Gravitational Wave Emission from Pulsars with Glitches

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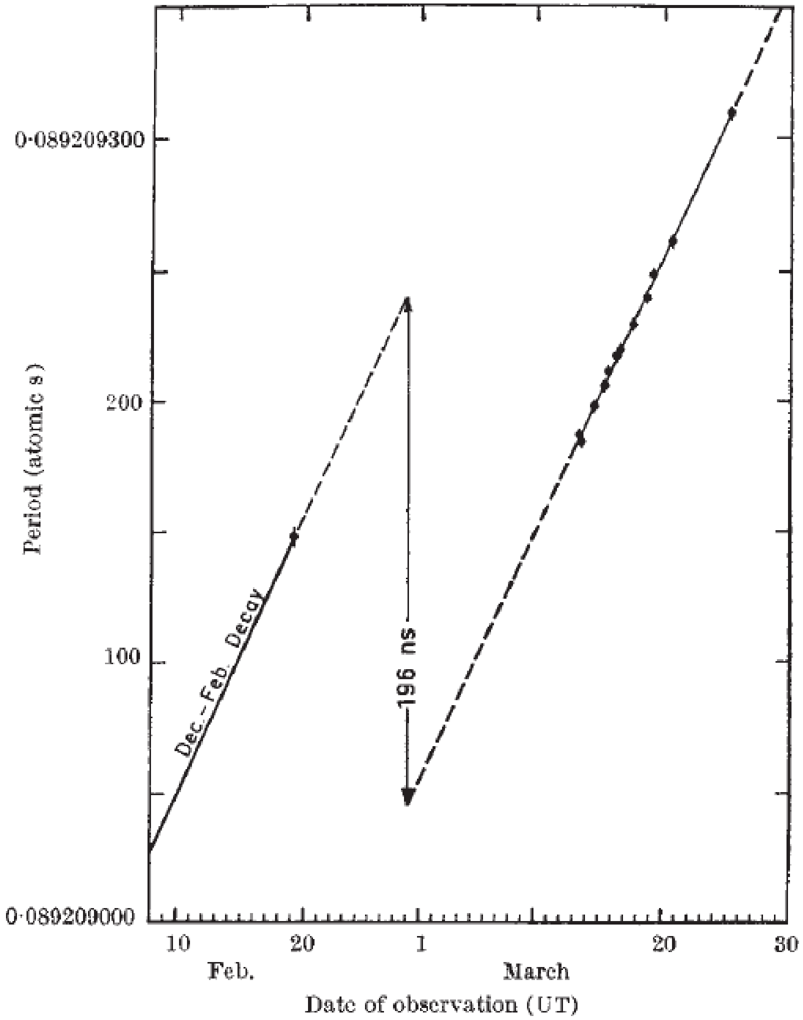
Pulsar Glitches

- Typical value of $\delta\nu/\nu$ is between 10^{-4} and 10^{-11}.
- Two possible mechanisms have been proposed
  - Star quake (Ruderman 1969)
  - Angular momentum transfer at the core (superfluid)-crust interface (Packard 1972; Anderson & Itoh 1975)
- Why are they so interesting? Because
  - They can be used to infer the neutron star's interior
  - They can give constraints of neutron star's equation of state
  - They also can excite some modes that can emit periodic gravitational waves.



Radharrishnan & Manchester (1969)

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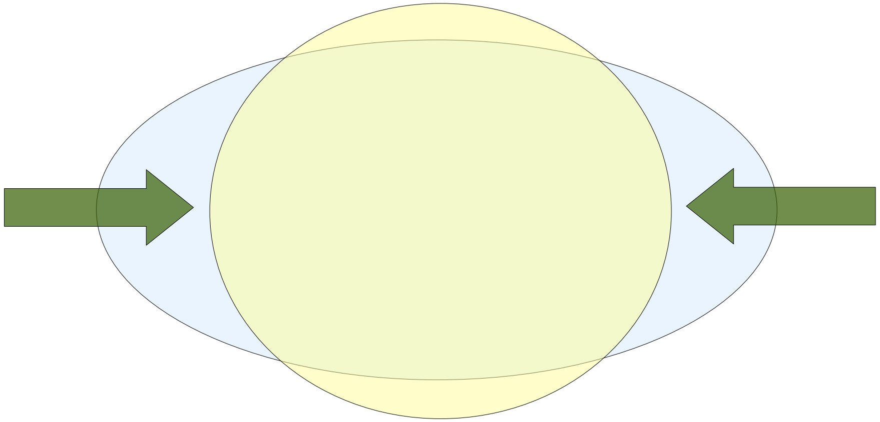
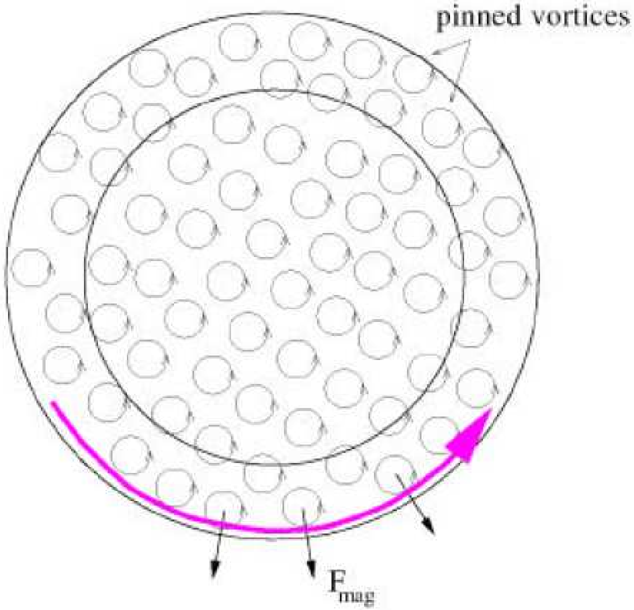


Star Quake Model



Vortex Unpinning Model

Previous Studies

- Pulsar glitch excites internal pulsation modes
  - Quasi radial mode (Sedrakian et. al. 2003, Sedrakian et. al. 2006)
  - f & p mode (Andersson & Comer 2001)
  - r-mode (Rezania & Jahan-Mari 2000)
  - meridional circulation (Bennett et. al. 2010)

Previous studies vs Our work

Linearized method	Hydrodynamical evolution
Amplitude of mode is obtained by the conservation of energy.	Amplitude can be evaluated directly. Energy is used for accuracy check.
Only 1 mode assume to be dominant.	Many different modes are excited simultaneously with different amplitudes.
Small size of glitch (realistic size) is possible.	Only large glitch size is possible. : extrapolation to smaller one

