Between Fisher and Monte Carlo:

Mapping the distribution of the maximum-likelihood estimator for GW source parameters

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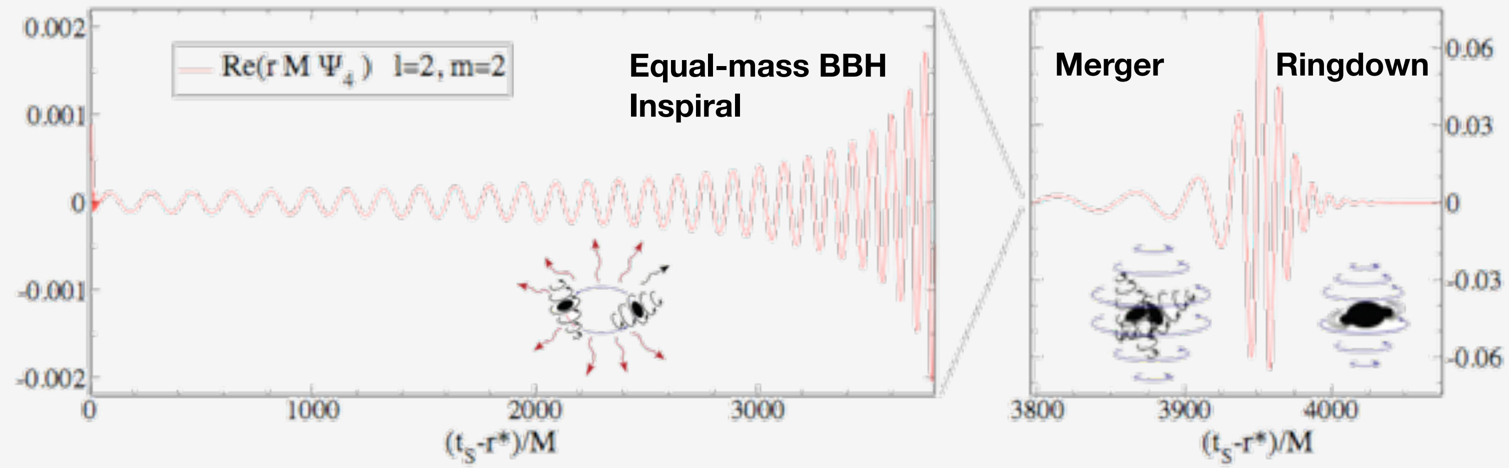
There's so much information in a gravitational waveform!

BH and NS astrophysics, cosmology, strong-field GR, BH structure, nuclear physics, alternative theories...

[Caltech/Cornell/CITA NR]



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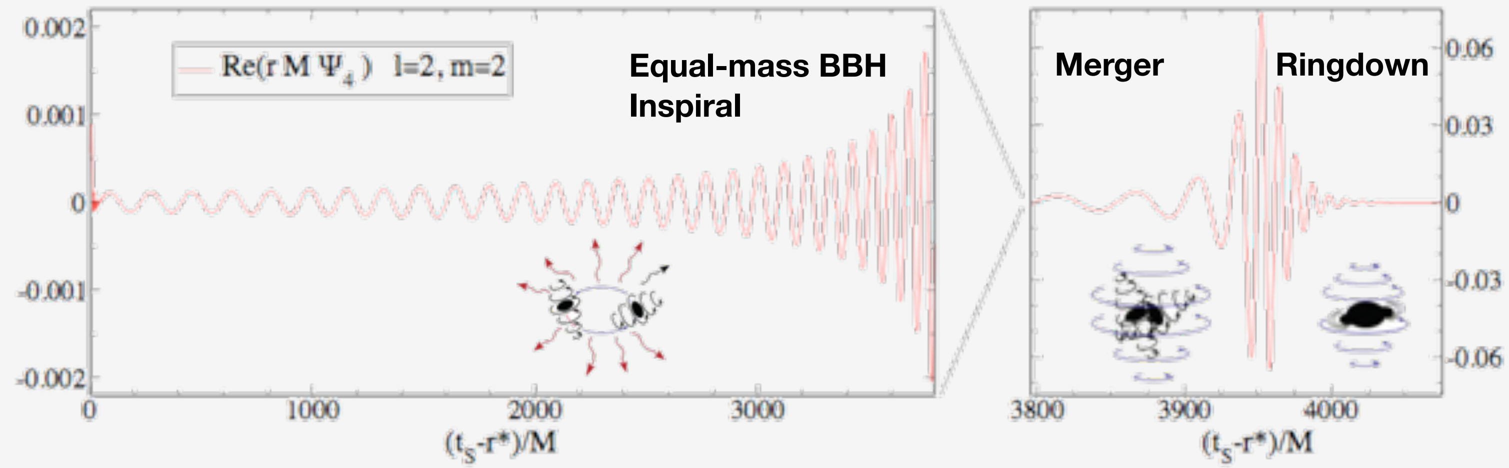

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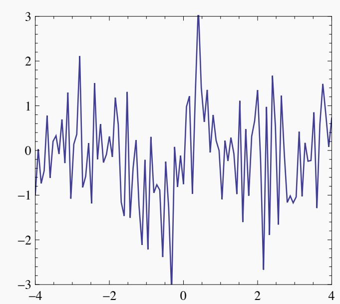

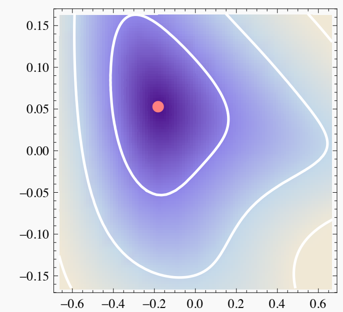

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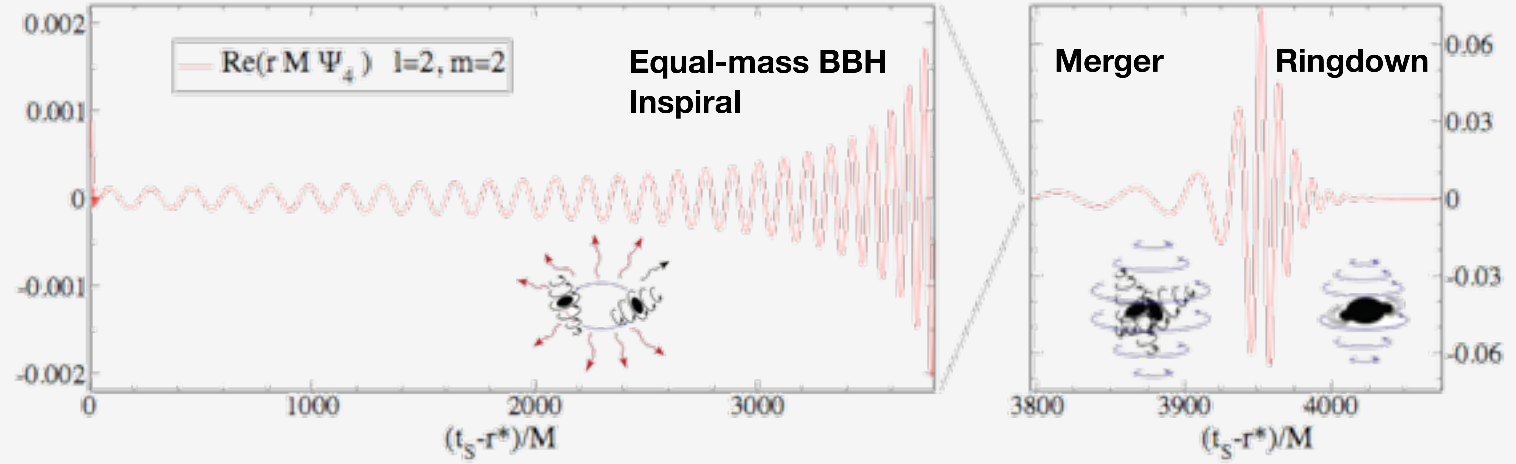

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Before detections, we want to know how well we can do

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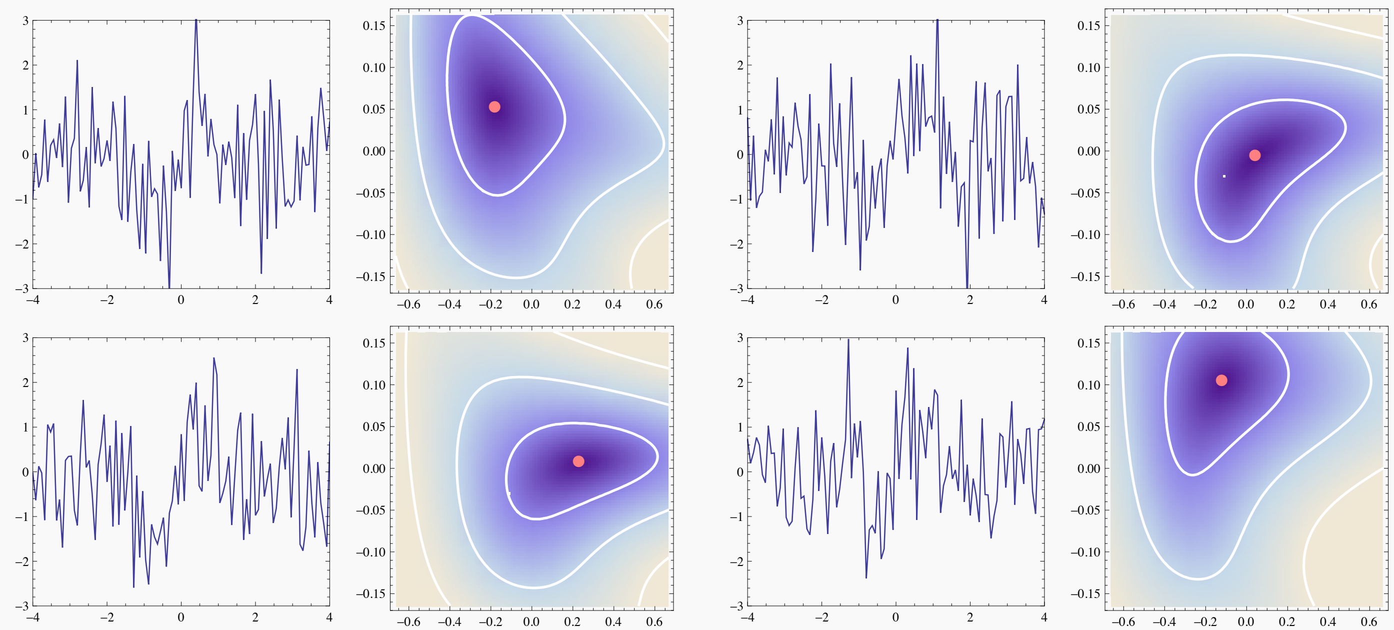

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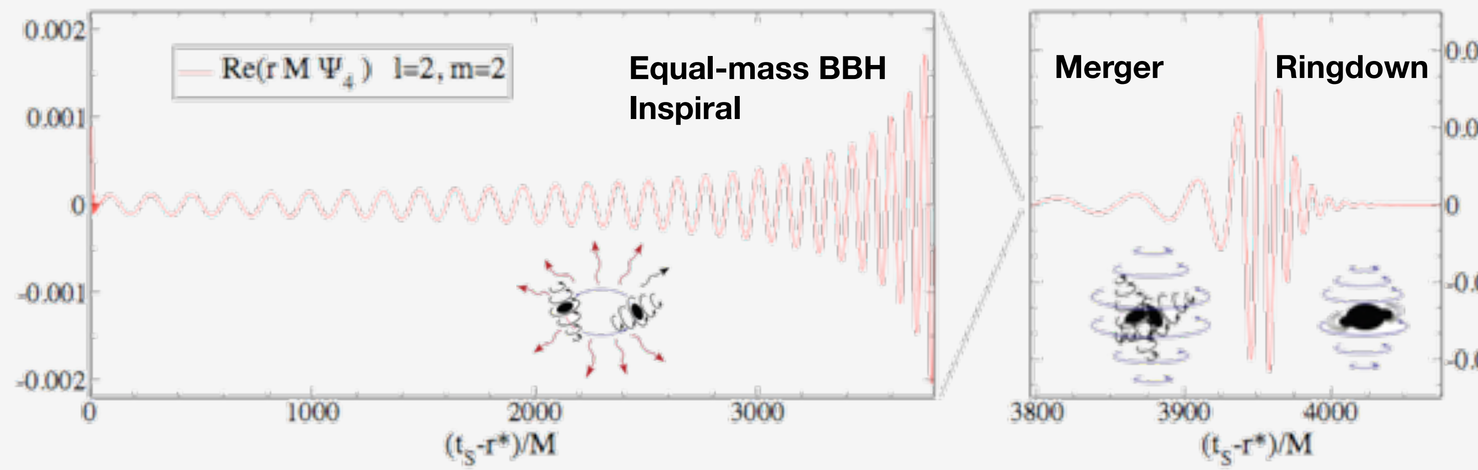

