# Measuring cosmological inhomogeneities with GW standard sirens

Atsushi Nishizawa (Kyoto Univ., Japan)

Collaborator:
Stefano Camera (Universidade Tecnica de Lisboa)

Jun. 4 - 7, 2012, GWPAW
@ Novotel Hannover, Germany

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

# Standard siren

[ Schutz 1986 ]

Gravitational waves from a compact binary

chirp mass

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$\dot{f}(t) \propto [(1+z)M_c]^{5/3} f^{11/3},$$

GW waveform

$$h_{+\times}(t) \propto \frac{[(1+z)M_c]^{5/3} f^{2/3}}{D_L},$$

From observational data,

$$h_{+\times}, f, \dot{f},$$

$$\Downarrow$$

redshifted chirp mass

$$M_z \equiv (1+z)M_c$$

$$\Downarrow$$

luminosity distance

$$D_L$$

Luminosity distance is determined by GW observation.

# Standard siren

The redshift and chirp mass are degenerated, though the degeneracy can be broken in some cases.

Assuming the redshift is determined by EM observation of the host galaxy,

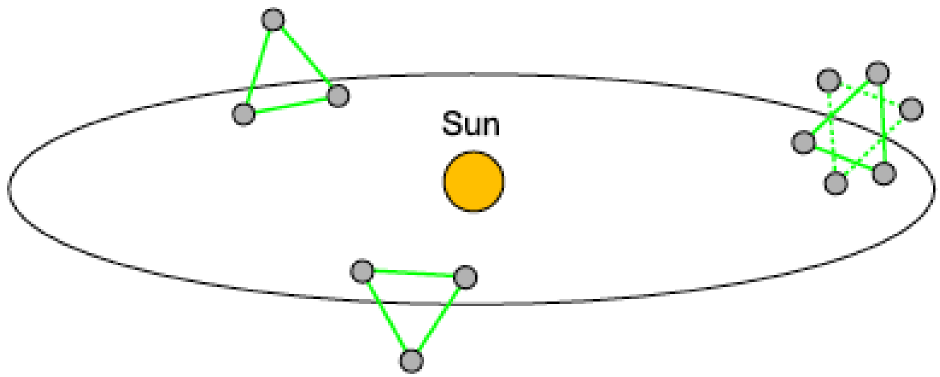
| |
|---|
| $M_c$ can be determined. |
| z-dL relation can be obtained. |

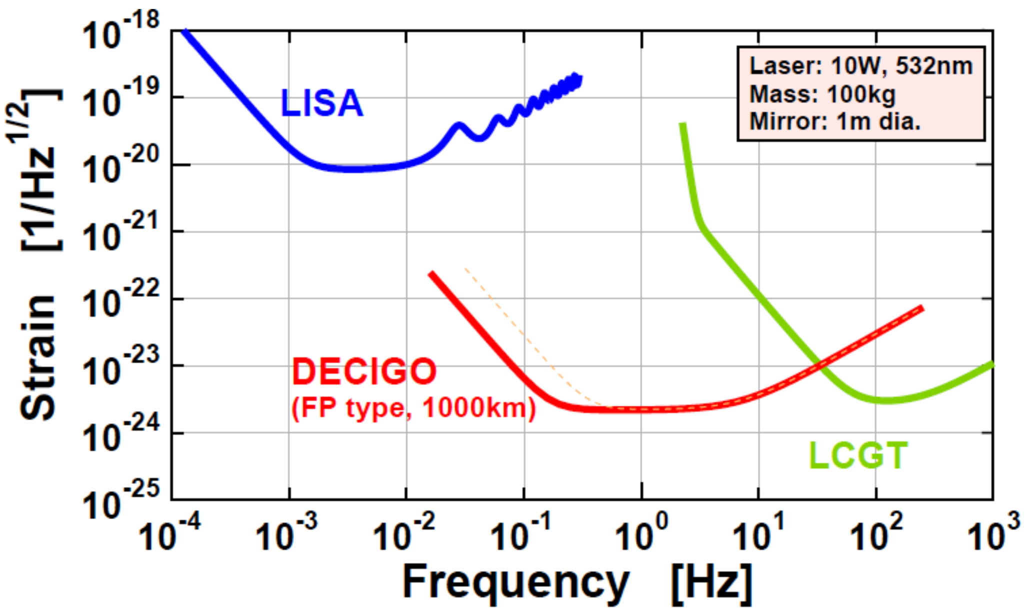
⇩

Advantages of cosmological-expansion measurement by GWs

- No need of distance ladder.
- Consistency test of SNe observation.
- New cosmological observation tool.