Method

I. Time evolution of rotating stars with perturbations which mimic pulsar glitches

II. Extraction of the time series of quadrupole moment

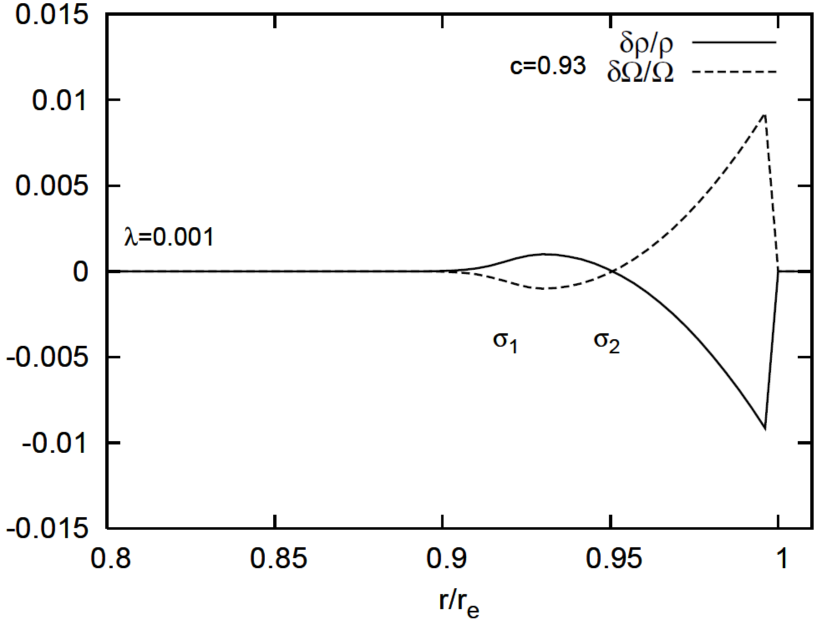
III. Fourier transformation

IV. Estimation of GW strain amplitude

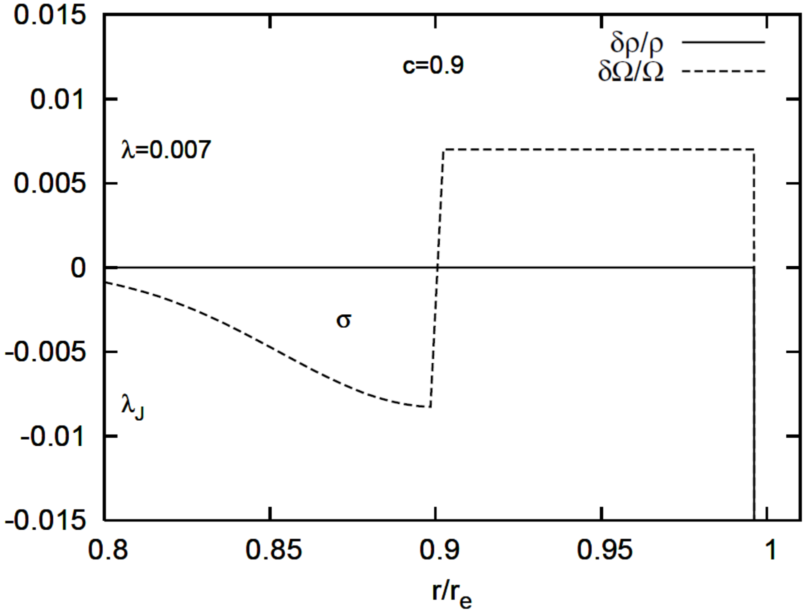
Imposed Perturbations

- We assume that
  - the depth of the neutron star's crust is 10% of its radius.
  - the effects of crust due to the hardness such as fractures are neglected.
- All perturbations should obey two constraints: total mass and total angular momentum conservations i.e.,

$$M_0=\int \rho_0 W\,dV=\text{constant}\,,\ J=\int T^0_\phi\,dV=\text{constant}\,.$$



1) Star Quake Model



2) Superfluid Model

Pseudo-Newtonian Approach

- Taking Newtonian limit

$$ds^2 = -(1+2\Phi)dt^2 + \frac{1}{1+2\Phi}\delta_{ij}dx^i dx^j$$

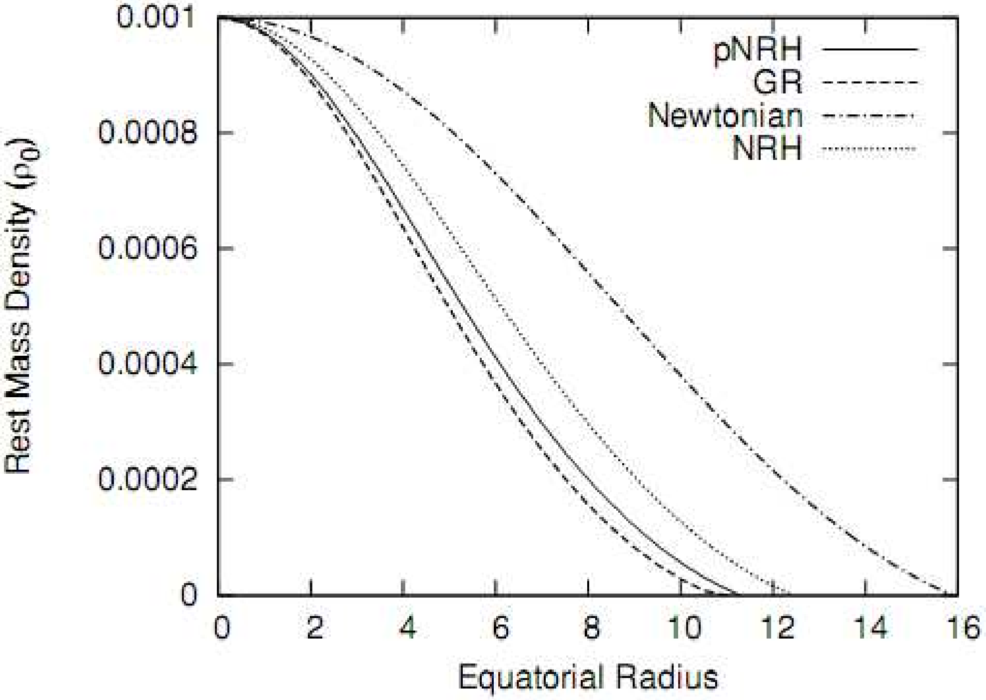
- If the metric is given, hydrodynamics equation can easily be written in standard formulation
- Einstein equation → 2nd order approximation of (v/c) → equation for gravitational potential : Poisson equation $\nabla^2\Phi = 4\pi\rho_{active}$
  - Note : source term in Poisson equation is 'Active Mass Density' not just baryon density or total mass density
  - Active mass density contains all forms of energy ingredients (baryon number density as well as enthalpy, pressure and velocity)

$$\rho_{\text{active}} = \rho_0 h \frac{1+v^2}{1-v^2} + 2\mathrm{P}$$

Pseudo-Newtonian Approach

- Density profile of the spheroidal and quasi-toroidal shape
  - $P = K\rho_0^{1+1/N}$ (N=1, K=100) and $\rho_{max} = 0.001$

Axis ratio = 0.75



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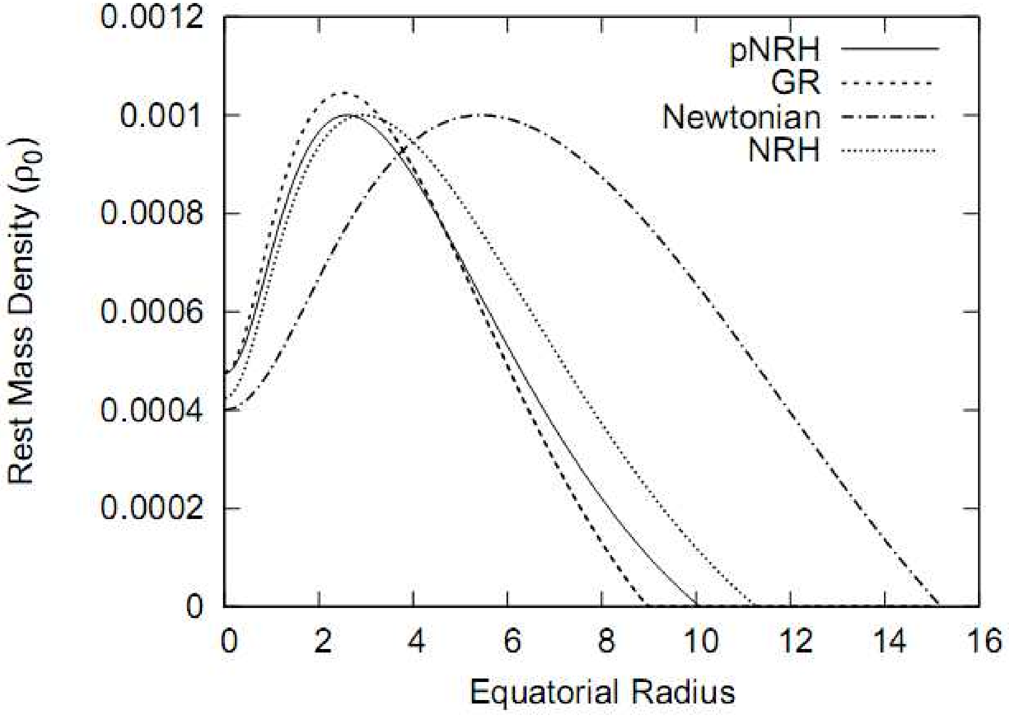