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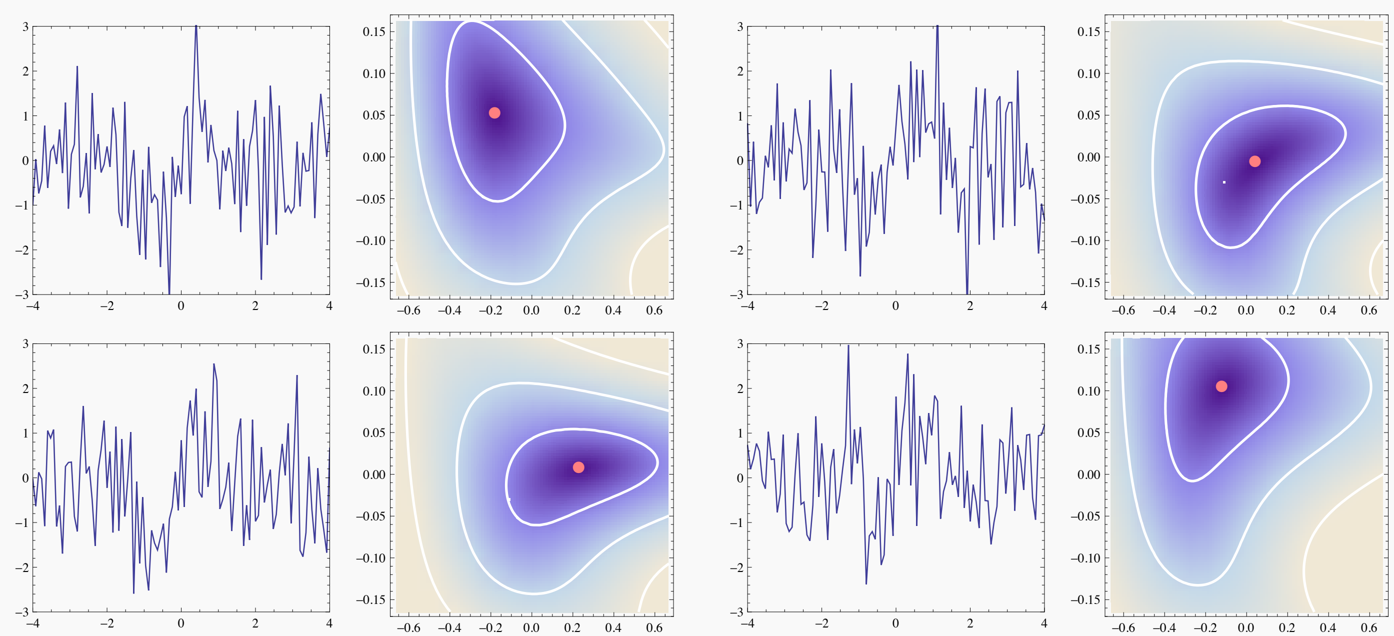


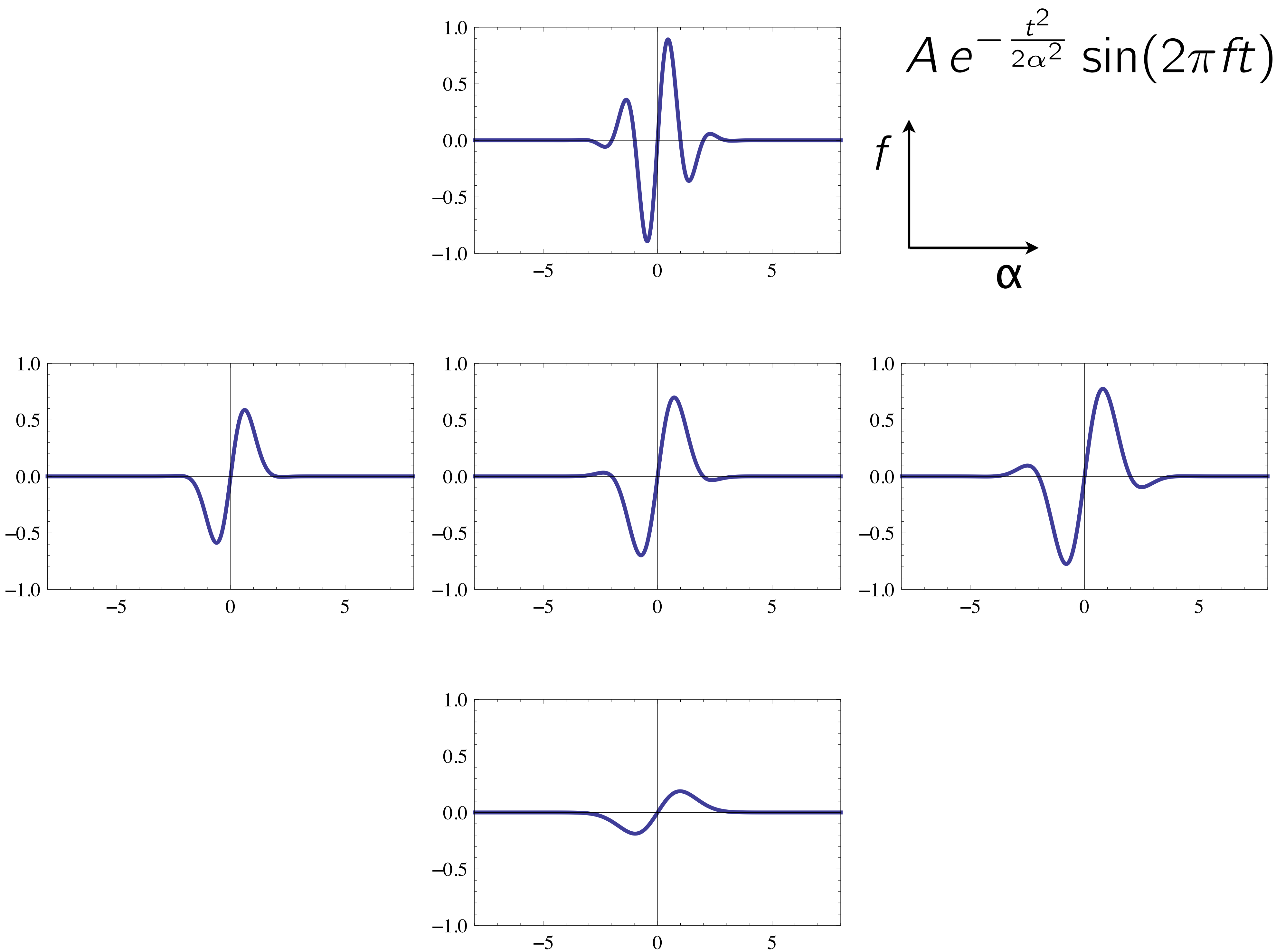
In this talk: a new efficient way to map the distribution of the maximum-likelihood estimator (●) across noise realizations

Bonus: a gentle intro to ML estimation

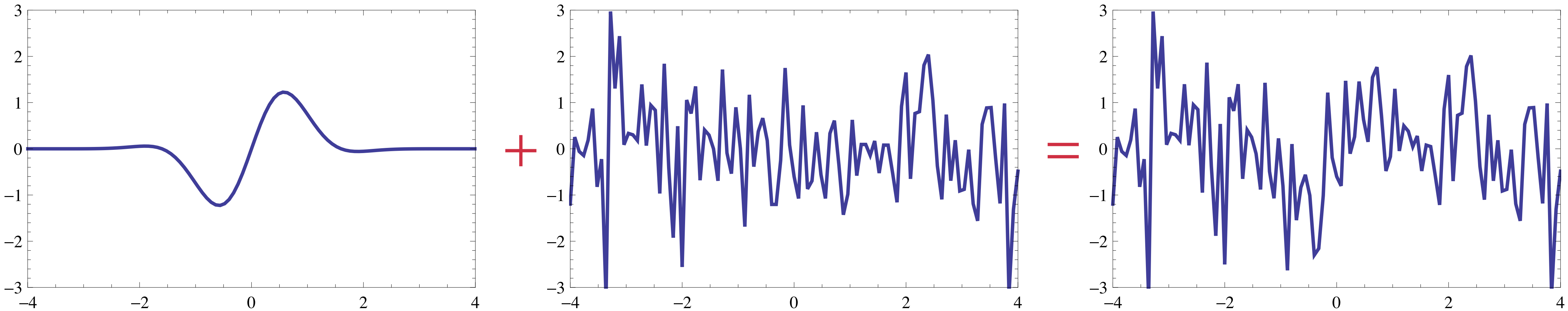
$$p(\theta) = \frac{e^{-(\Delta h, h_i)(F^{-1})^{ij}(\Delta h, h_j)/2}}{\sqrt{(2\pi)^d |F_{ij}|}} \times \frac{1}{\sqrt{(2\pi)^{d(d-1)/2} |D_{\mu\nu}|}} \times \int \left| F_{ij} + (\Delta h, h_{ij}) - M_{(ij)} \right| e^{-M_\mu (D^{-1})^{\mu\nu} M_\nu /2} dM_\mu$$

GW science =
learn about sources by estimating their parameters

$$A\,e^{-\frac{t^2}{2\alpha^2}}\,\sin(2\pi f t)$$

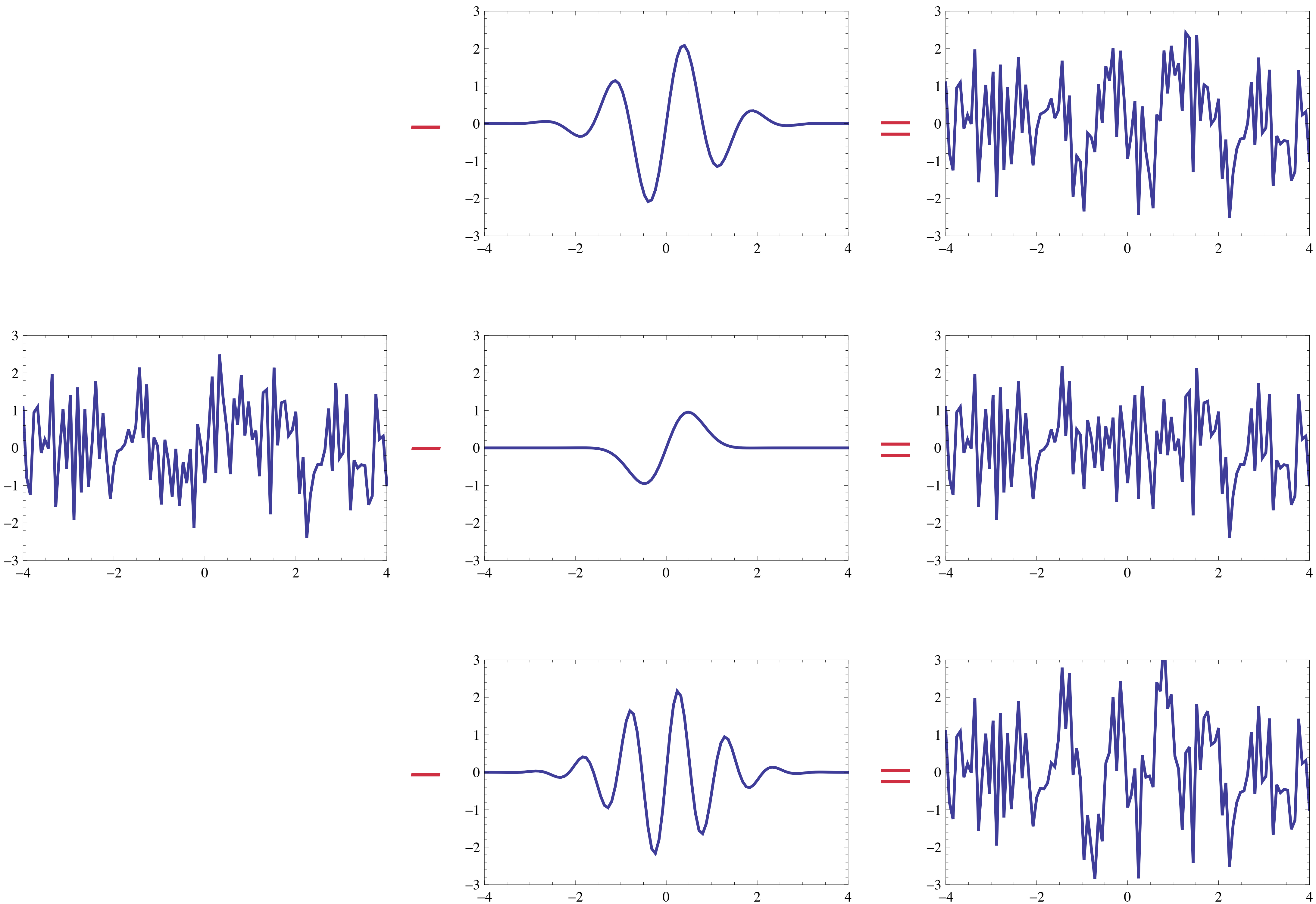


data = signal + noise



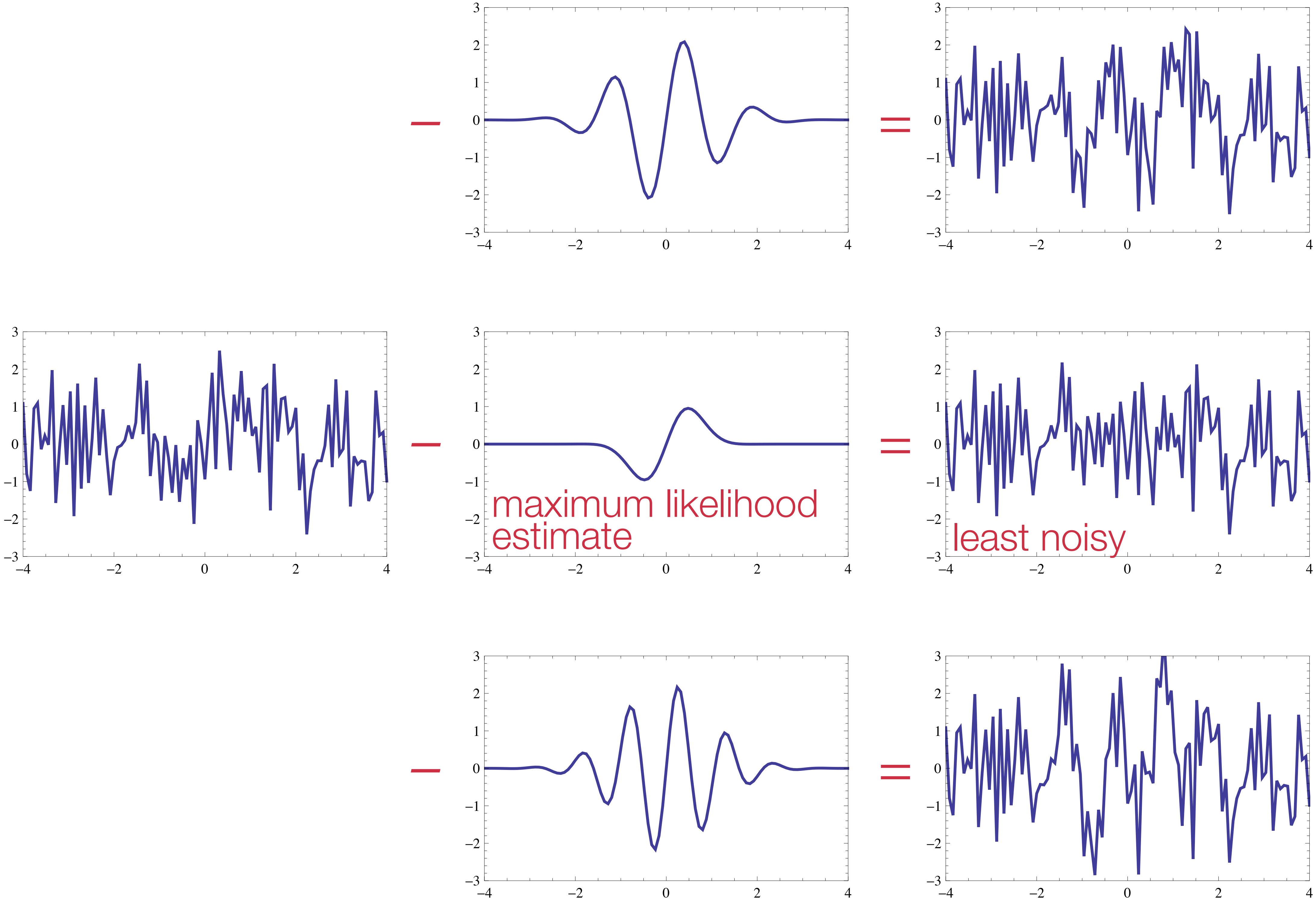
(SNR = 5)

noise = data – signal



noise = data – signal

hence p(signal parameters) = p(noise residual)



