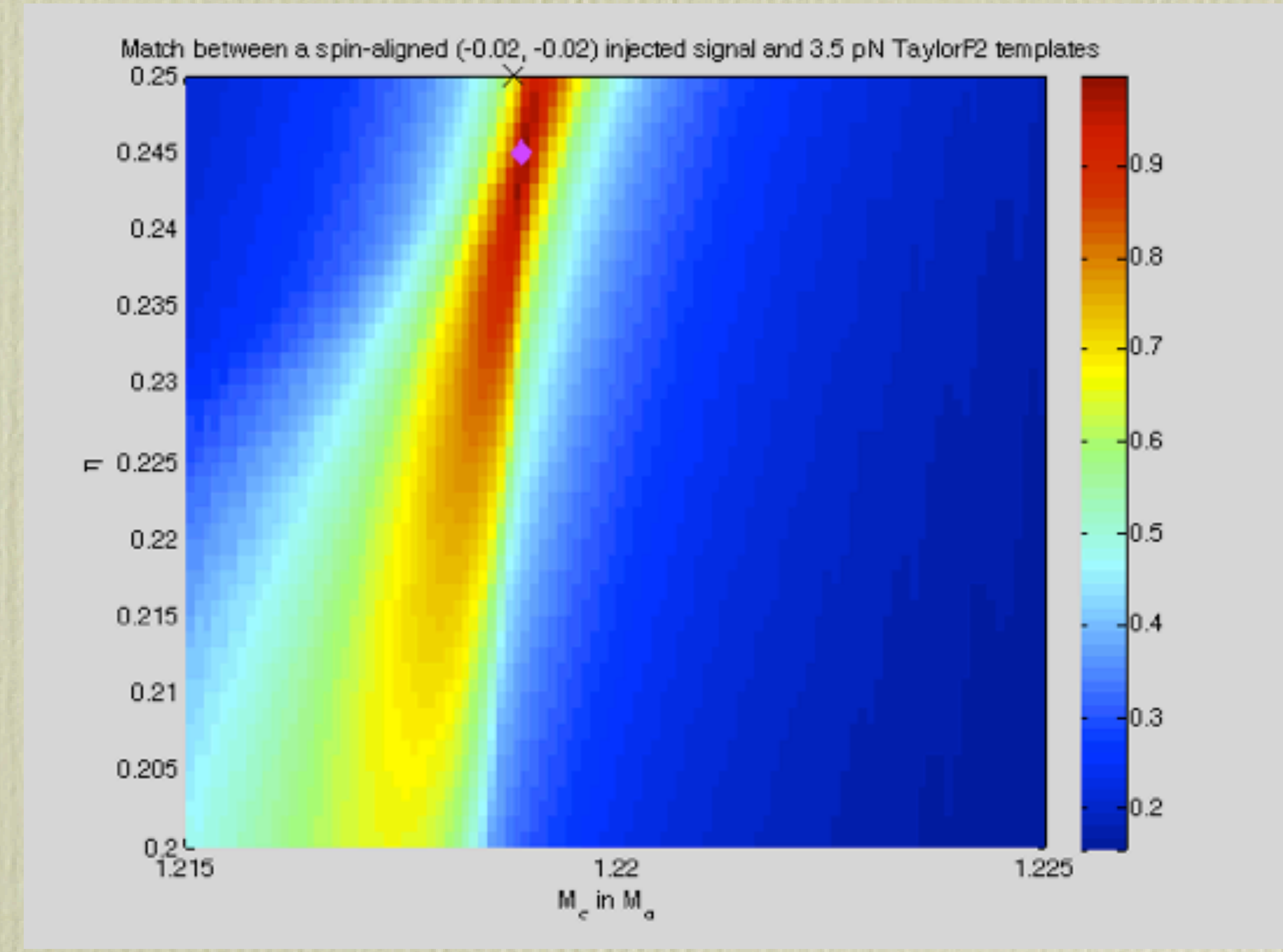
Tidal effects





- 3.5pN+tidal terms injection, 3.5 pN recovery
- Use single-parameter parametrization for tidal deformability [Hinderer et al., 2009], valid to ~500 Hz

Effect of spins



- 3.5pN injection with non-precessing aligned spins (-0.02,-0.02), 3.5 pN recovery
- Best-fit masses are 1.6+1.2 solar masses
- Preliminary results, need to carefully evaluate spin effects

What do ~~data analysts~~ I want?

- A family of IMR waveforms with two spinning, precessing components: hybridized with NR results
- Need practical confidence statements, not just match to NR in regime of matching
- Request: provide several approximate waveform families that are within systematic uncertainty in fits to NR
- *Direct use of NR waveforms for parameter estimation?*

What can data analysts contribute?

- Studies of systematic impacts of variations in waveform families (e.g., NINJA context)
- Improved (relative to the overly strict $|dh|<1$) accuracy requirements on numerical and approximate waveforms
- Accounting for waveform uncertainty directly in Bayesian parameter-estimation methods