Kim, J., Kim, H. I. & Lee, H. M. (2009)

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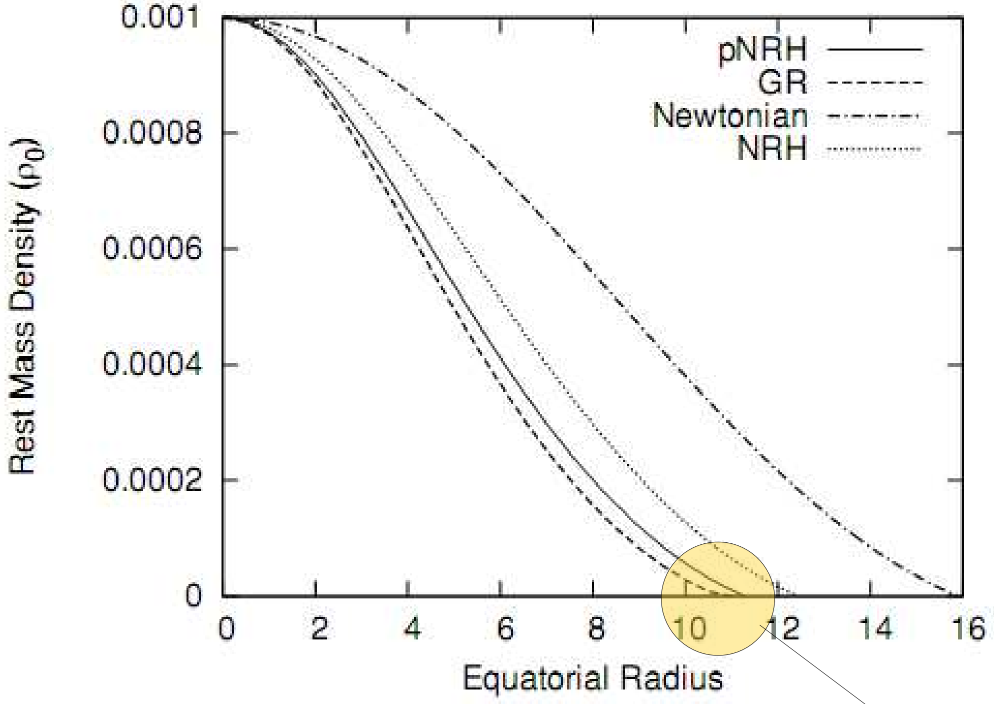

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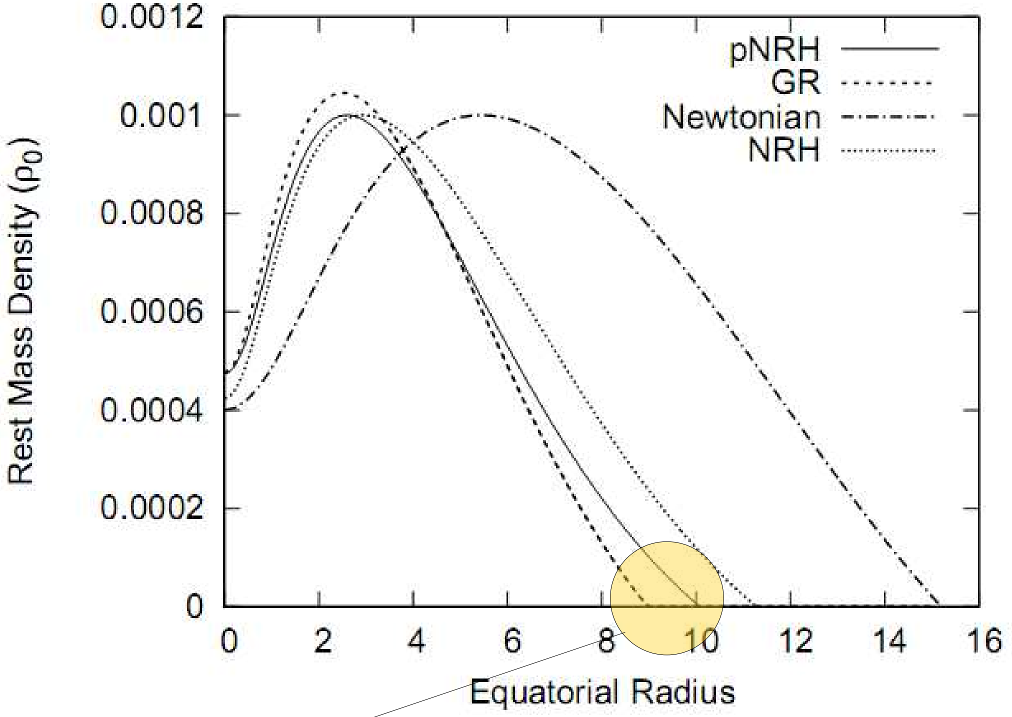

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Less than 5% difference!