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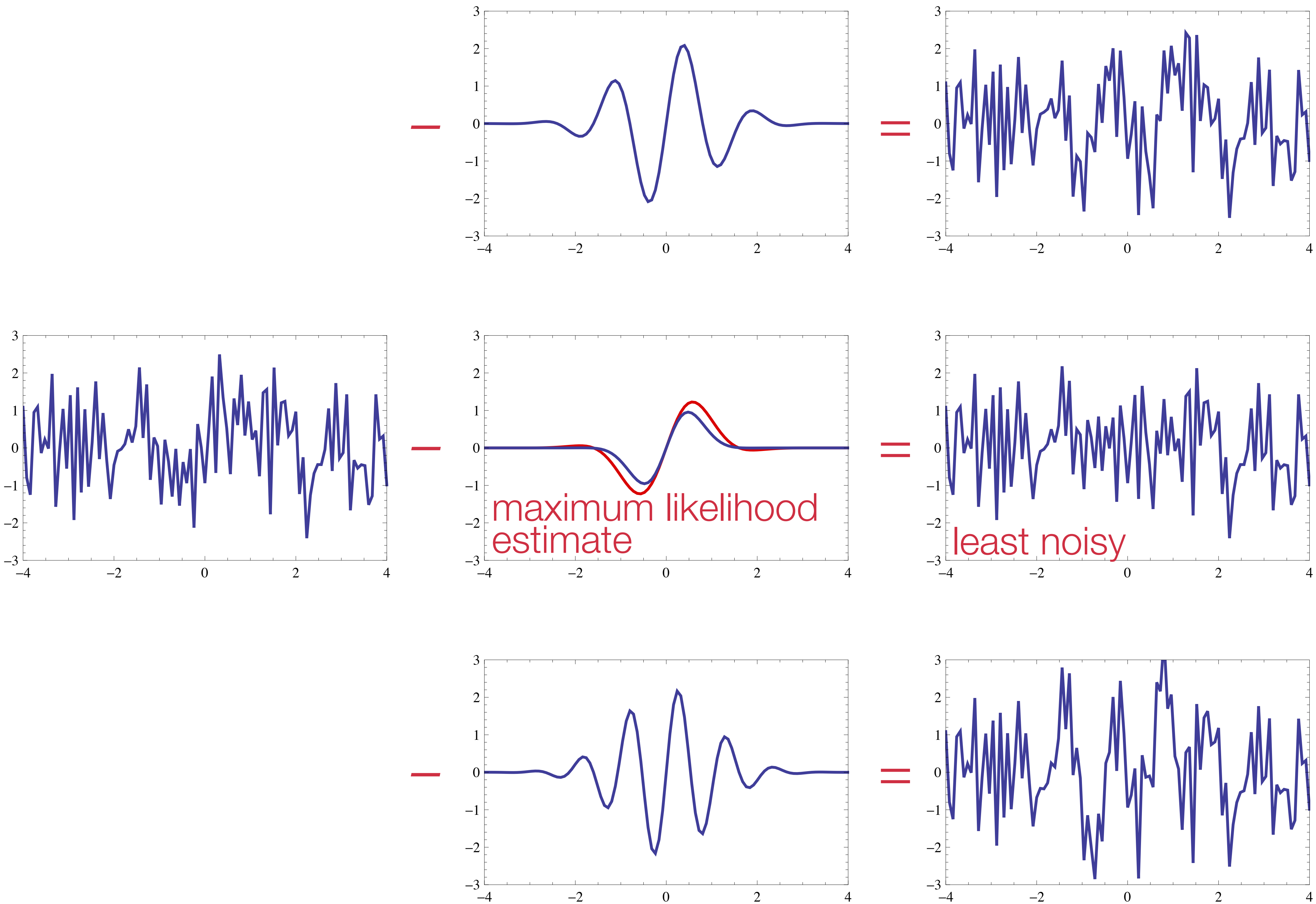
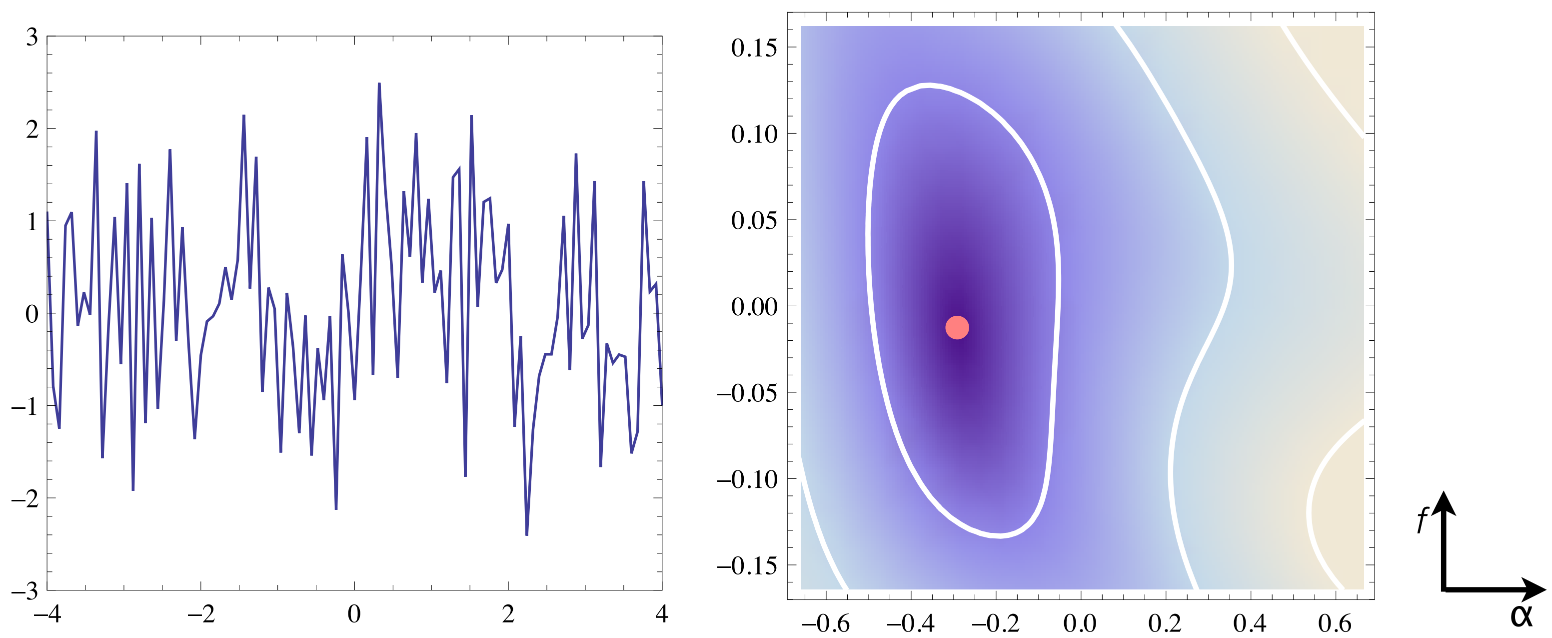


for a single noise realization, we make a likelihood map...



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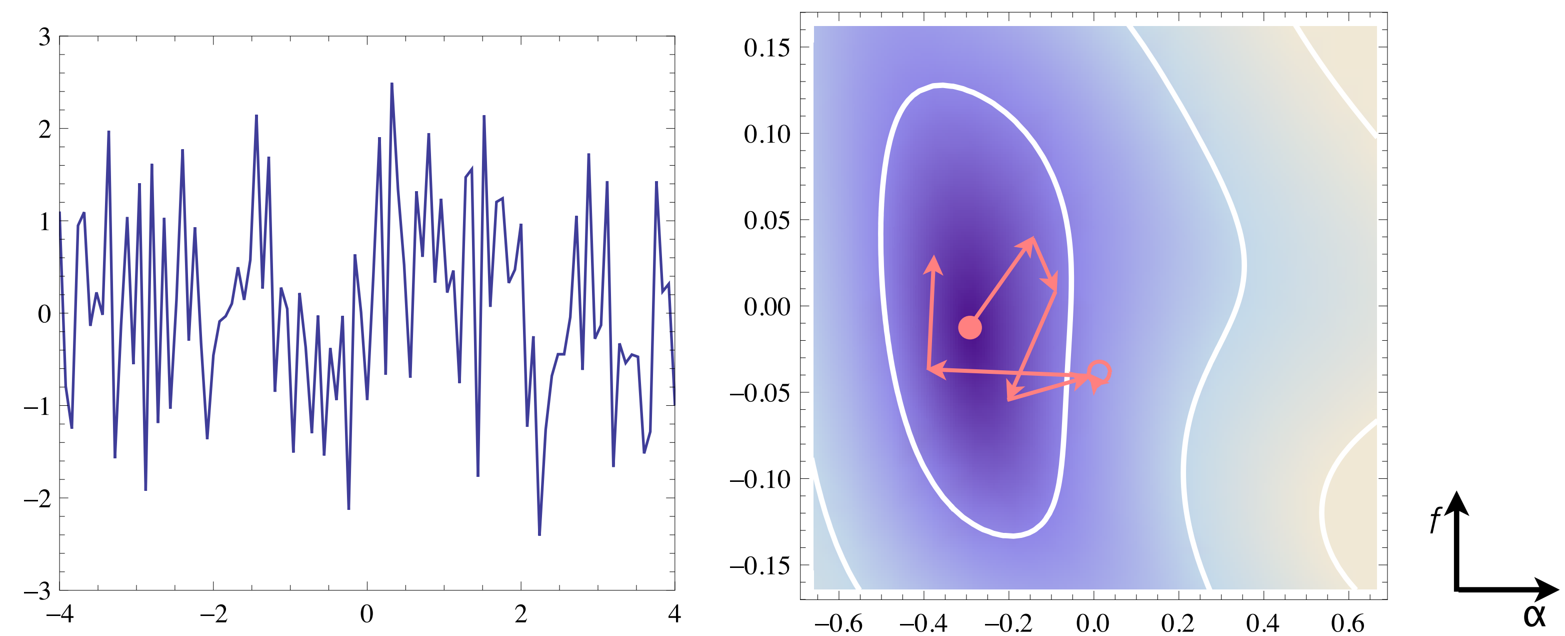
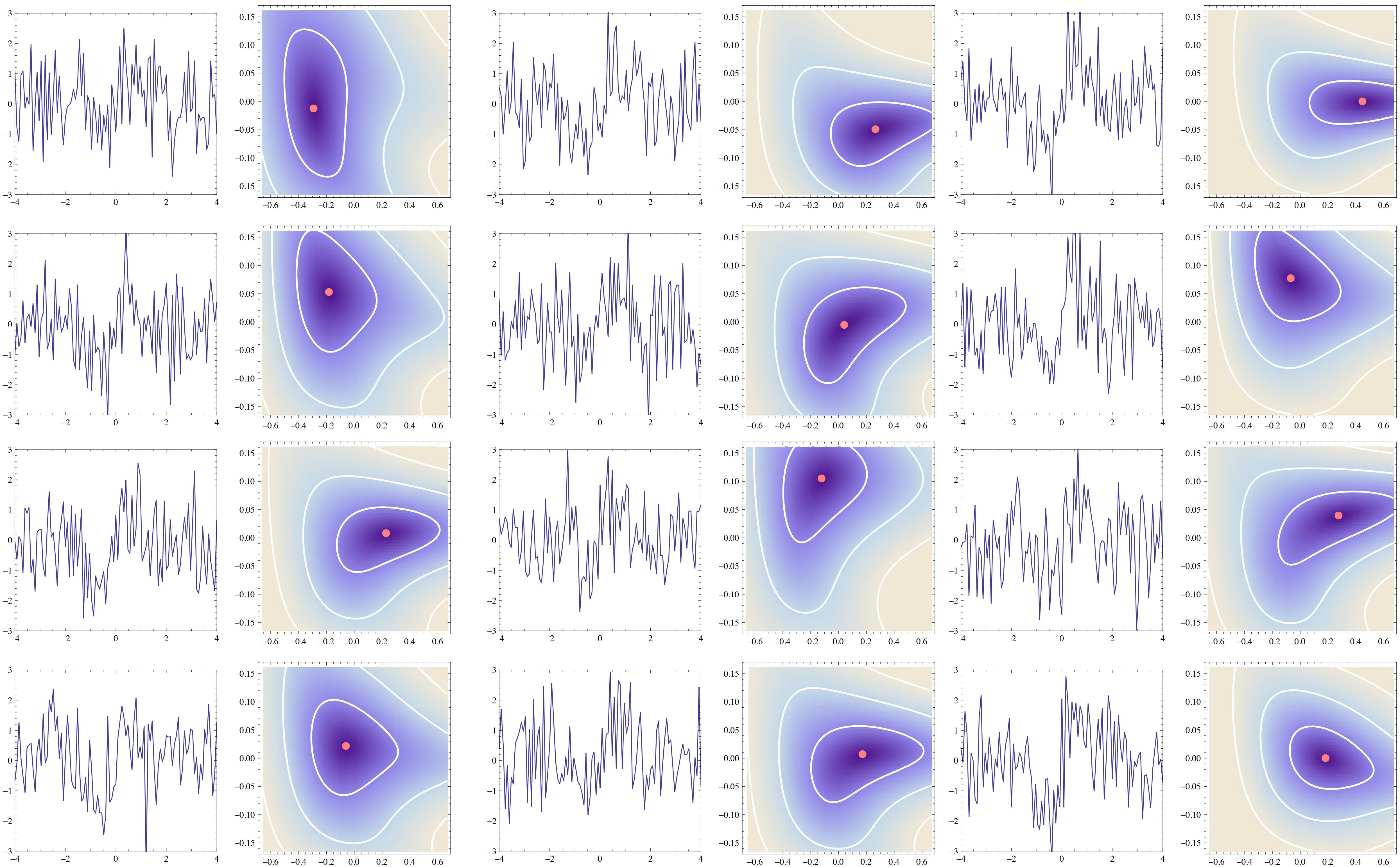