# DECIGO



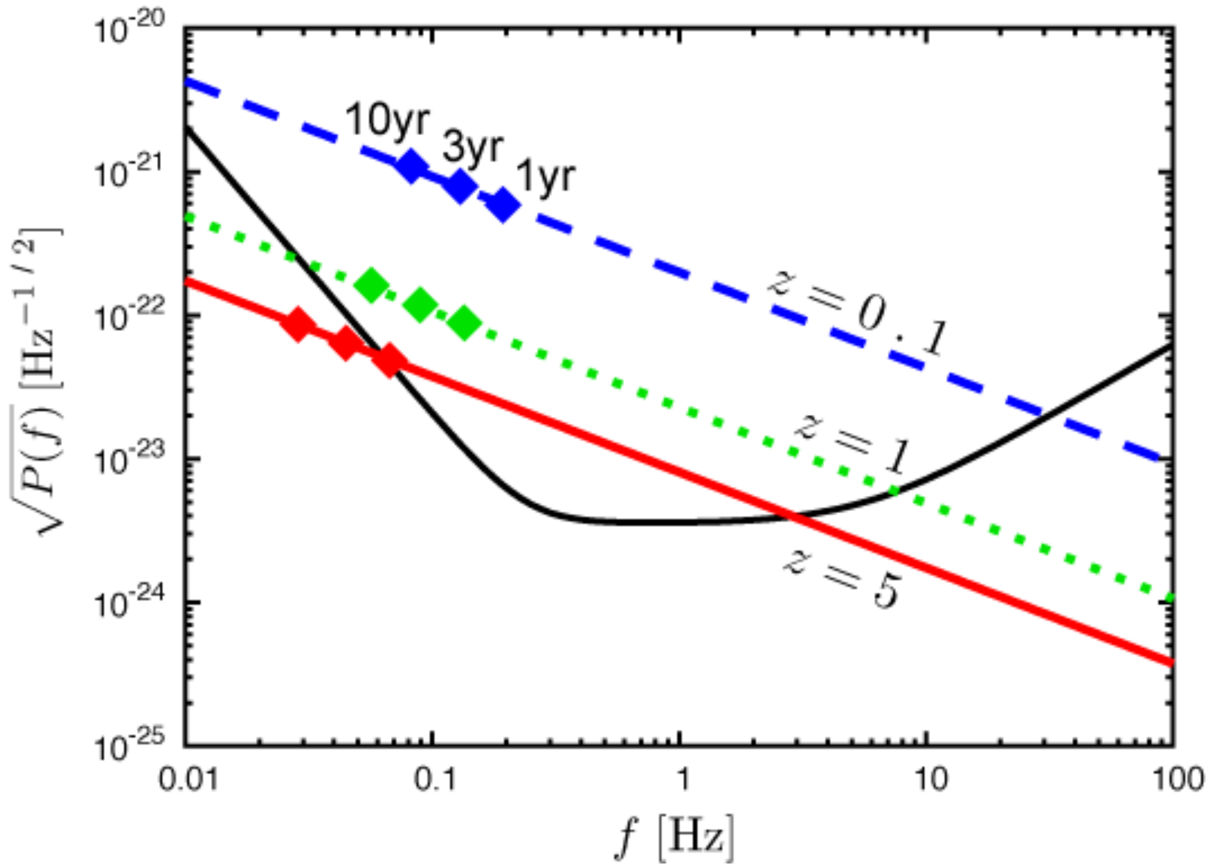

Deci-hertz Interferometer
Gravitational-wave
Observatory

- Launch 2028-
- 4 clusters
- 8 independent interferometers
- arm length： 1000 km
- Fabry-Perot cavity
- freq. band $0.01 - 100\,\mathrm{Hz}$
- main targeted sources: inflationary GWB, IMBH binaries, NS binaries

# NS binaries seen by DECIGO

z=0.1 ⇨ SNR~700
z=1 ⇨ ~80
z=10 ⇨ ~20

$\sim 10^6$ NS binaries
up to z~10 are observable
with high SNR.