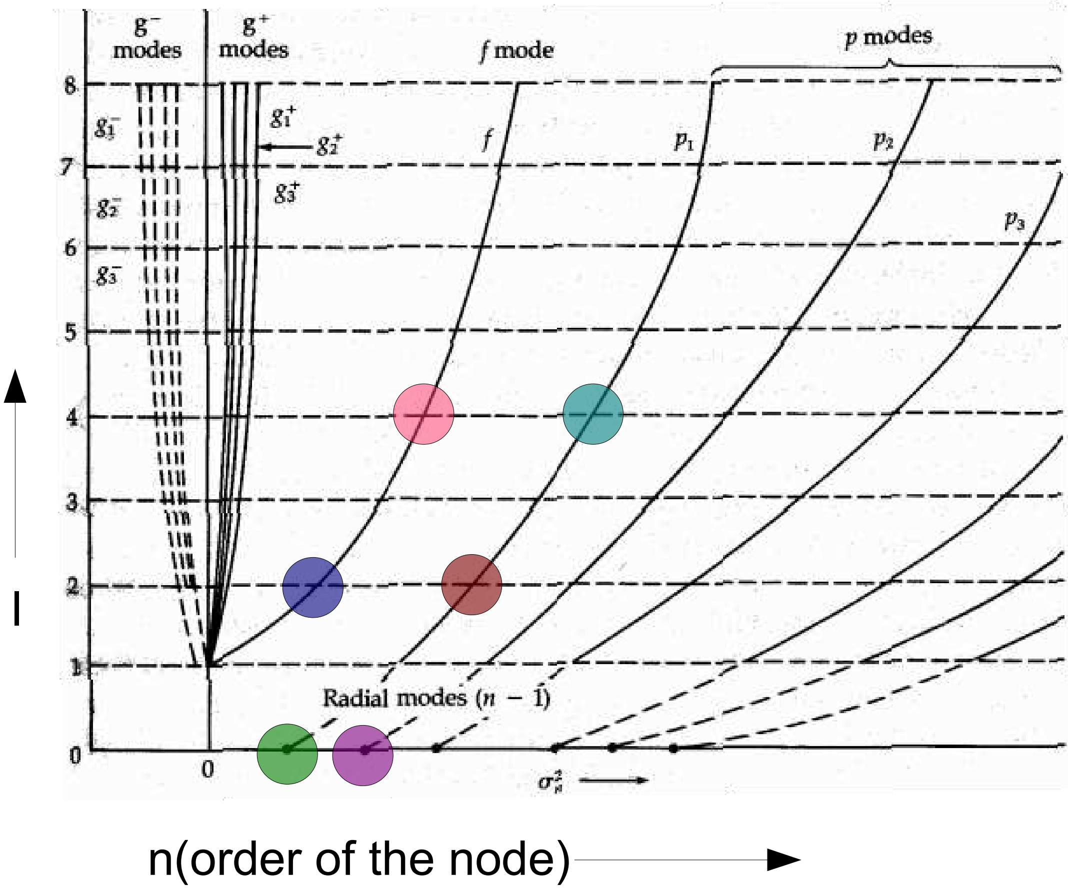
Mode Analysis & Excited Modes

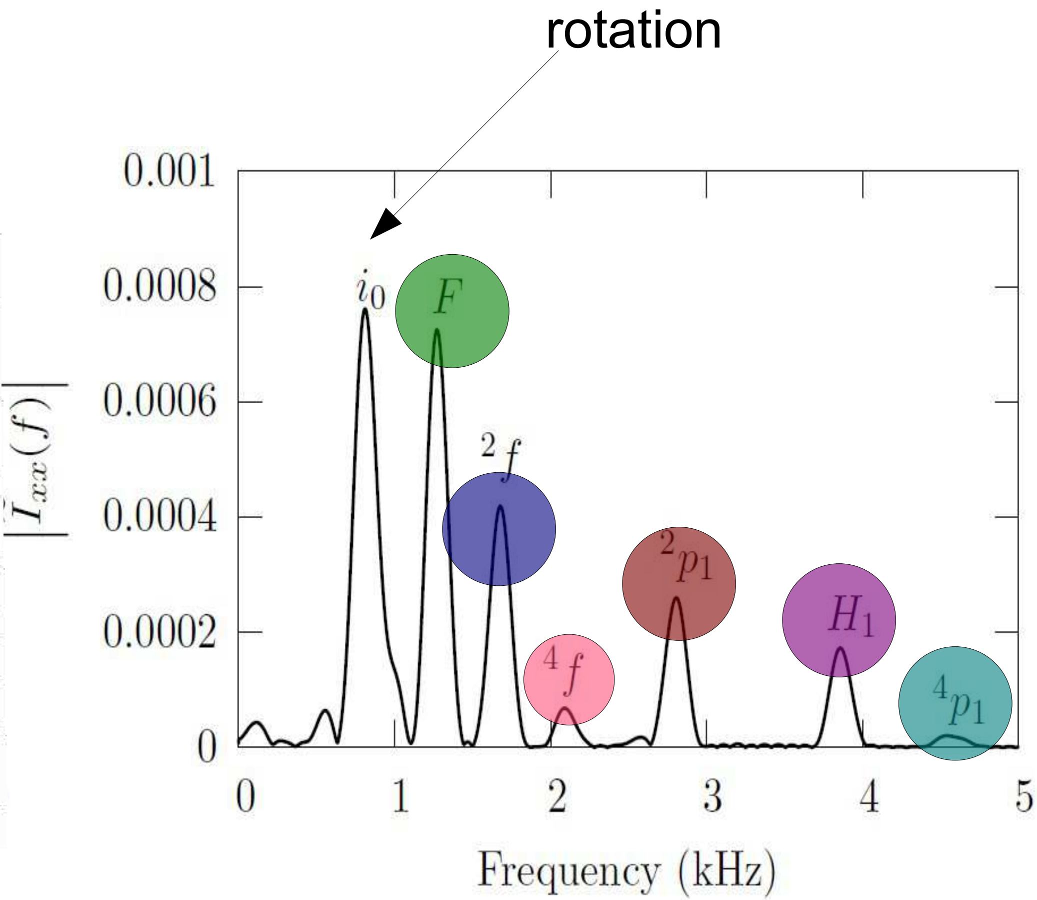
- In order to extract the mode which can produce gravitational wave, we use the time series of quadrupole moment in the simulations.
- The quadrupole moments in our approach are

$$I_{xx}=\int \rho\left(x^2-\frac{1}{3}r^2\right)dV \ , \quad I_{zz}=\int \rho\left(z^2-\frac{1}{3}r^2\right)dV \quad .$$

- To identify specific modes, we compare with the Newtonian and (approximated) general relativistic (Font et. al., 2001; Demmelmeier et. al., 2006; Yoshida & Eriguchi, 2000) ones.
- We also find out the eigenfunctions of modes by the mode recycling method.

Mode Identification





•Schematic view of the modes from Cox (1970)

- Sinusoidal amplitudes excited by perturbation 2 (superfluid model)

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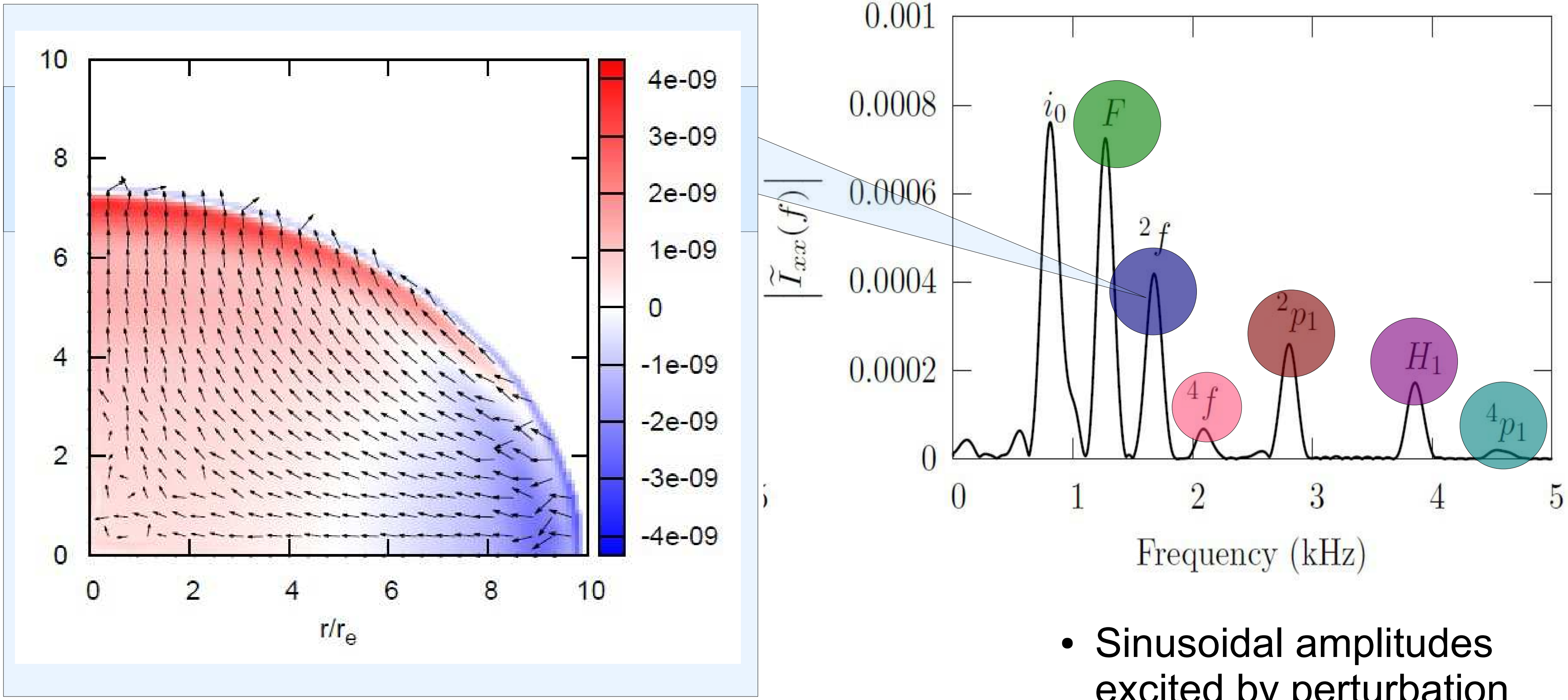