Markov-chain Monte Carlos
explore probability density
with a controlled random walk

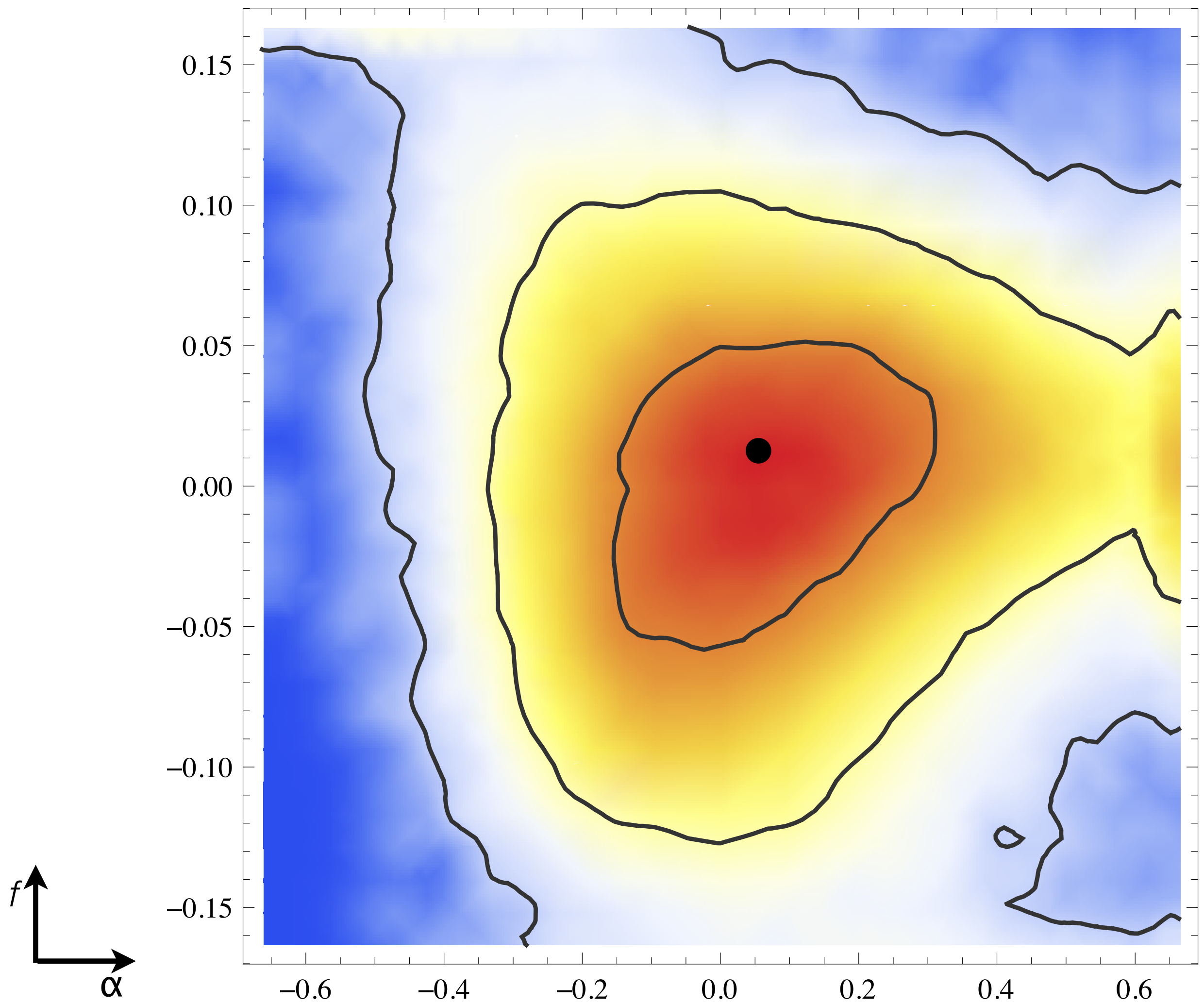
the Holy Grail: a Monte Carlo of Monte Carlos

[10^6 maps x 10^6 parameter sets x 10^6-point FFT = exaproblem]



Monday, June 11, 2012

not as holy, still a challenge: map the distribution of the maximum-likelihood estimator (●)



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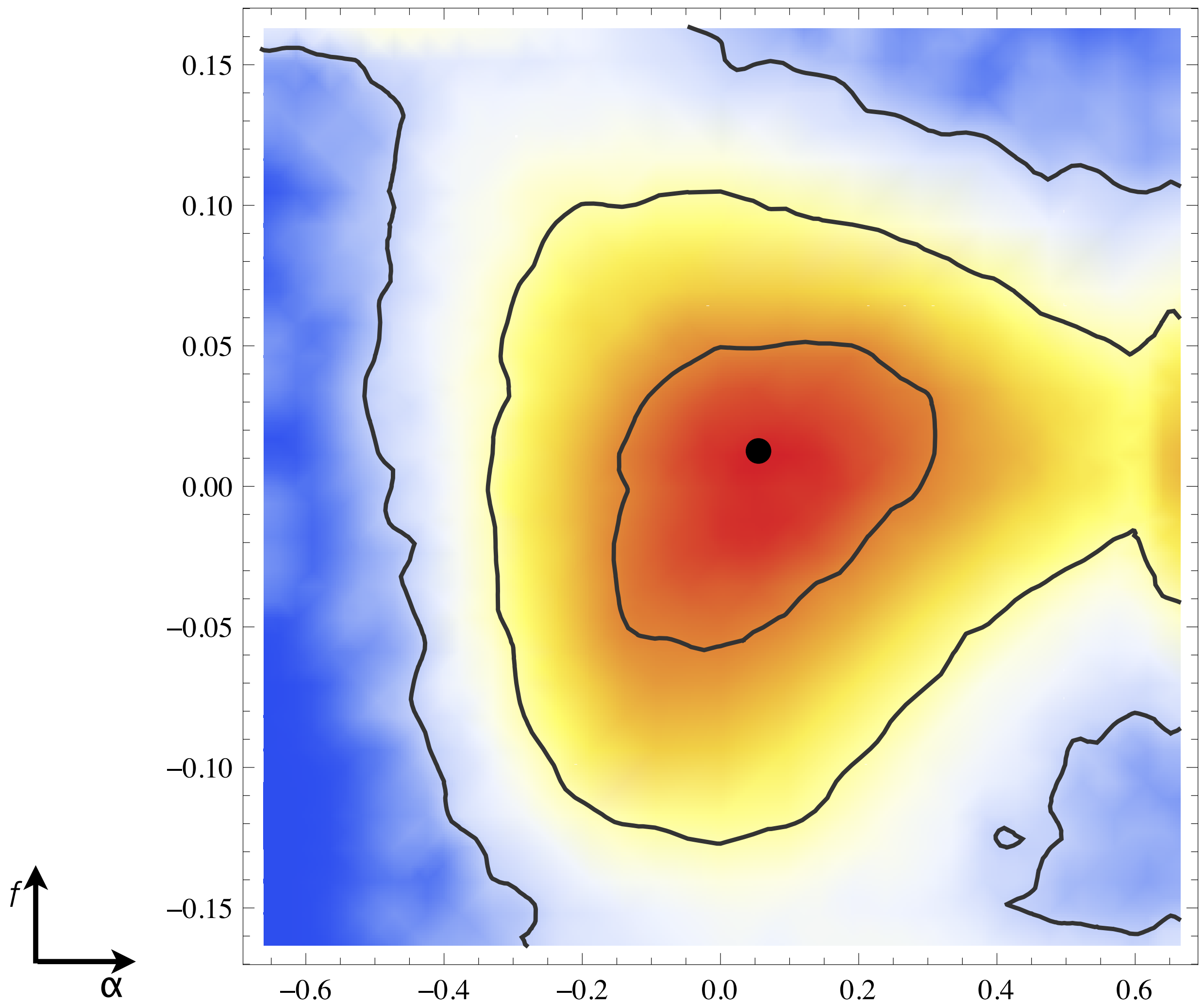

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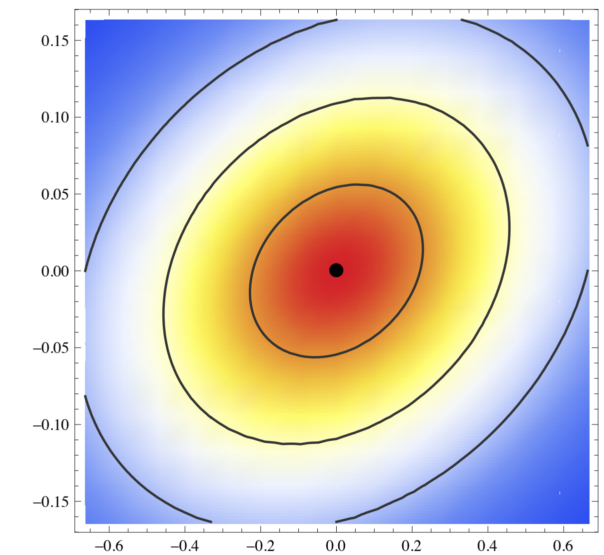

Fisher-matrix "prediction"

$$p(\Delta\theta) = \frac{e^{-\Delta\theta^i F_{ij} \Delta\theta^j / 2}}{\sqrt{(2\pi)^d |F_{ij}^{-1}|}}$$

$$F_{ij} = (\partial_i h, \partial_j h)$$

aside: how you check if you can use the Fisher matrix!

[MV, PRD 77, 042001 (2008)]

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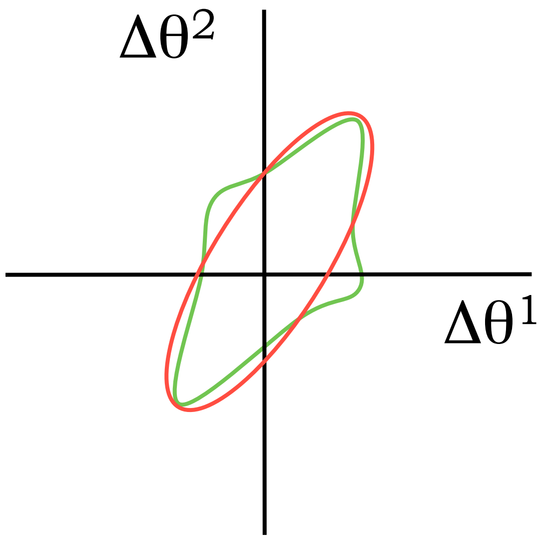
- explore the 1σ probability surface predicted by the Fisher matrix
- evaluate the ratio of the exact likelihood to the linearized likelihood

$$|\log r(\theta, A)| = \left(\theta_j h_j - \Delta h(\theta), \theta_k h_k - \Delta h(\theta)\right)/2$$



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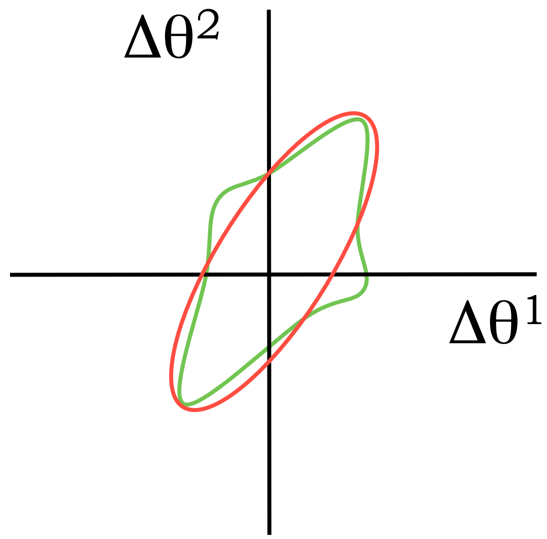
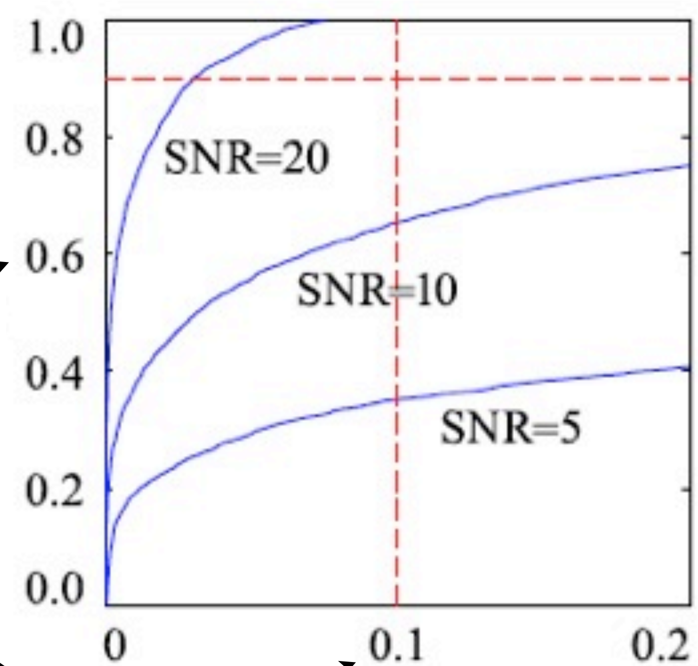