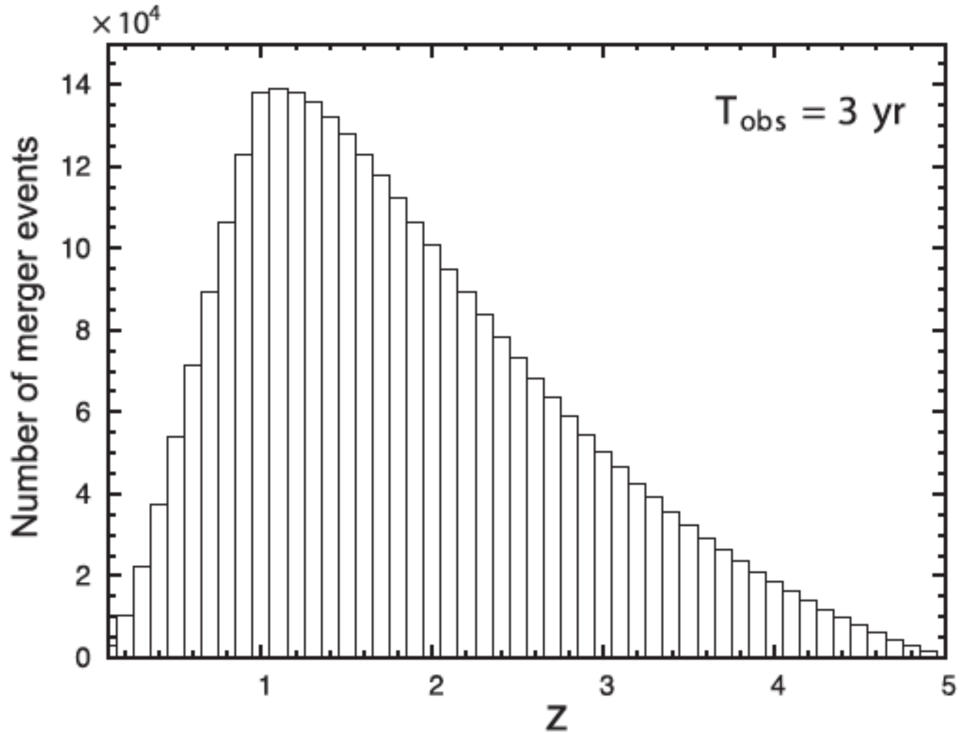
# z distribution of NS binaries

Linear fitting to the estimation by Schneider+ 2001
[ Cutler & Harms 2006 ]

NS binary merger rate

$$\dot{n}_0 = 10^{-6}\,\mathrm{Mpc}^{-3}\,\mathrm{yr}^{-1}$$

$$\begin{aligned} \dot{n}(z) &= \dot{n}_0\, r(z)\,, \\ r(z) &= \begin{cases} 1+2z & (z \le 1) \\ \frac{3}{4}(5-z) & (1 < z \le 5) \\ 0 & (5 < z) \end{cases} \end{aligned}$$

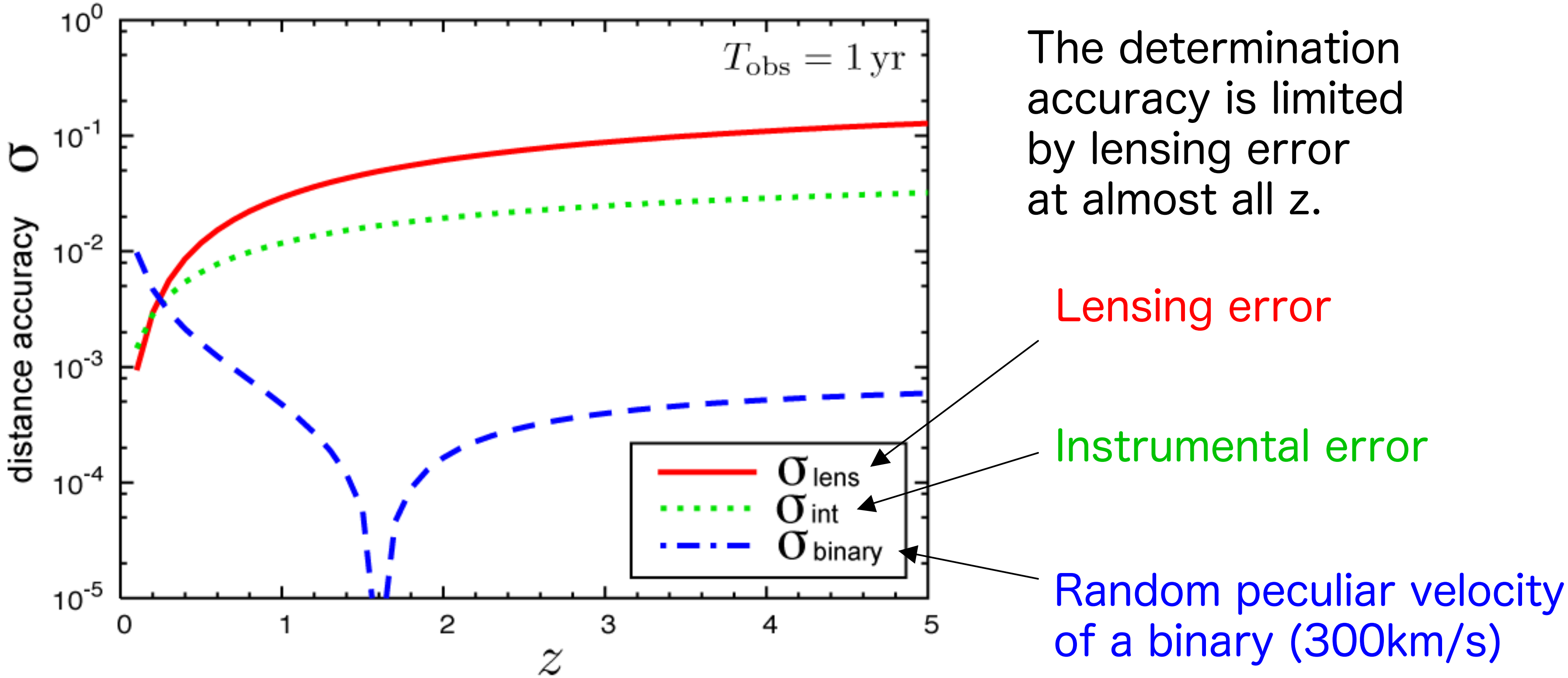



NS binary merger distribution
= event distribution observed by DECIGO

# Measurement accuracy of distance

For a single NS binary

$$\left[\frac{\Delta d_L^{(0)}(z)}{d_L^{(0)}(z)}\right]^2 = \sigma_{\rm int}^2(z) + \sigma_{\rm lens}^2(z) + \sigma_{\rm binary}^2(z)$$




The determination accuracy is limited by lensing error at almost all z.