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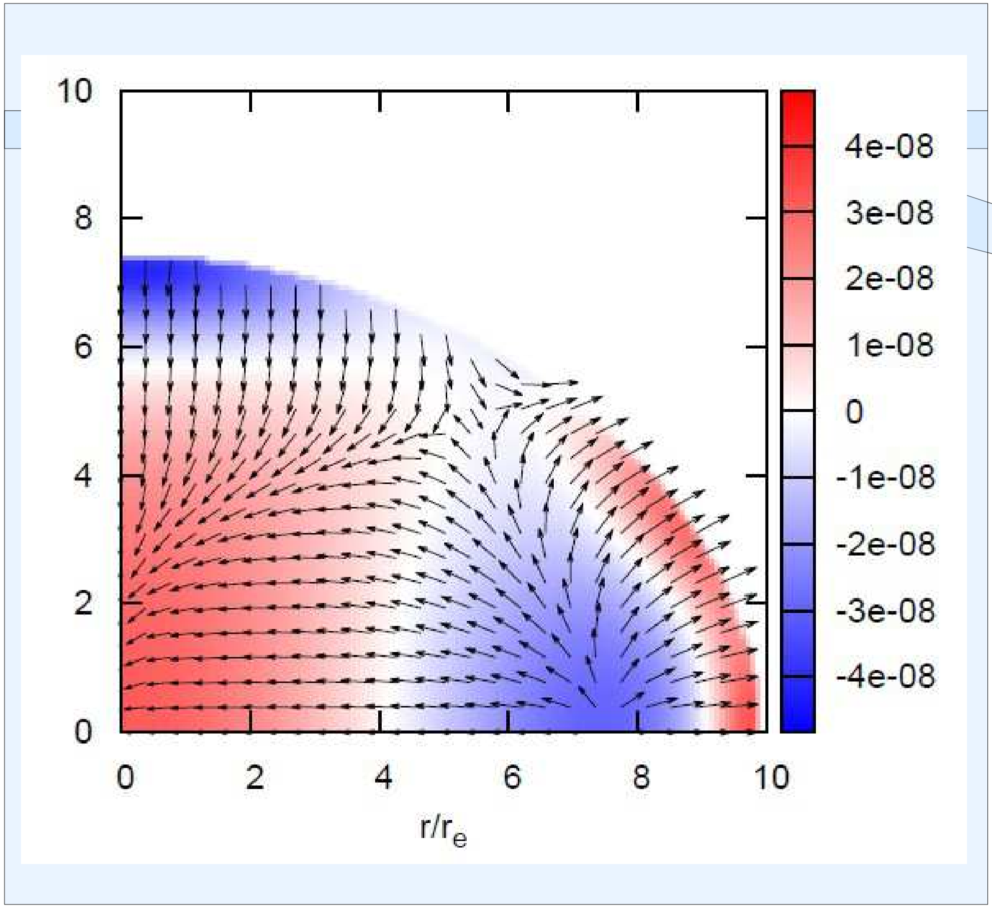

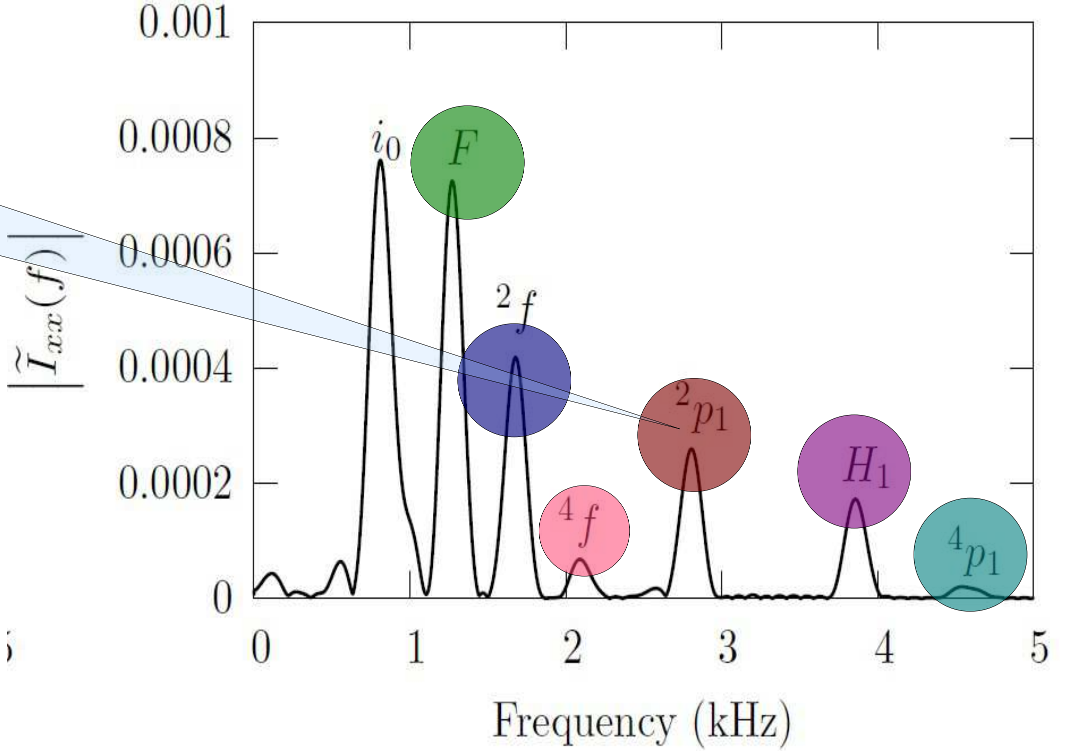

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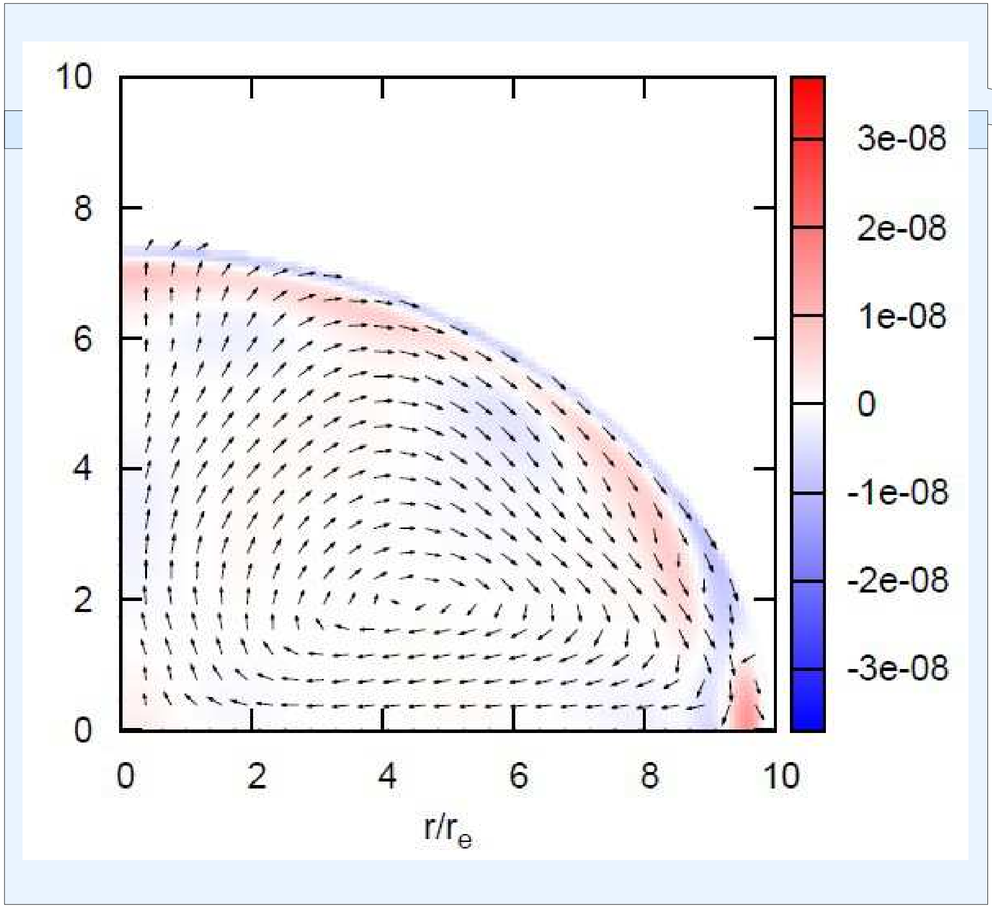