BNS inspiral



fraction of 1σ surface with likelihood exact to

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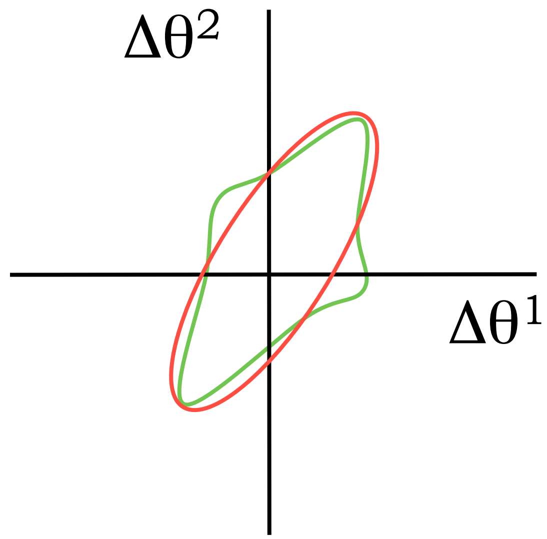

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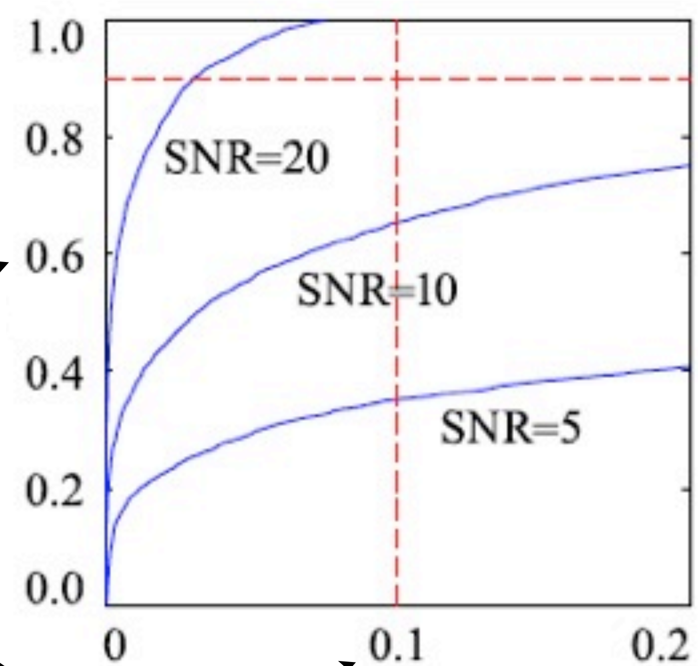
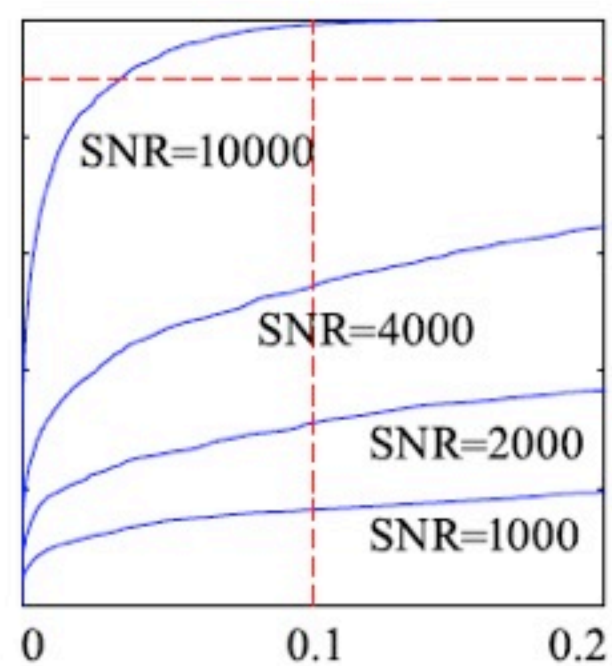


+ 2 spin parameters



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let's find the distribution of the ML estimator from first principles:

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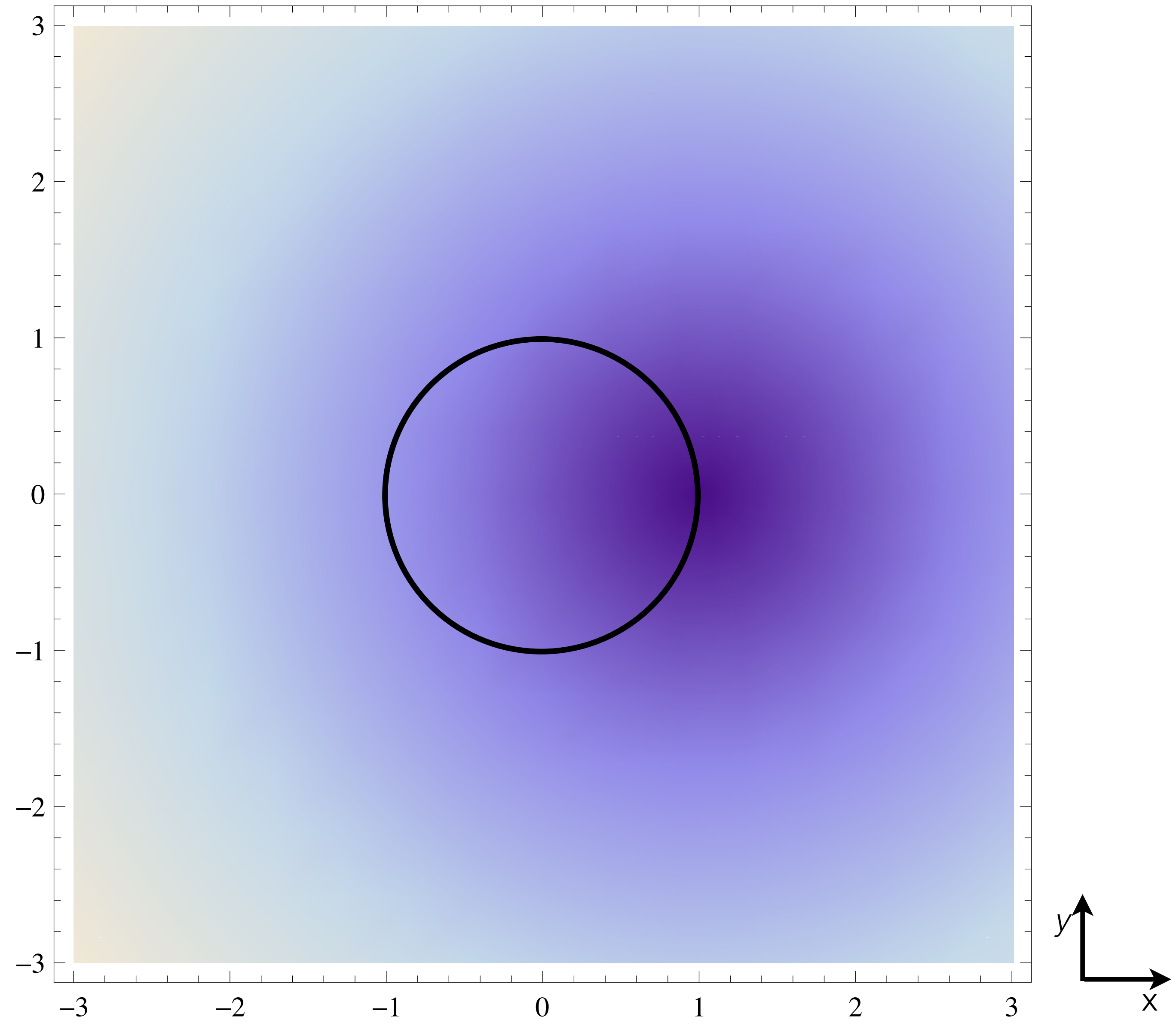
- thus: $$p(\theta) = \mathcal{N} \int \delta(\mathrm{ML}_1(n)) \cdots \delta(\mathrm{ML}_d(n))\, |\partial \mathrm{ML}_i / \partial \theta_j|\, e^{-(n,n)/2} dn$$

  which is really an integral over ~ d^2, not N, dimensions!

for instance, with one parameter and two noise dimensions:

$$h(\theta) = (\cos\theta, \sin\theta)$$
$$\theta_{\text{true}} = 0$$
$$s = (n_x + 1, n_y)$$
$$p(n) = \mathcal{N} \exp -(n_x^2 + n_y^2)/2$$



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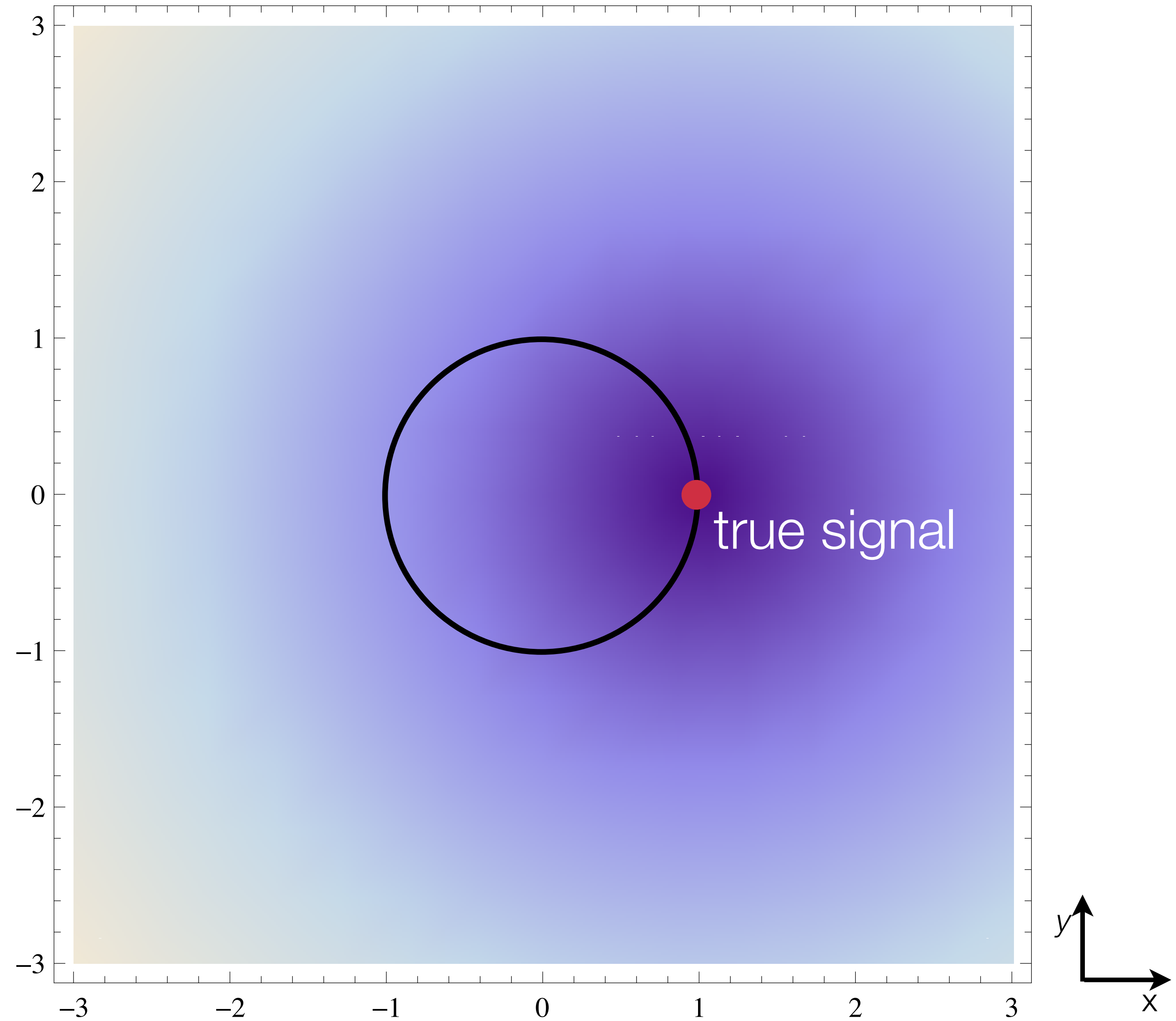

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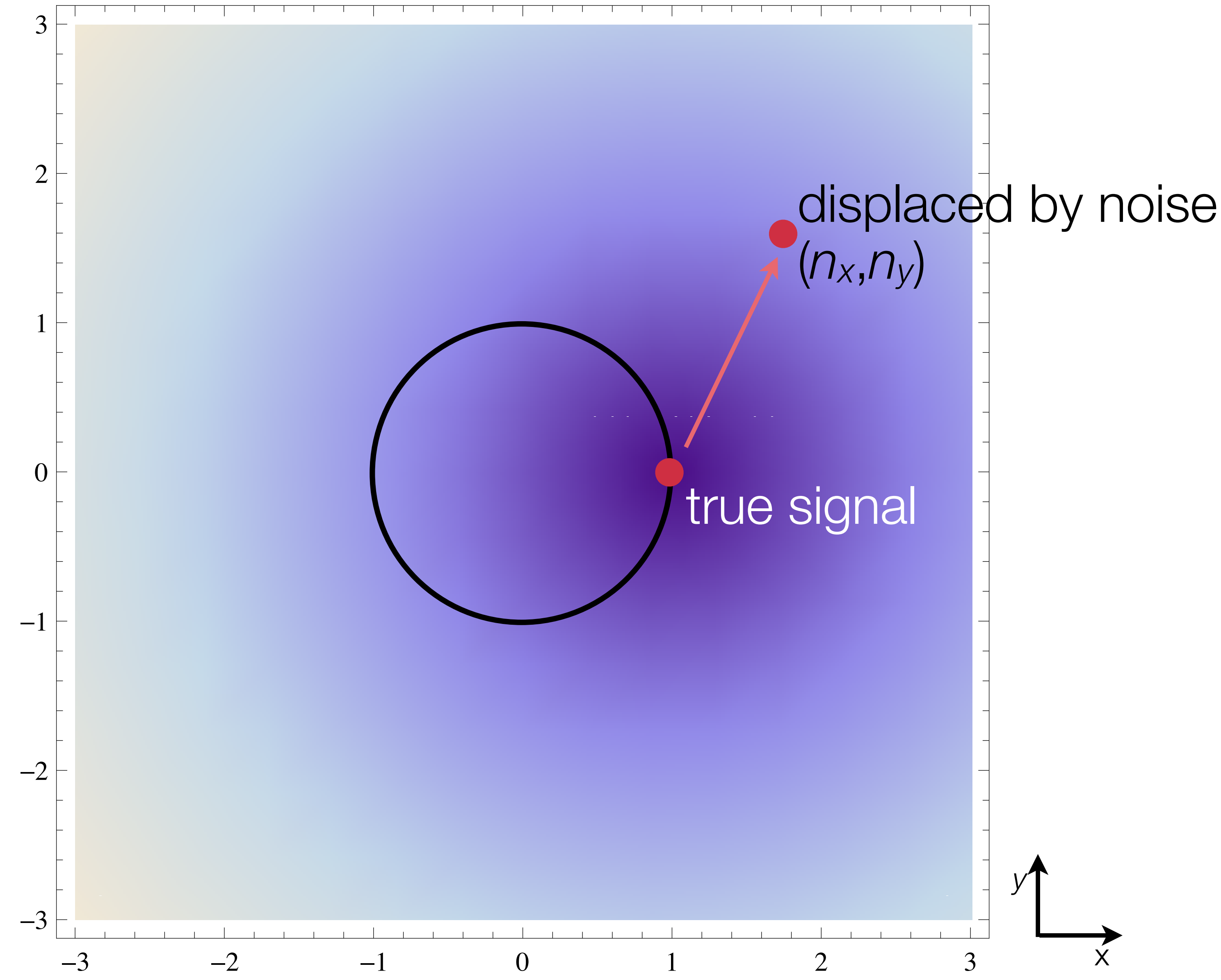