Random error can be reduced by observing a large number of binaries.

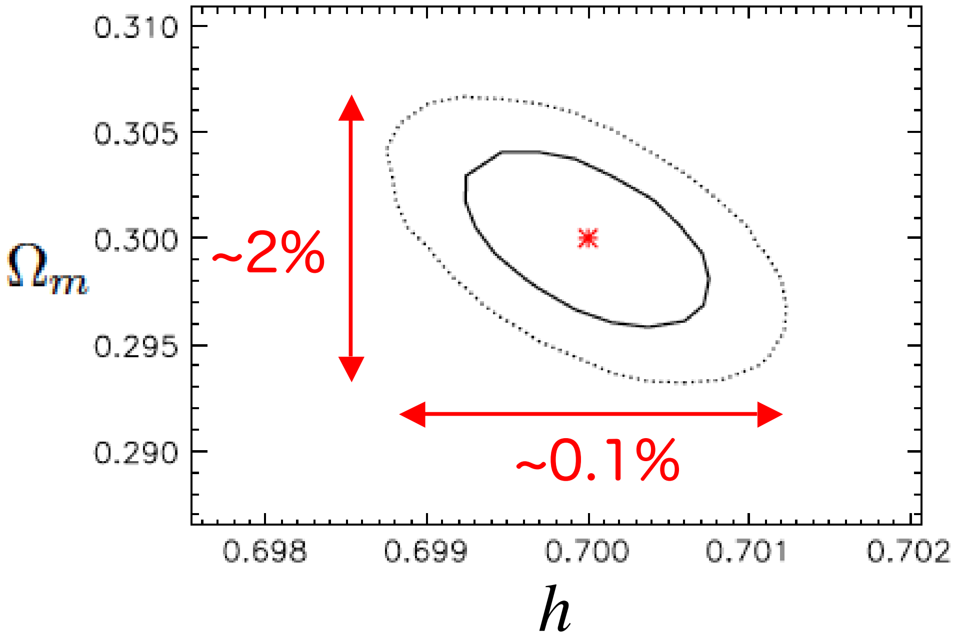
# Measurement accuracy of cosmo. parameters

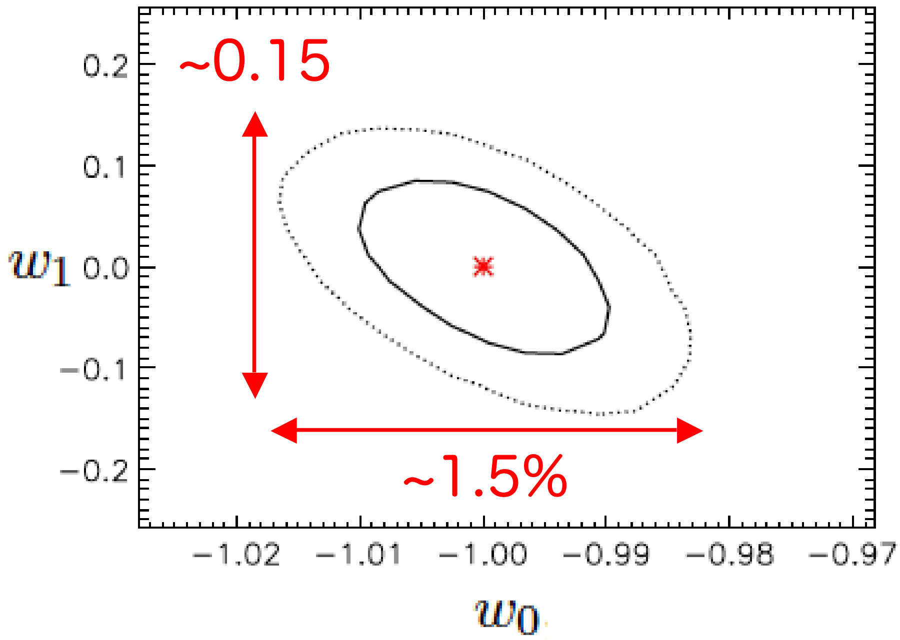
[ Cutler & Holz 2009 ]

Using all NS binaries observed by BBO.
Assuming redshifts of all binaries.

$$w(z) = w_0 + w_1 \frac{z}{1+z}$$

flat universe, obs. time 3yr




These accuracies are very good, compared with future EM observations !!

# Other methods of standard siren

## Measuring the Hubble parameter at each redshift

[ Nishizawa, Taruya, Saito 2011 ]

Dipole moment of luminosity distance is inversely proportional to the Hubble parameters. H(z) can be directly measured.