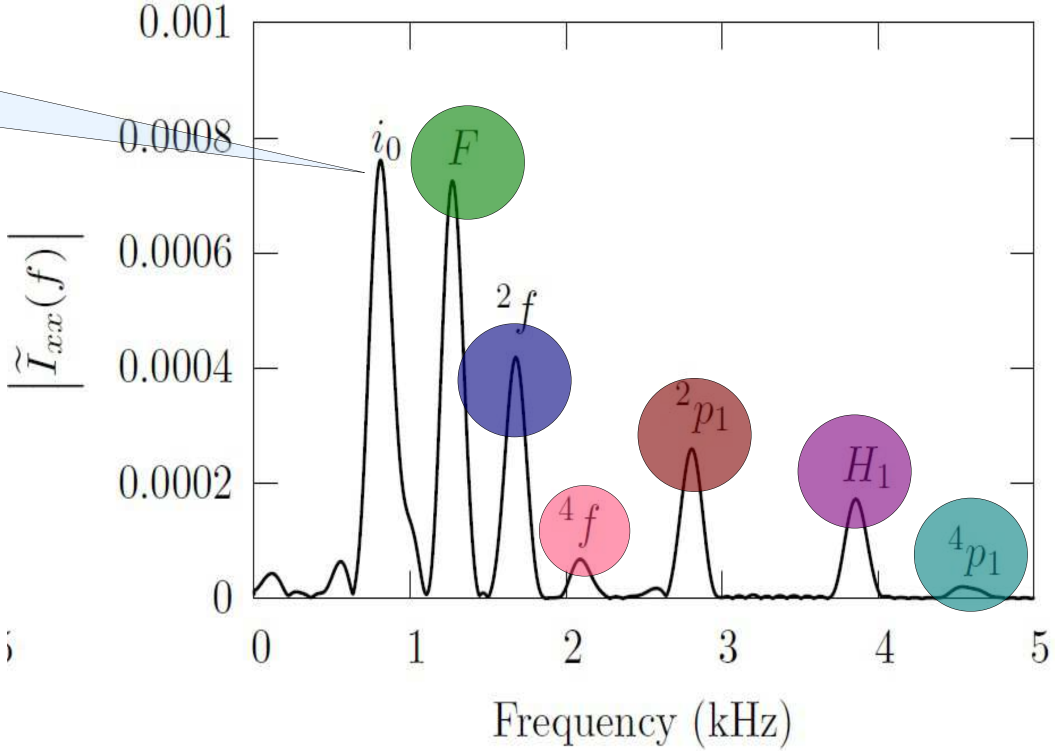

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Gravitational Wave From the Glitching Pulsar

- Strain amplitude of gravitational wave at a distance d can be written as

$$h_{xx} \simeq \frac{8\pi^2}{d} f^2 \tilde{I}_{xx},\ h_{zz} \simeq \frac{8\pi^2}{d} f^2 \tilde{I}_{zz},$$

where $\tilde{I}$ is amplitude of oscillating quadrupole moment I.

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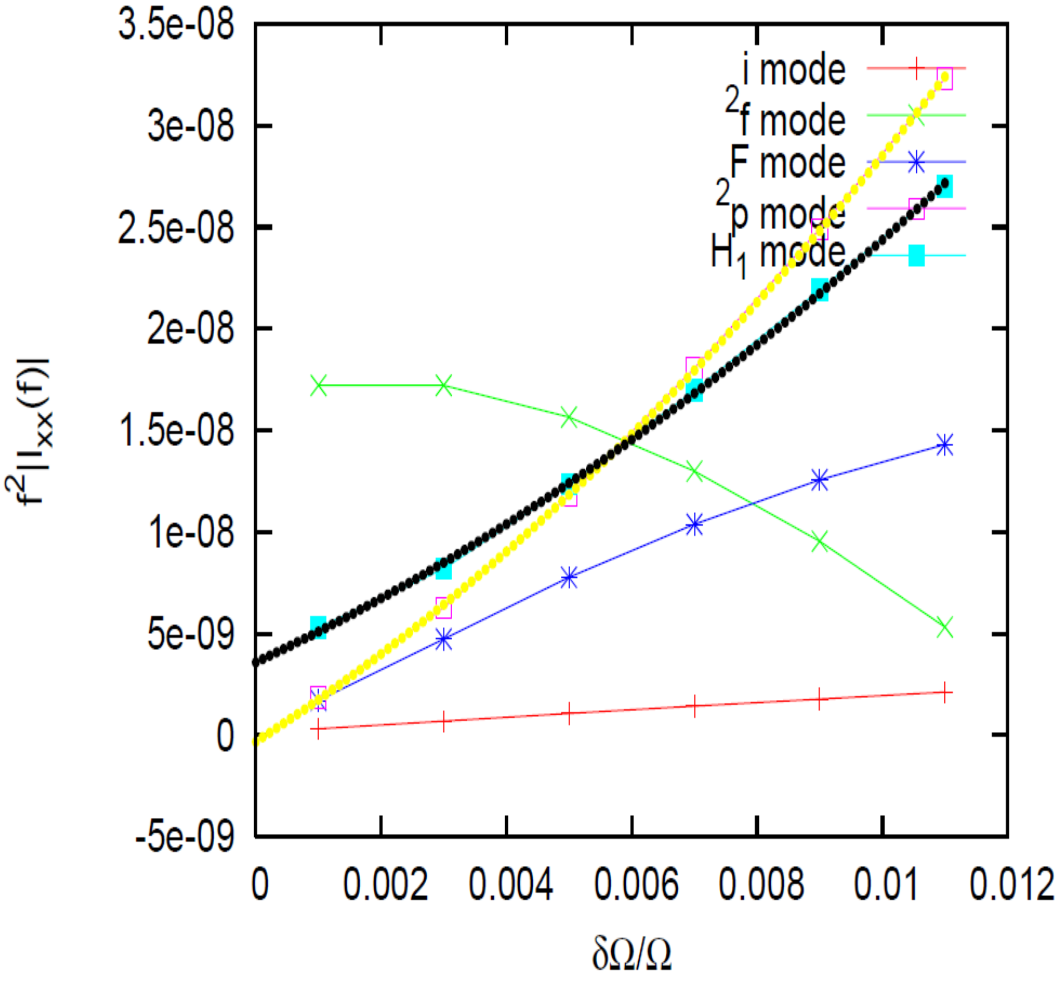