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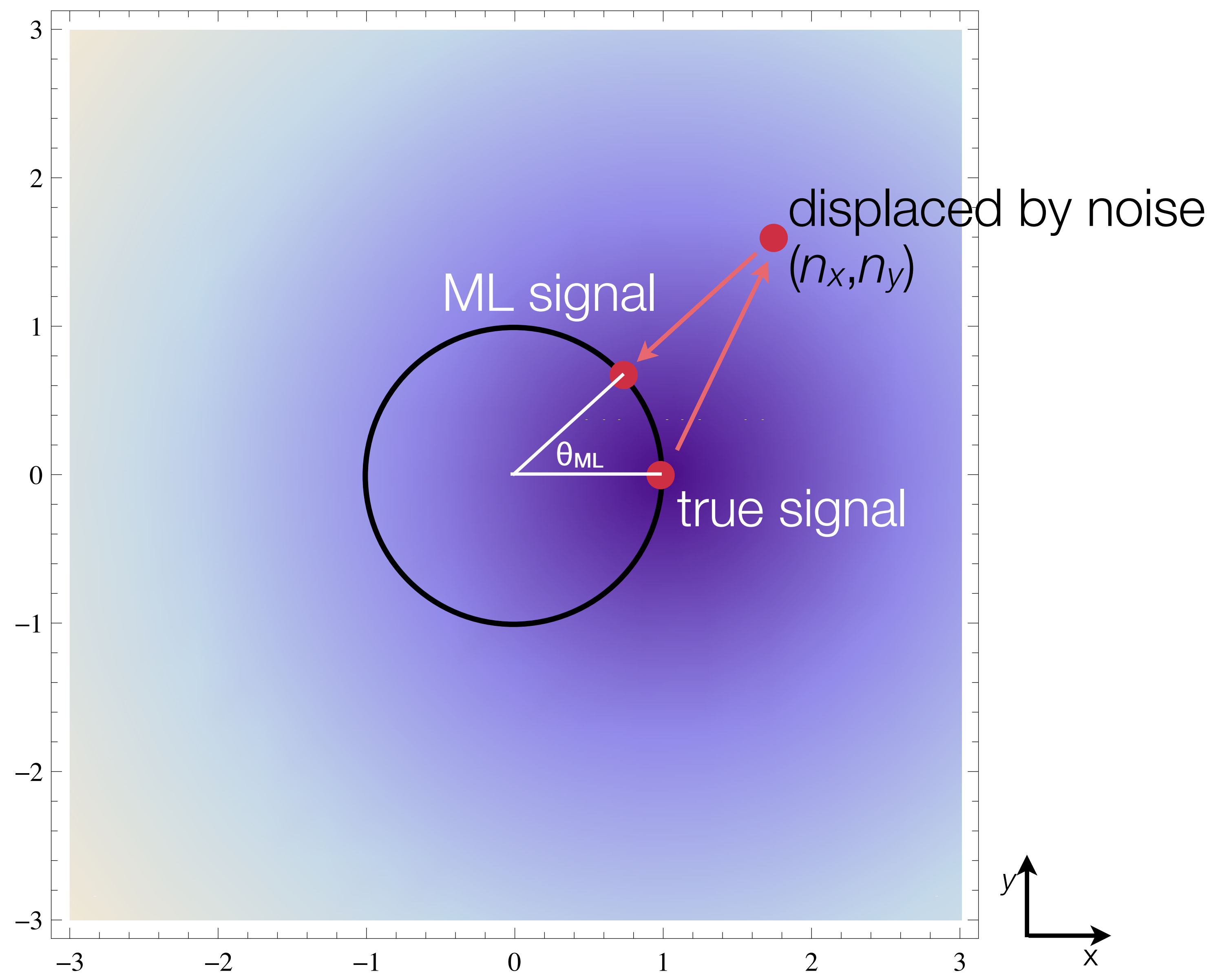

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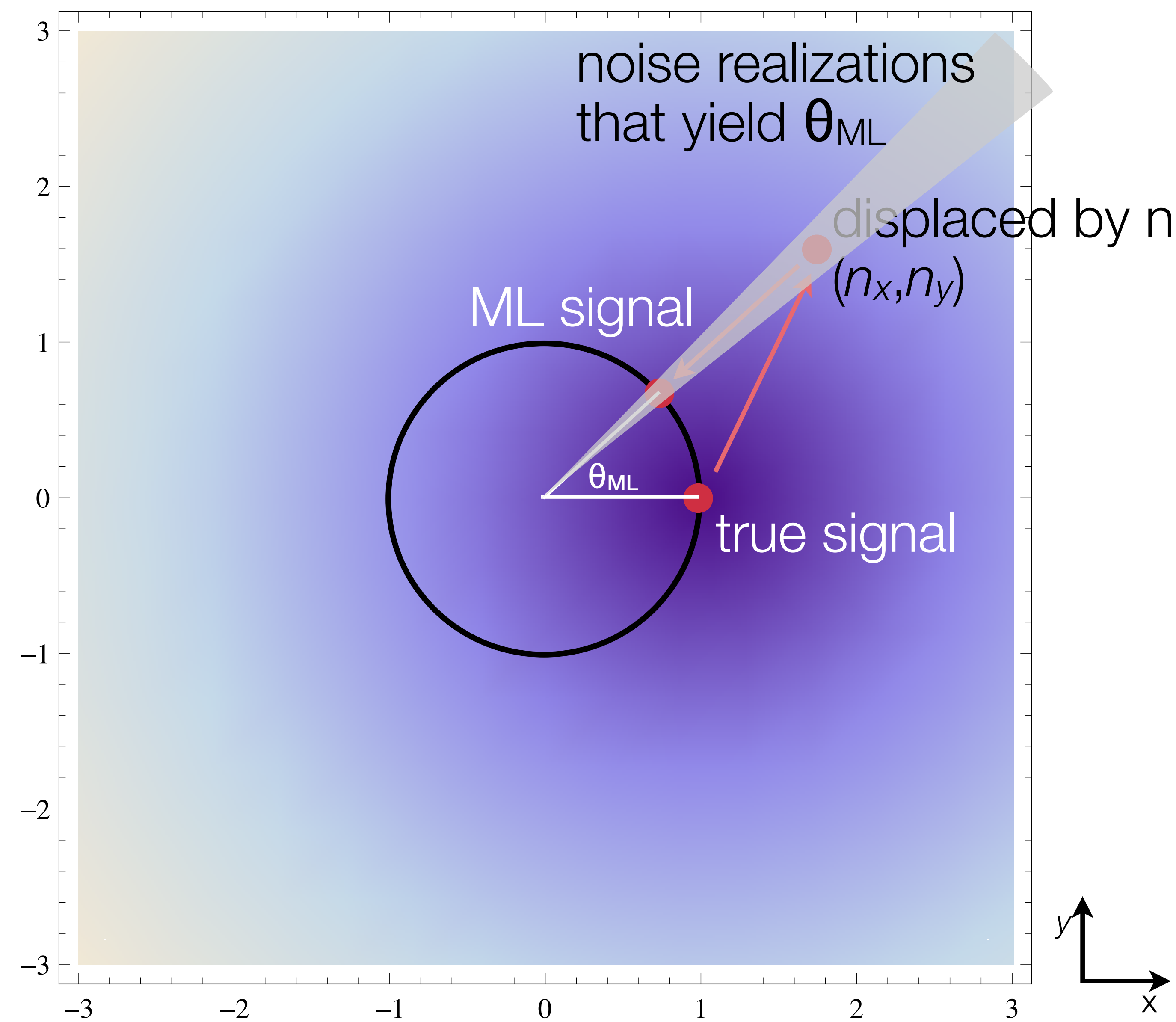

$$p(\theta) = \mathcal{N} \int \delta(\mathrm{ML}_1(n)) \cdots \delta(\mathrm{ML}_d(n)) \, |\partial \mathrm{ML}_i / \partial \theta_j| \, e^{-(n,n)/2} dn$$

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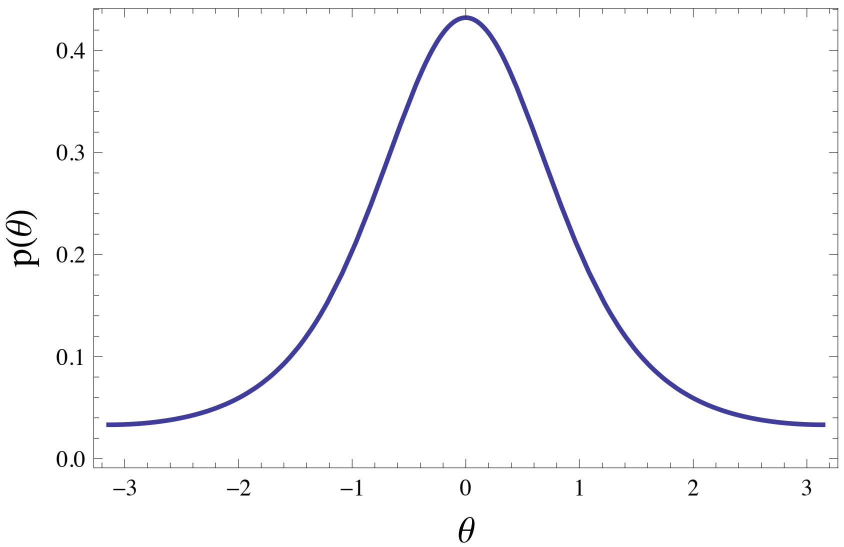

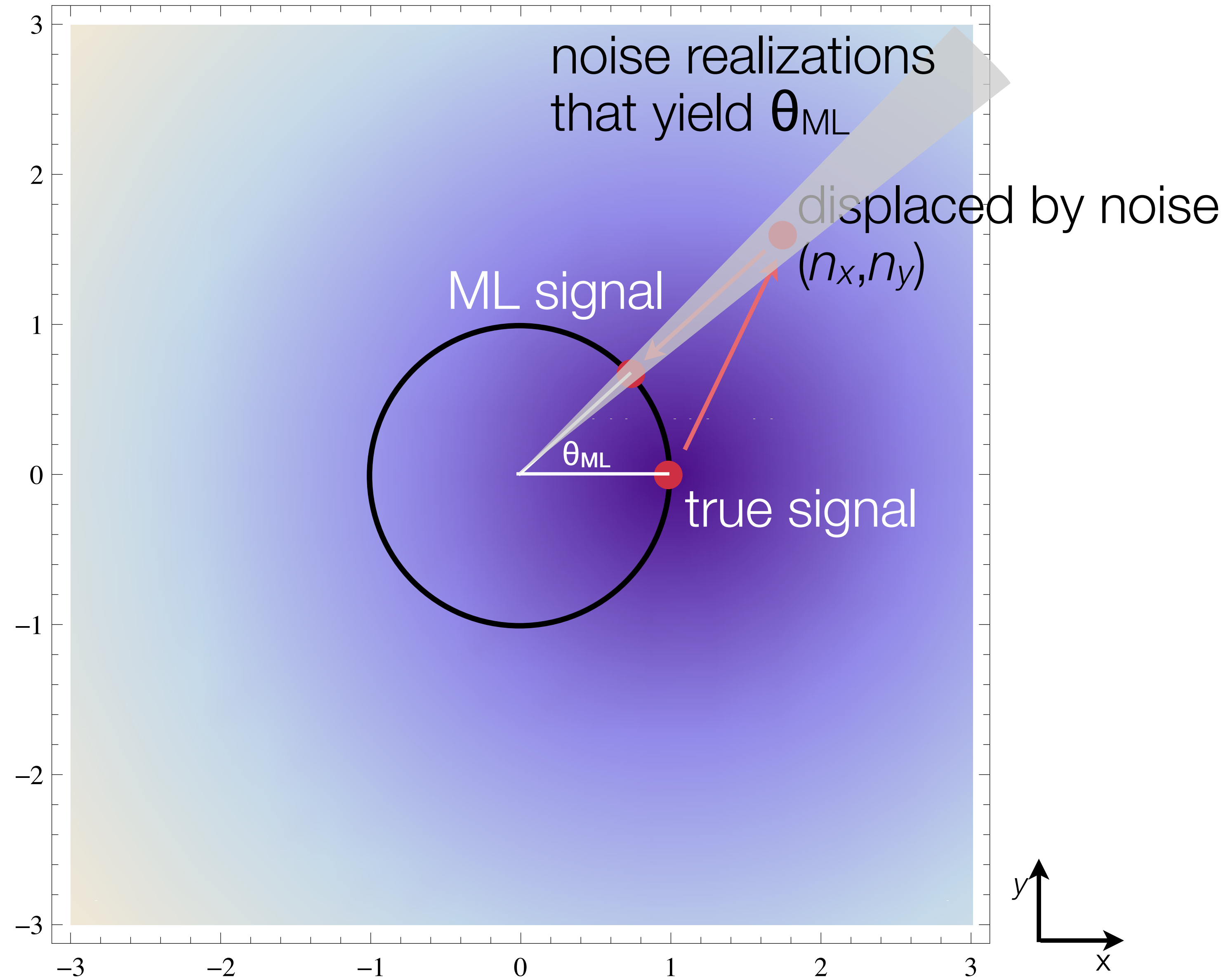

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- the inner product in the noise sampling probability can be written with respect to any basis...