## Standard siren without redshift identification

- statistical distribution of source redshift, assuming narrow distribution of NS mass
  [ MacLeod & Hogan 2008; Petiteau, Babak, Sesana 2011; Taylor, Gair, Mandel 2012 ]

- tidal deformation of NS, assuming EOS of NS
  [ Messeger & Read 2012 ]

- phase shift due to cosmic accelerating expansion
  [ Nishizawa et al. 2012 ]

All these methods focus on cosmic expansion at background.

# Beyond background cosmology

Various models for cosmic accelerating expansion

- smooth component with negative pressure (scalar field etc.)
- modification of gravity at large scale
- cosmological constant (vacuum energy)
- void model

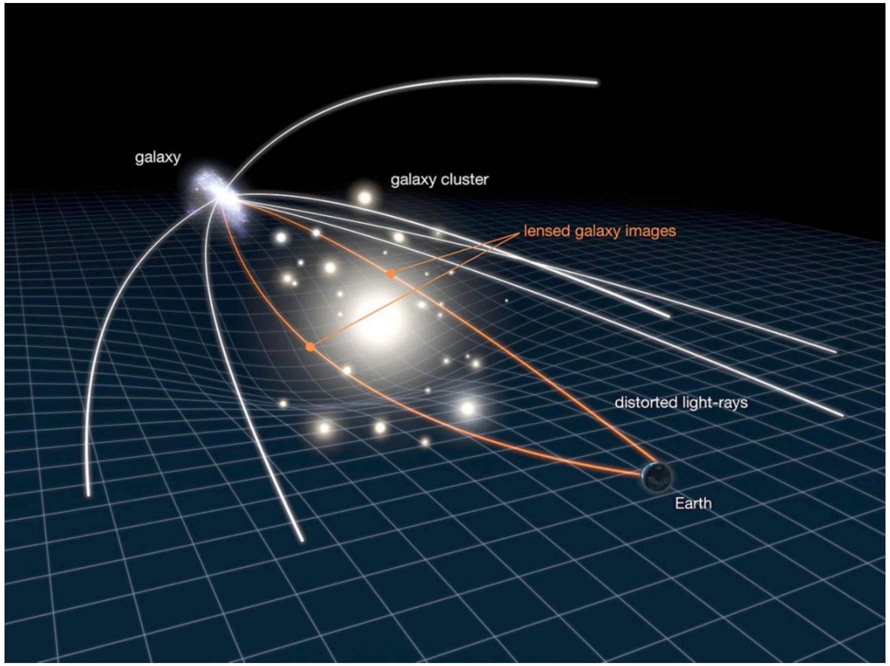
Current observational data (SNe + BAO + CMB + … ) are consistent with the cosmological constant.

The problem is that most of the models can mimic the LCDM model as a special case.

⇩

To discriminate the models in the future observation,
it is important to look at the perturbation level.
(growth of matter power spectrum, gravitational lensing, etc.)

# Gravitational lensing of GW




- GW traces its null geodesic and is lensed by galaxy and galaxy clusters.
- Source is a compact binary.

⇩

No shear (too small image), but “brightness” of GW is magnified.

- Apparent luminosity distance

$$D_{\rm app} = \frac{D}{\sqrt{\mu}}$$

Observing $D_{\rm app}$ and $z$ for each source gives $\mu$ as a function of cosmological parameters.

- From the magnification distribution on the sky, we can probe the large scale structure of the universe.