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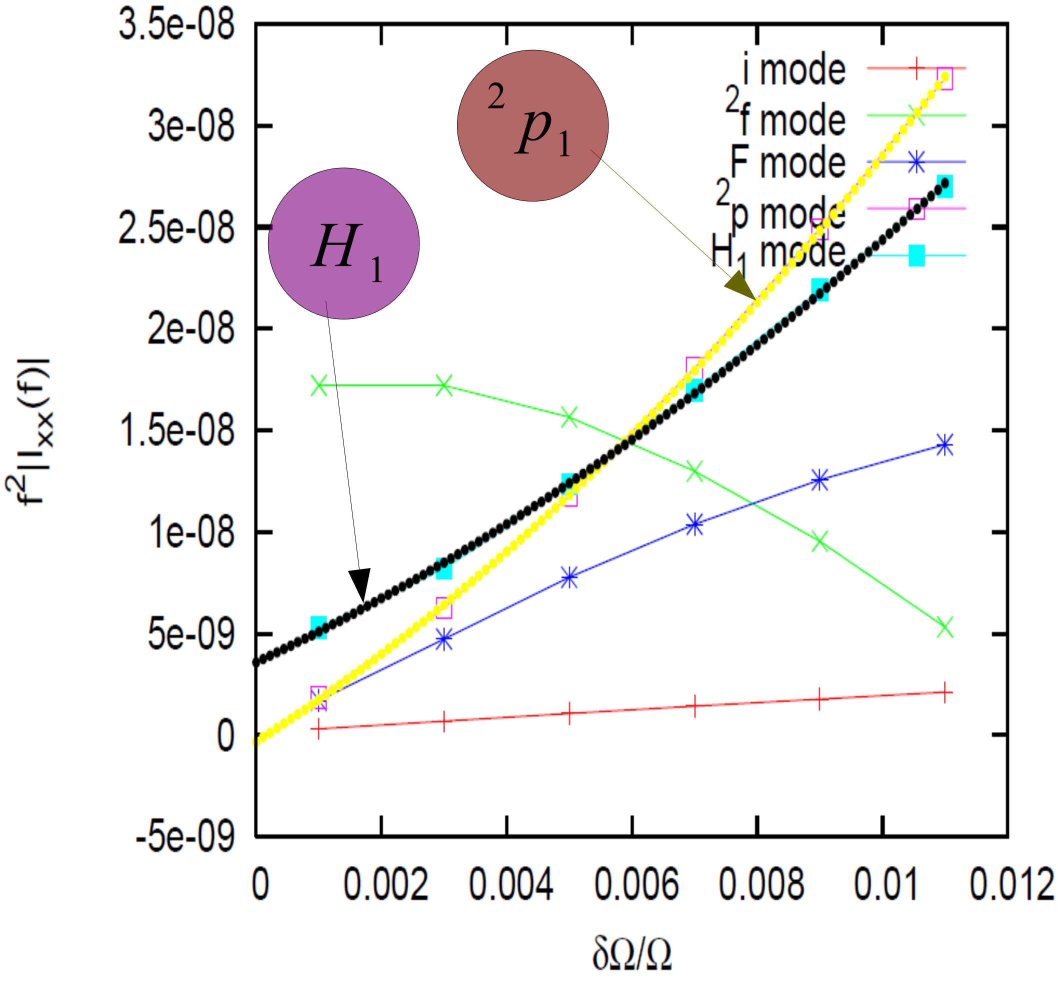

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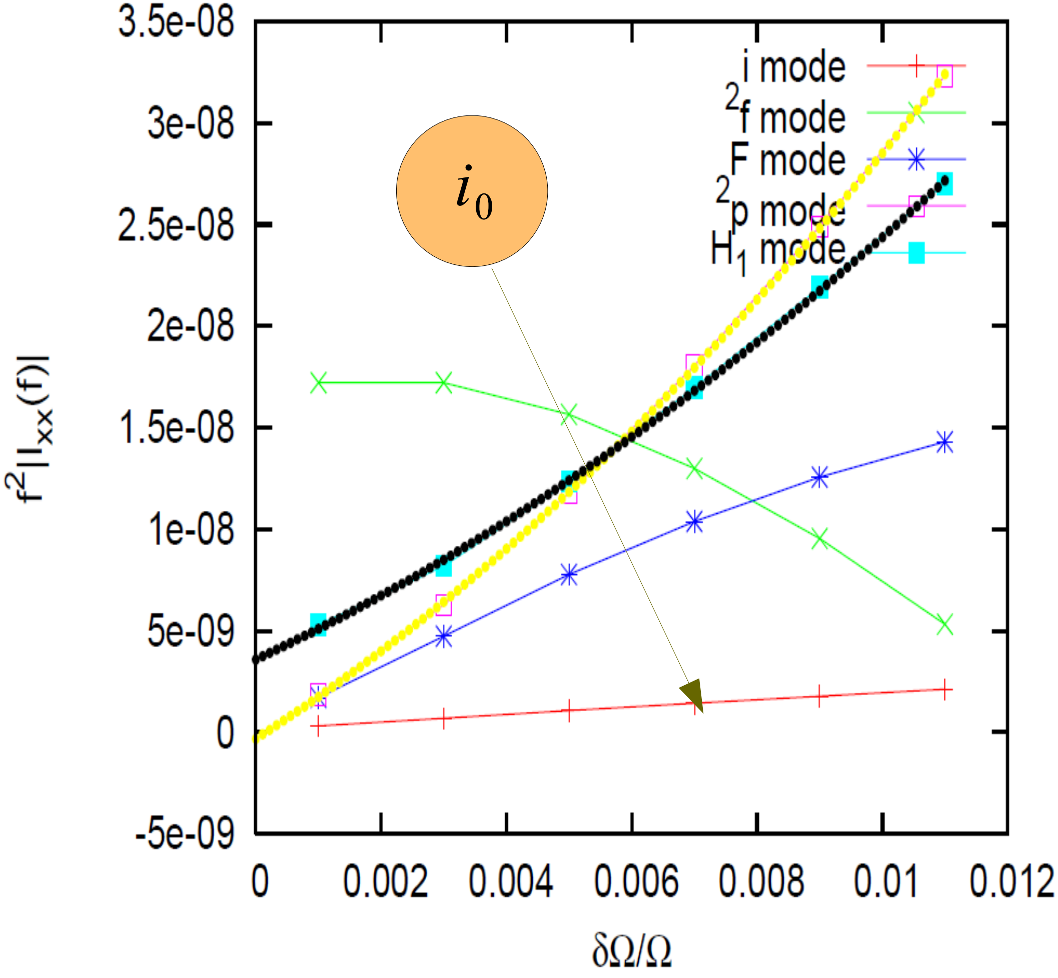

- We found that the strongest and second strongest modes are $^2p_1$ and H_1, contrary to the usual assumption of the ^2f mode as the strongest mode.

- The amplitude of inertial mode is not very strong but it may be able to become non-axisymmetric r-mode which can emit stronger gravitational wave.