$$p(n) = \mathcal{N} \exp -(n, n)/2$$
$$= \mathcal{N} \exp\Big\{ -\sum_k N^k N_k/2 \Big\}$$

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- the inner product in the noise sampling probability can be written with respect to any basis...
- ...such as one that **orthonormalizes** the $\partial_i h$ and $\partial_{ij} h$ (needed for $\partial \mathrm{ML}_i / \partial \theta_j$):

$$p(n) = \mathcal{N} \exp -(n, n)/2$$
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$$\hat{n}_1 \propto h_1,$$
$$\hat{n}_2 \propto (1 - \hat{n}_1 \otimes \hat{n}_1) h_2,$$
$$\ldots$$
$$\hat{n}_{d+1} \propto (1 - \hat{n}_1 \otimes \hat{n}_1 - \cdots) h_{11},$$
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- the integrand is a function only of the first $d + d(d+1)/2$ coefficients N^k (i.e., the projections of the noise over the first and second signal derivatives); all the others integrate out

$$n = N^k \hat{n}_k,$$
$$h_i = C_i^k \hat{n}_k,$$
$$h_{i,j} = C_{ij}^k \hat{n}_k,$$
$$\mathrm{ML}_i = C_i^k N_k - C_i^k (\Delta h, \hat{n}_k),$$
$$\mathrm{ML}_{i,j} = C_{ij}^m N_m - C_{ij}^m (\Delta h, \hat{n}_m) - C_i^k C_{jk}$$

$$\Delta h = h(\theta) - h(\theta_{\mathrm{true}})$$

let's show that $p(\theta) = \mathcal{N} \int \delta(\mathrm{ML}_1(n)) \cdots \delta(\mathrm{ML}_d(n)) \, |\partial \mathrm{ML}_i / \partial \theta_j| \, e^{-(n,n)/2} dn$

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$$\begin{aligned} n &= N^k \hat{n}_k, \\ h_i &= C_i^k \hat{n}_k, \\ h_{i,j} &= C_{ij}^k \hat{n}_k, \\ \mathrm{ML}_i &= C_i^k N_k - C_i^k (\Delta h, \hat{n}_k), \\ \mathrm{ML}_{i,j} &= C_{ij}^m N_m - C_{ij}^m (\Delta h, \hat{n}_m) - C_i^k C_{jk} \end{aligned}$$

let's show that $p(\theta) = \mathcal{N} \int \delta(\mathrm{ML}_1(n)) \cdots \delta(\mathrm{ML}_d(n)) \, |\partial \mathrm{ML}_i / \partial \theta_j| \, e^{-(n,n)/2} dn$

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- furthermore, the deltas satisfy integration over the first d N_ks:

"h_{ij}" degrees of freedom only!

$$p(\theta) = \frac{e^{-\Sigma_{k=1}^{d} (\Delta h, \hat{n}_k)^2/2}}{(2\pi)^{d(d+3)/4} \prod_{k=1}^{d} C_k^k} \int \left| C_{ij}^m N_m - C_{ij}^m (\Delta h, \hat{n}_m) - C_i^k C_{jk} \right| e^{-\Sigma_m (N_m)^2/2} dN_m$$

- the integrand is a function only of the first $d + d(d+1)/2$ coefficients N^k (i.e., the projections of the noise over the first and second signal derivatives); all the others integrate out

$$\begin{aligned} n &= N^k \hat{n}_k, \\ h_i &= C_i^k \hat{n}_k, \\ h_{i,j} &= C_{ij}^k \hat{n}_k, \\ \mathrm{ML}_i &= C_i^k N_k - C_i^k (\Delta h, \hat{n}_k), \\ \mathrm{ML}_{i,j} &= C_{ij}^m N_m - C_{ij}^m (\Delta h, \hat{n}_m) - C_i^k C_{jk} \end{aligned}$$

$\Delta h = h(\theta) - h(\theta_{\mathrm{true}})$

let's show that $p(\theta) = \mathcal{N} \int \delta(\mathrm{ML}_1(n)) \cdots \delta(\mathrm{ML}_d(n)) \, |\partial \mathrm{ML}_i / \partial \theta_j| \, e^{-(n,n)/2} dn$ is really low-dimensional!

- furthermore, the deltas satisfy integration over the first d N_ks:

"h_{ij}" degrees of freedom only!

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⇓

$$p(\theta) = \frac{e^{-(\Delta h, h_i)(F^{-1})^{ij}(\Delta h, h_j)/2}}{\sqrt{(2\pi)^d |F_{ij}|}} \times \frac{1}{\sqrt{(2\pi)^{d(d-1)/2} |D_{\mu\nu}|}} \int \left| F_{ij} + (\Delta h, h_{ij}) - M_{(ij)} \right| e^{-M_\mu (D^{-1})^{\mu\nu} M_\nu / 2} dM_\mu$$

- ...where the $M_\mu \equiv M_{ij}$ are normal random variables with covariance matrix given by $D_{\mu\nu} \equiv (\tilde{h}_{ij}, \tilde{h}_{kl})$, where ~ denotes projection orthogonal to the h_k

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- ...where the $M_\mu \equiv M_{ij}$ are normal random variables with covariance matrix given by $D_{\mu\nu} \equiv (\tilde{h}_{ij}, \tilde{h}_{kl})$, where ~ denotes projection orthogonal to the h_k
- this $d(d+1)/2$-dimensional integral is trivial numerically. All that's needed are F_{ij} and $D_{\mu\nu}$, which require ~ $d^4/8$ inner products

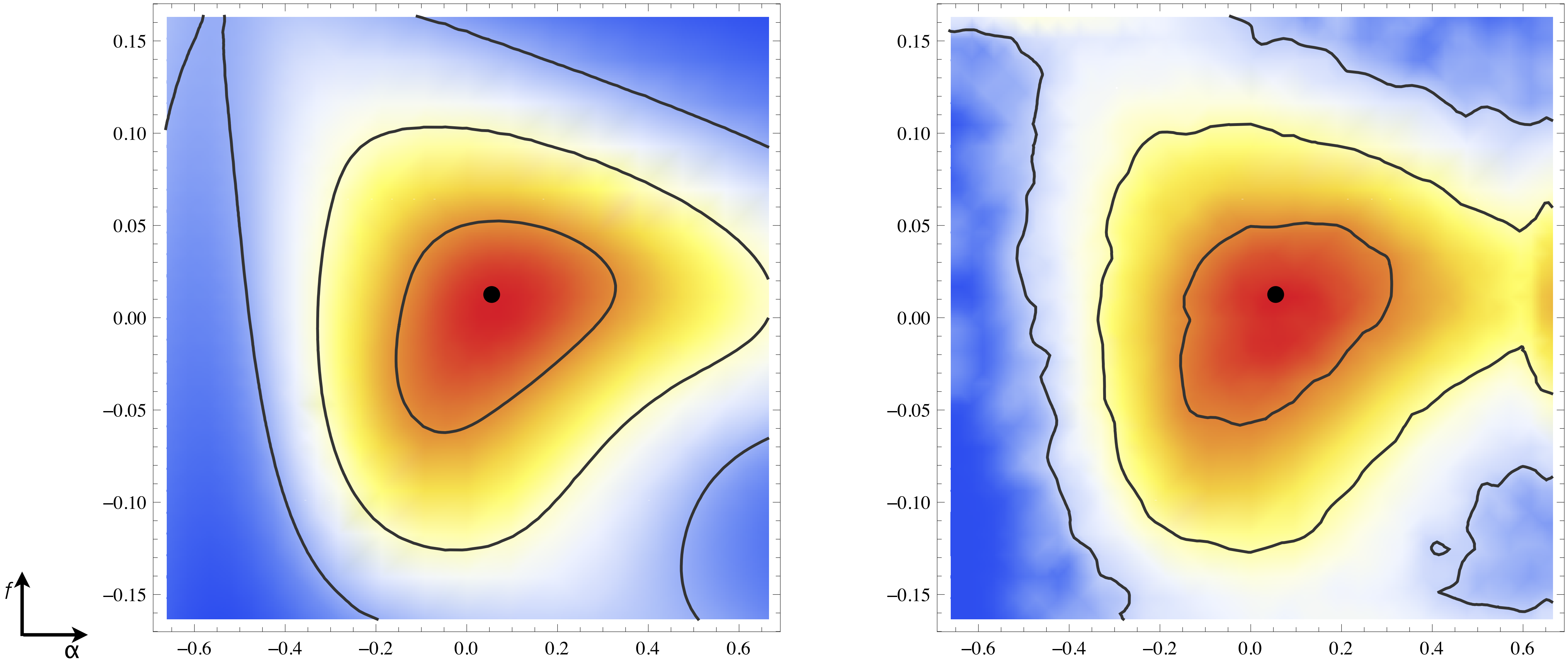
check: the leading-order Fisher-matrix result follows from neglecting the second derivatives of the signal

$$p(\theta) = \frac{e^{-(\Delta h, h_i)(F^{-1})^{ij}(\Delta h, h_j)/2}}{\sqrt{(2\pi)^d |F_{ij}|}} \times$$
$$\frac{1}{\sqrt{(2\pi)^{d(d-1)/2}|D_{\mu\nu}|}} \int \left| F_{ij} + \cancel{(\Delta h, h_{ij}) - M_{(ij)}} \right| e^{-M_\mu (D^{-1})^{\mu\nu} M_\nu /2} dM_\mu$$

⇩

$$p(\Delta\theta) = \frac{e^{-\Delta\theta^i F_{ij} \Delta\theta^j/2}}{\sqrt{(2\pi)^d |F_{ij}^{-1}|}}$$

check: evaluating the mapping integral recovers the distribution from the numerical search

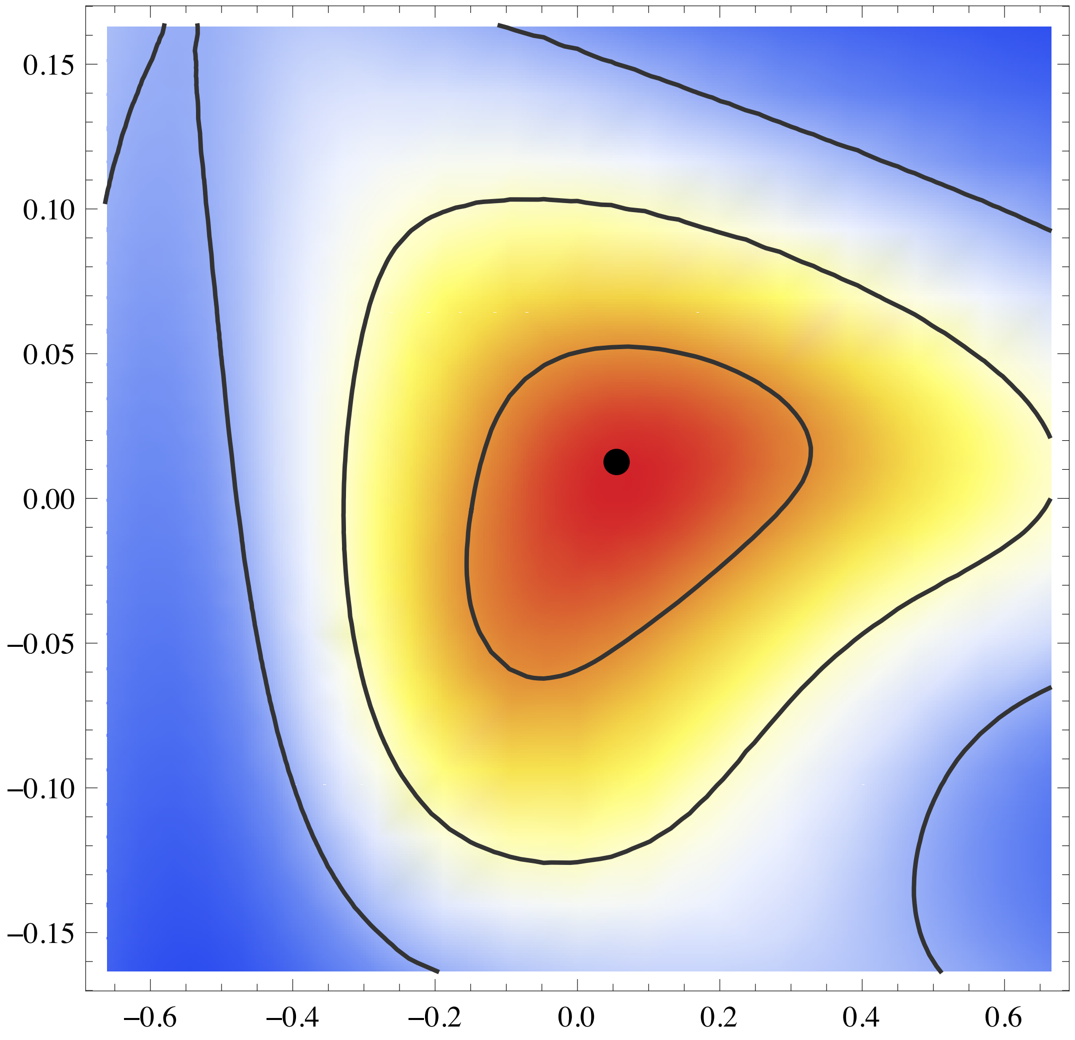


integral formula,
100x100 grid

numerical search,
100,000 noise realizations

go read about it and use it!