# Magnification angular power spectrum

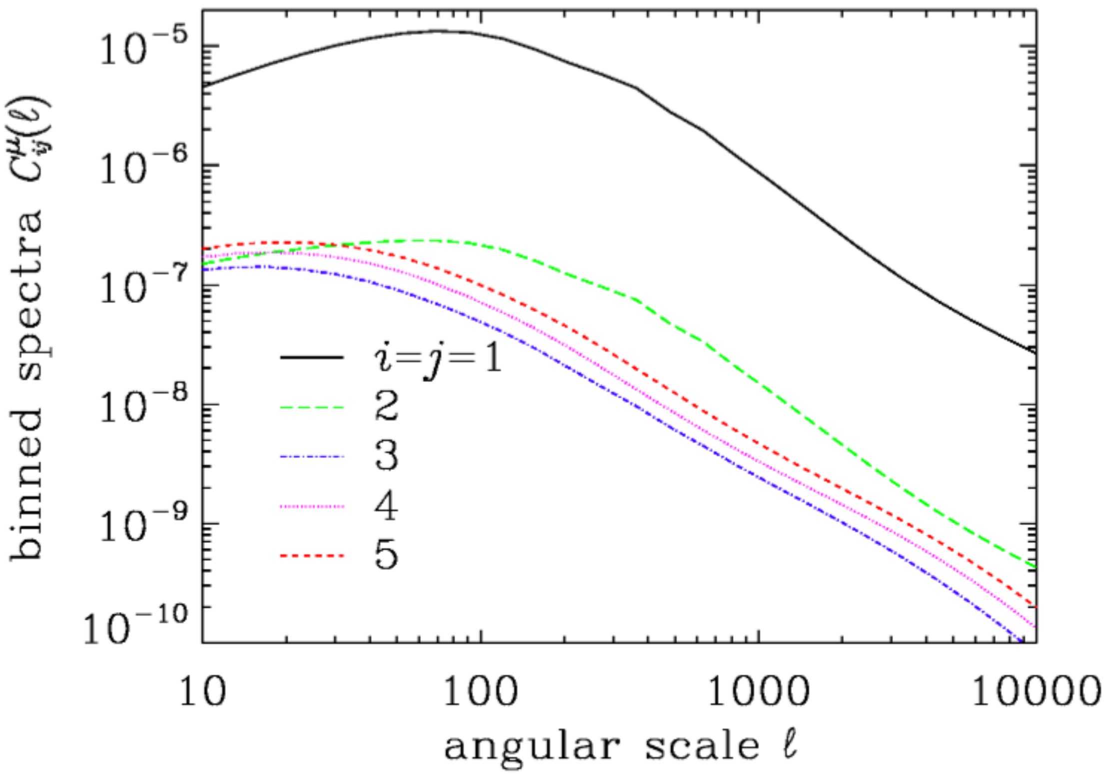
magnification angular power spectrum

$$C^{\mu}_{ij}(\ell) = \int \frac{\mathrm{d}z}{H(z)} \frac{W_i(z)W_j(z)}{\chi^2(z)} \underline{P^{\delta}\left[k = \frac{\ell}{\chi(z)}, z\right]}$$

matter power spectrum

weight function

$$W_i(z) = 3{H_0}^2\Omega_m(1+z)\chi(z)\int_z^{z_H} \mathrm{d}z' \frac{\chi(z')-\chi(z)}{\chi(z')} \frac{\mathrm{d}N^{(i)}_{\mathrm{NS}}}{\mathrm{d}z'}$$




- maximum redshift is set to 2
- 5 redshift bins in the range $z = 0.1 - 2$ ( $\Delta z = 0.38$ )

# Analysis method

Fisher matrix

$$\mathbf{F}_{\alpha\beta} = \sum_{\ell} \frac{2\ell+1}{2} f_{\rm sky} \frac{\partial C^{\mu}_{ij}}{\partial \vartheta_\alpha}(\ell)\, [\mathbf{C}^{\mu,\mu}_{\ell}]^{-1}_{ijkl} \frac{\partial C^{\mu}_{kl}}{\partial \vartheta_\beta}(\ell)$$

$$\vartheta_\alpha = \{\Omega_m,\, \Omega_b,\, n_s,\, \sigma_8,\, w_0,\, w_a\}$$ $h_0$ is fixed.

covariant matrix

$$[\mathbf{C}^{\mu,\mu}_{\ell}]_{ijkl} = \underbrace{C^{\mu}_{ik}(\ell) C^{\mu}_{jl}(\ell)}_{\text{signal}} + \underbrace{C^{\mu}_{il}(\ell) C^{\mu}_{jk}(\ell)}_{\text{statistical shot noise}} + \underbrace{\delta_{ijkl} \left[\frac{{\sigma_\mu}^2(z_i)}{N^{(i)}_{\rm NS}}\right]^2}_{\text{instrumental noise}}$$

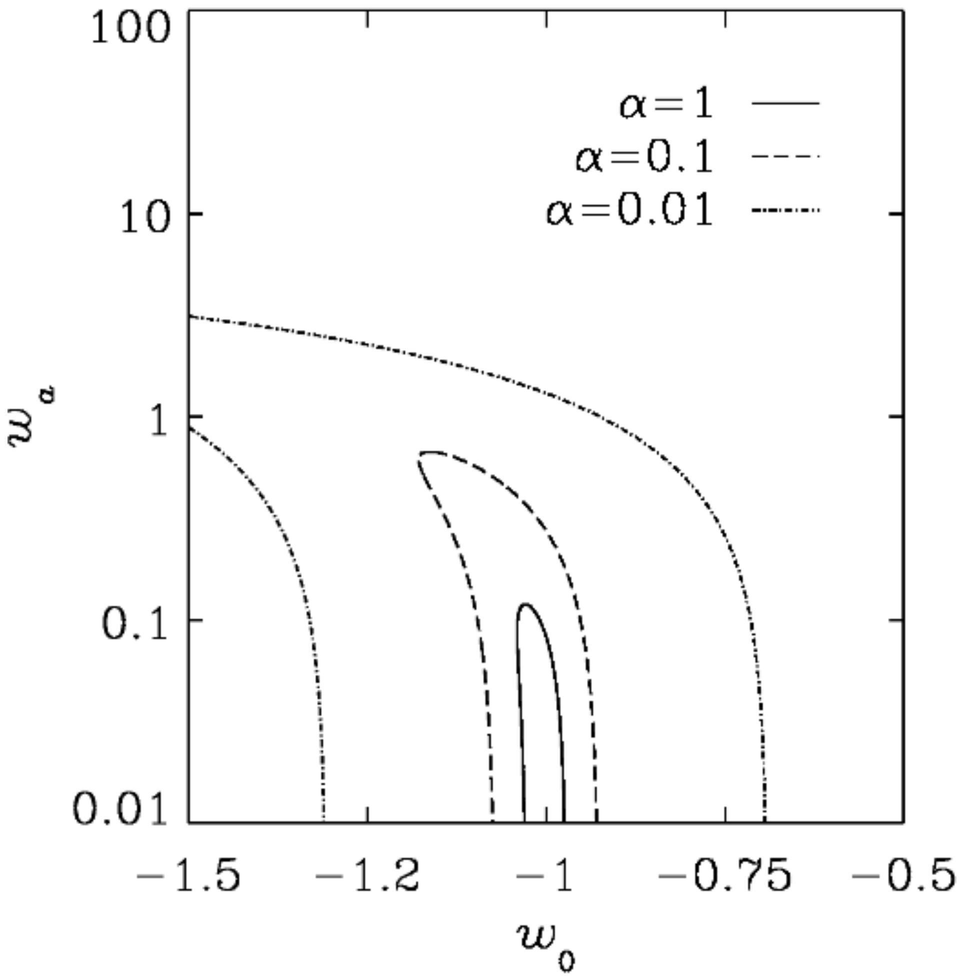
Note that the lensing effect is not noise but signal in this case.