Characteristic strain (h_c) of gravitational wave from the glitching pulsar which has $\Delta\Omega/\Omega$ and is located at a distance d is

$$h_c = w\left(\frac{\Delta\Omega/\Omega}{1\times10^{-5}}\right)\left(\frac{d}{1\ \text{kpc}}\right)^{-1}$$

		Star quake					Superfluid		
		perturbation 1 $(\times10^{-25})$					perturbation 2 $(\times10^{-25})$		
		i_0	F	^2f	$^2p_1$	H_1	i_0	$^2p_1$	H_1
$\rho_0^{\text{max}}=1.28\times10^{-3}$ sequence	AU1	0.00752	-	0.282	2.31	0.373	0.145	3.37	0.975
	AU2	0.0731	8.97	6.86	5.03	3.52	1.01	7.49	1.28
	AU3	0.367	11.9	15.8	8.64	4.55	3.07	14.2	11.5
	AU4	1.39	14.7	24.7	9.00	5.66	8.10	13.9	3.55
$M_0=1.4\,M_{\text{sun}}$ sequence	BU1	0.00321	3.00	0.128	0.964	0.371	0.0943	2.19	0.746
	BU2	0.0130	5.65	0.967	2.30	1.17	0.560	4.86	1.49
	BU3	0.0491	6.47	2.39	2.58	1.45	1.61	7.39	3.37
	BU4	-	8.28	5.09	2.39	1.30	3.61	8.71	0.0356

Ω increases

w values of various models

Energy of the Each Modes

- We used Newtonian definition of Kinetic energy which is written as $T=\frac{1}{2}\int \rho_0 v^2 dV$

Mode	Energy of Mode $(\times 10^{-8})$
i_+	4.39
i_0	38.2
^2f	0.00
F	0.91
$^2p_1$	0.146
H_1	4.32
$^2p_2$	2.98
H_2	0.722
$^4p_1$	0.00
total	51.7

Energy of the each specific mode for the $2\mathrm{M}_{\mathrm{sun}}$ neutron star with $\delta\Omega/\Omega = 1.1\times 10^{-2}$

Total input energy given by perturbation $=41.9\times 10^{-8}$

Difference = 19%

↓

Difference in strain amplitude = 9.1%

Details of Inertial Modes

- Inertial modes
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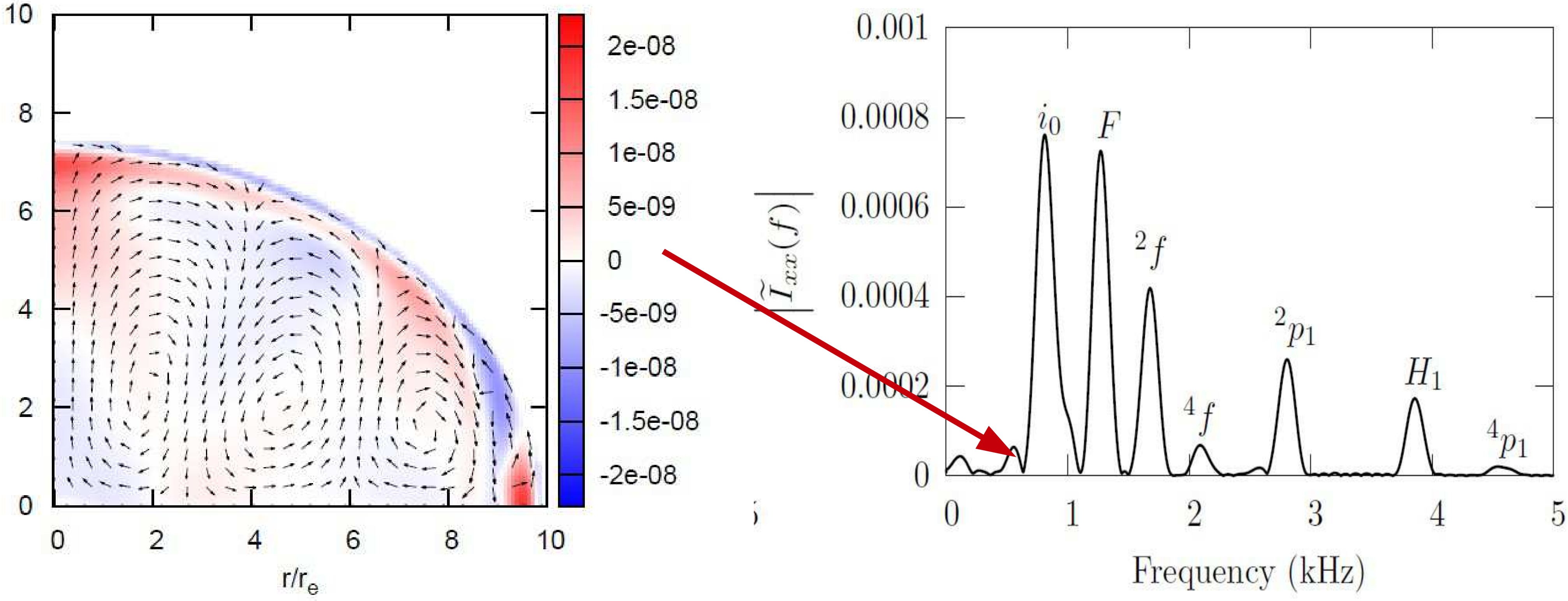

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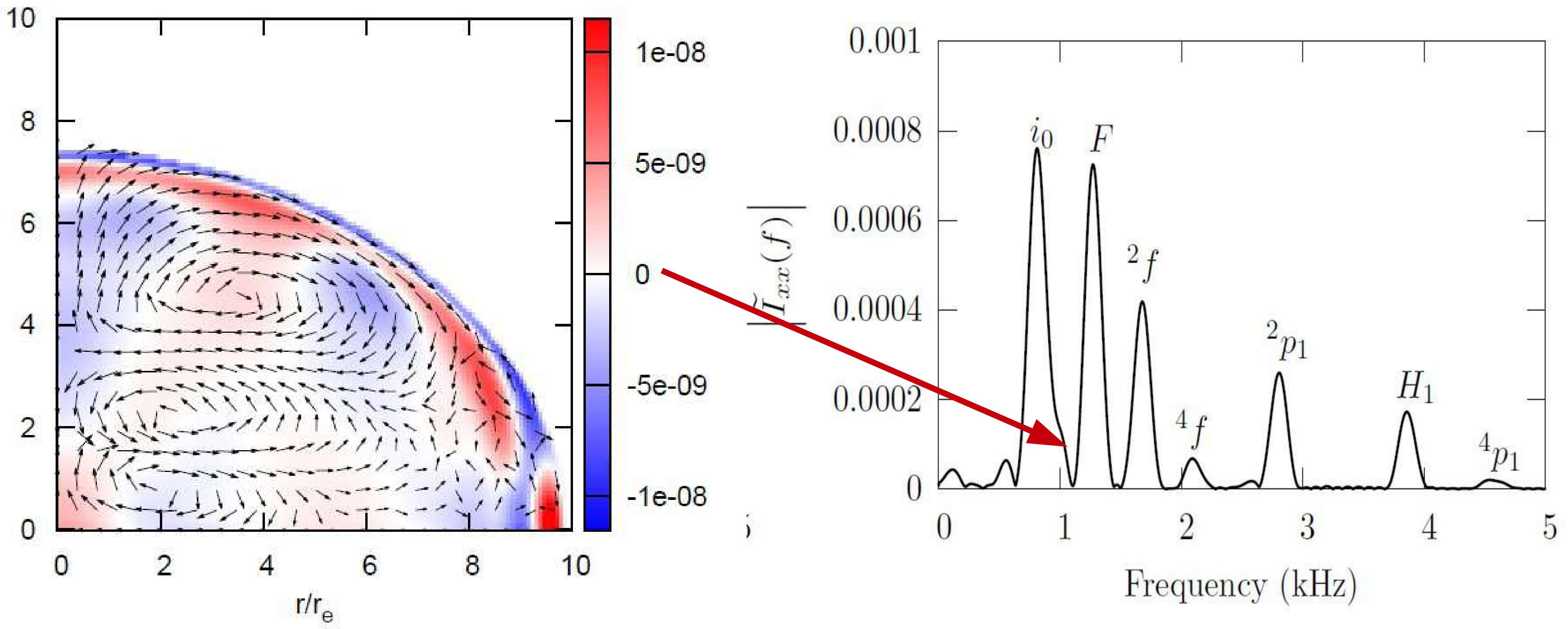

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  - contain most of kinetic energy : long decay time.
  - have a lot to do with r-mode