[MV, PRL 107, 191104 (2011)]



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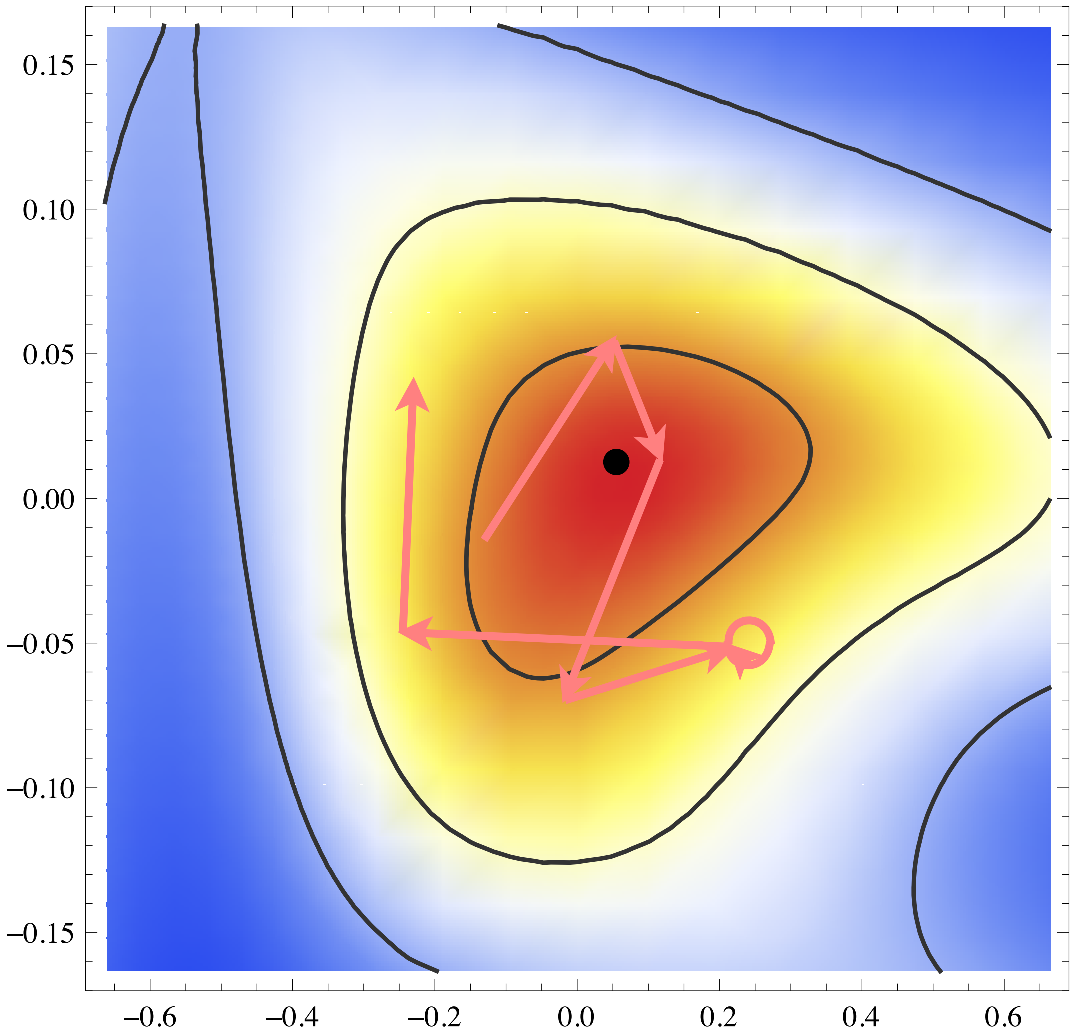

- in d dimensions, a Markov Chain can be used to sample p(θ)

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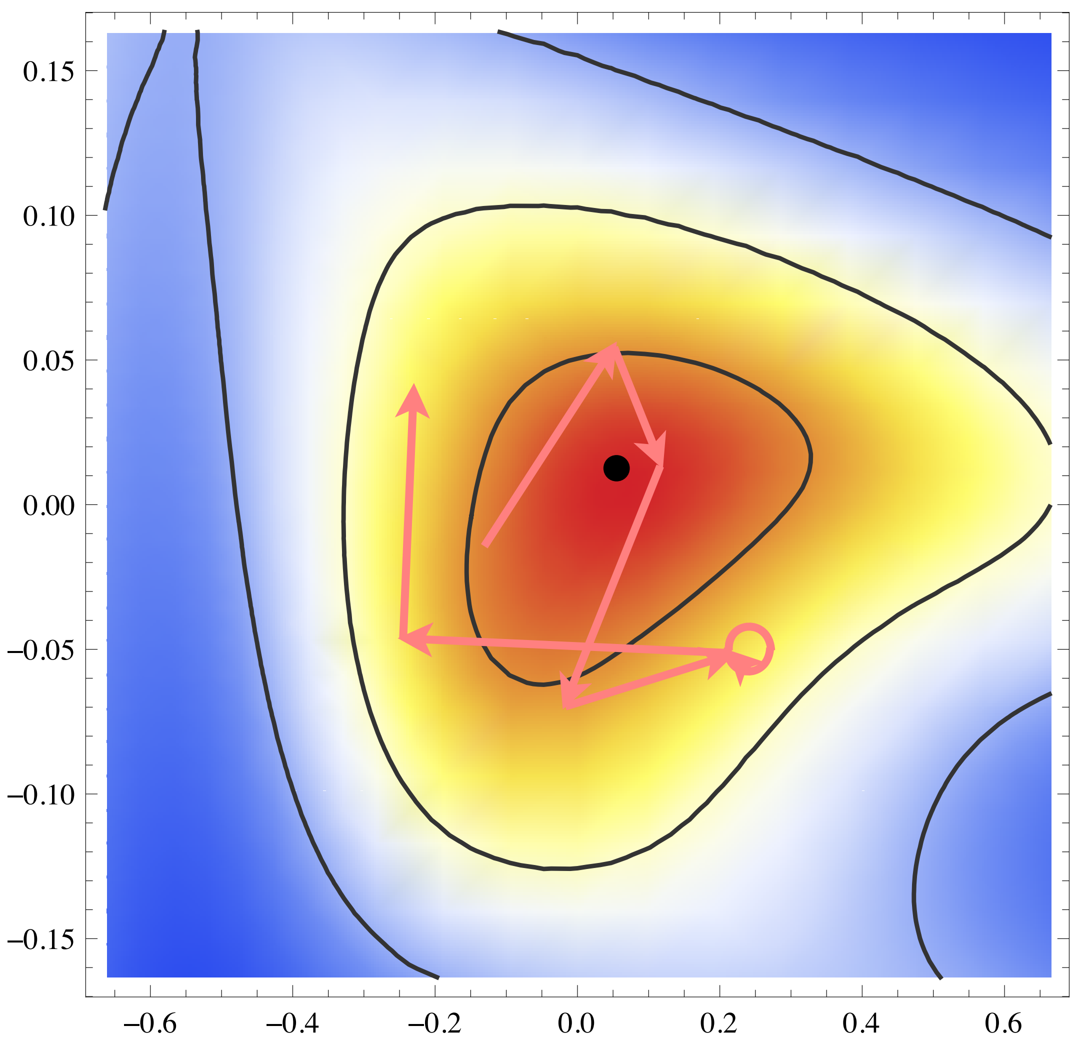
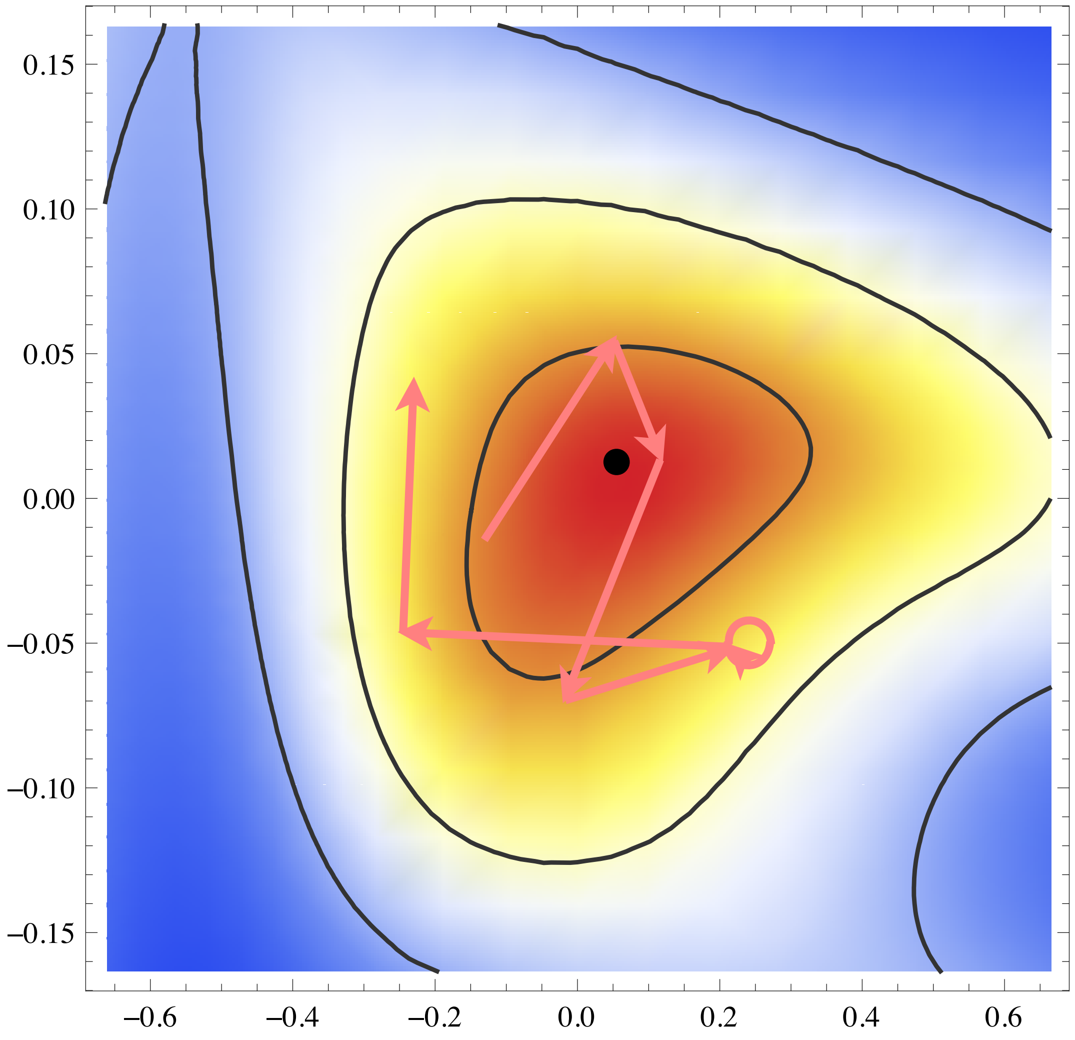


- in d dimensions, a Markov Chain can be used to sample p(θ)
- [10^6 samples x $d^4/8$ x 10^6-point FFT = 1000x cheaper for d = 10, still very parallelizable]

go read about it and use it!

[MV, PRL 107, 191104 (2011)]



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- this techniques generates exact estimates of the frequentist error for the ML estimator for any SNR

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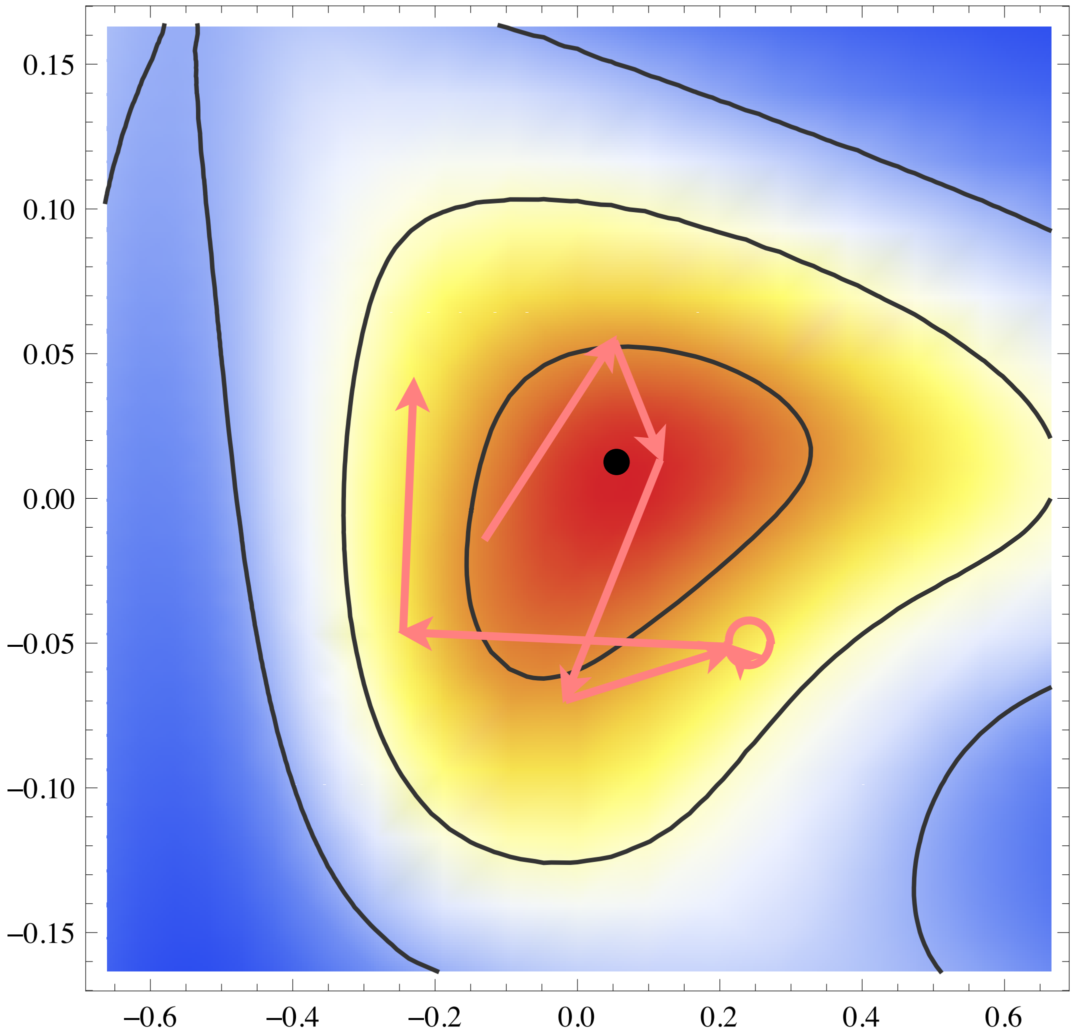

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- [10^6 samples x $d^4/8$ x 10^6-point FFT = 1000x cheaper for d = 10, still very parallelizable]
- this techniques generates exact estimates of the frequentist error for the ML estimator for any SNR
- the resulting distribution can be used to seed Bayesian-inference Monte Carlos (and will include isolated secondary maxima)

Between Fisher and Monte Carlo:

Mapping the distribution of the maximum-likelihood estimator for GW source parameters

Michele Vallisneri
Jet Propulsion Laboratory

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