$$\boxed{\text{Estimated error} \quad \Delta\theta_\alpha = \sqrt{(\mathbf{F}^{-1})_{\alpha\alpha}}}$$

Strictly speaking, lensing magnification distribution is non-Gaussin.
[ Hirata, Holz, & Cutler 2010, Shang & Haiman 2011 ]

# DE parameters

$\alpha$ : fraction of redshift identification among all NS binaries

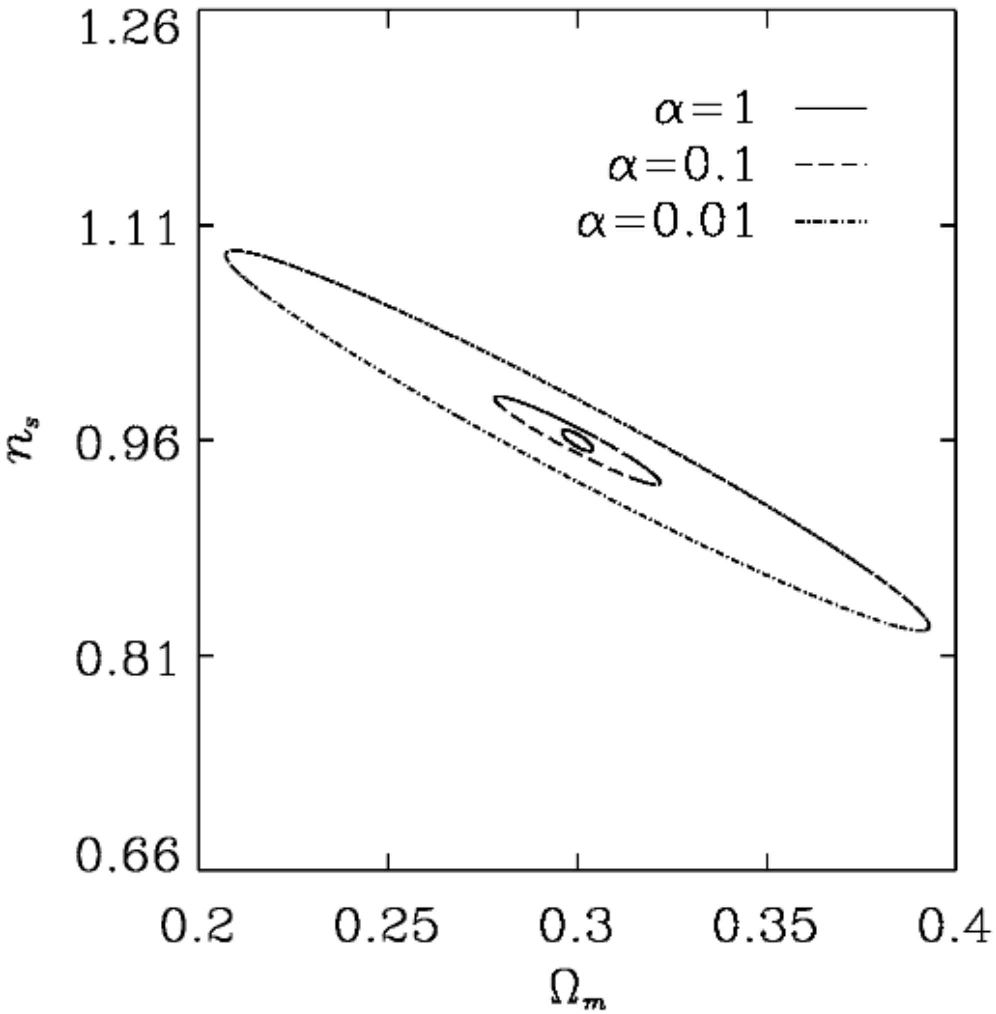



- Identifying all source redshift is rather strong assumption. We consider three cases:

$$\alpha = 1,\ 0.1,\ 0.01.$$

- If $\alpha = 1$ , constraints on the DE parameters are comparable to those in the background analysis of standard siren with lensing error.

# Other cosmological parameters




- This clearly shows that GW can probe the inhomogeneities of the universe.
- If $\alpha = 1$, sensitivity to $\Omega_m$ is comparable to the background analysis of standard siren.
- We can also measure $n_s$ and $\Omega_b$

# Parameterization of modified gravity (1)

Deviation from FRW metric can be parametrized as

$$ds^2 = -a^2(\tau)[(1+2\Psi)d\tau^2 - (1-2\Phi)d\vec{x}^2]$$

where $\Phi$ and $\Psi$ are functions of time and space.

$$\boxed{\begin{aligned} &\frac{\Phi}{\Psi} = \eta(a,k), \\ &k^2\Psi = -4\pi G a^2 \mu(a,k)\rho\Delta, \qquad \Delta \equiv \delta + 3\frac{aH}{k}v \end{aligned}}$$

In GR, $\eta(a,k) = \mu(a,k) = 1$

The two functions characterize the time- and scale-dependent effective strength of gravity, and determines the growth of large-scale structures.

# Parameterization of modified gravity (2)

As a simple case,

[ Zhao et al. 2010 ]

$$\mu(z) = \frac{1-\mu_0}{2}\left(1+\tanh\frac{z-z_s}{\Delta z}\right)+\mu_0,$$

$$\eta(z) = \frac{1-\eta_0}{2}\left(1+\tanh\frac{z-z_s}{\Delta z}\right)+\eta_0.$$

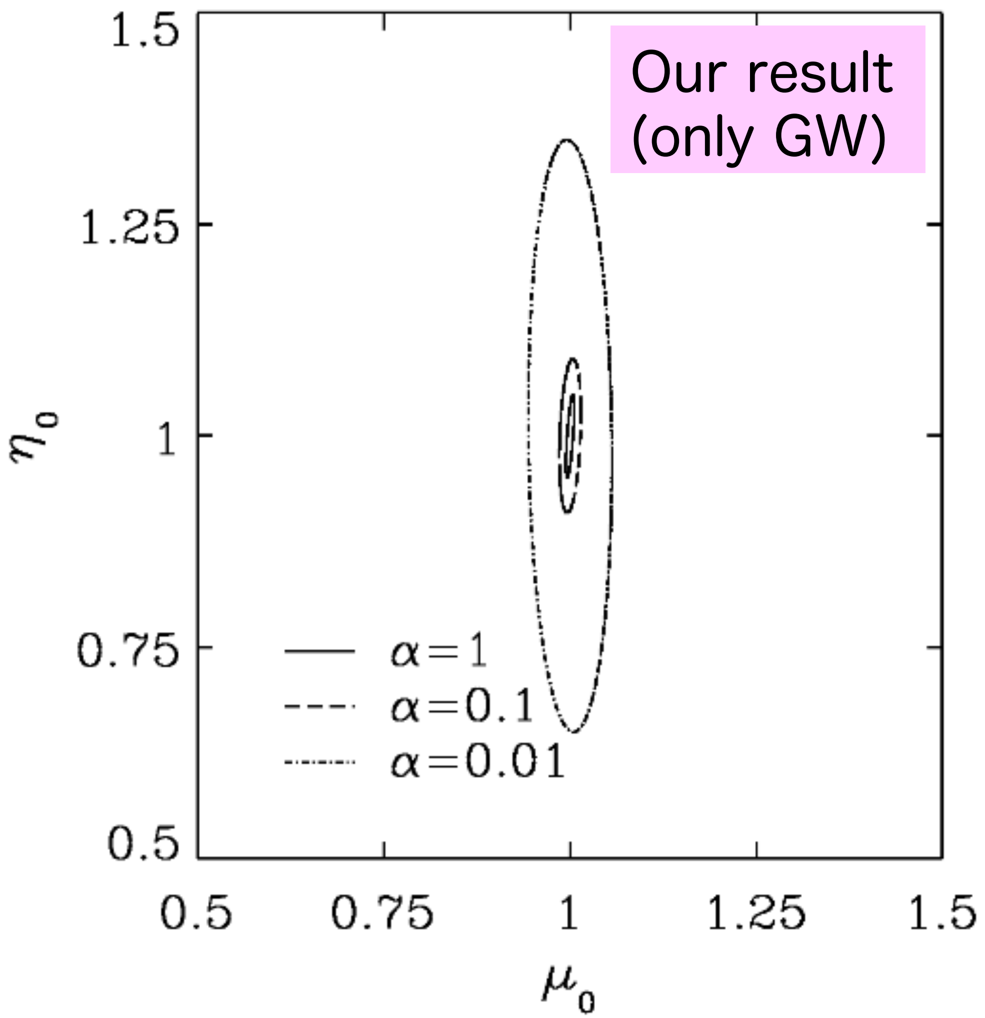
At present $z < z_s$ , $(\mu, \eta) = (\mu_0, \eta_0)$

At the past $z > z_s$ , $(\mu, \eta) = (1, 1)$

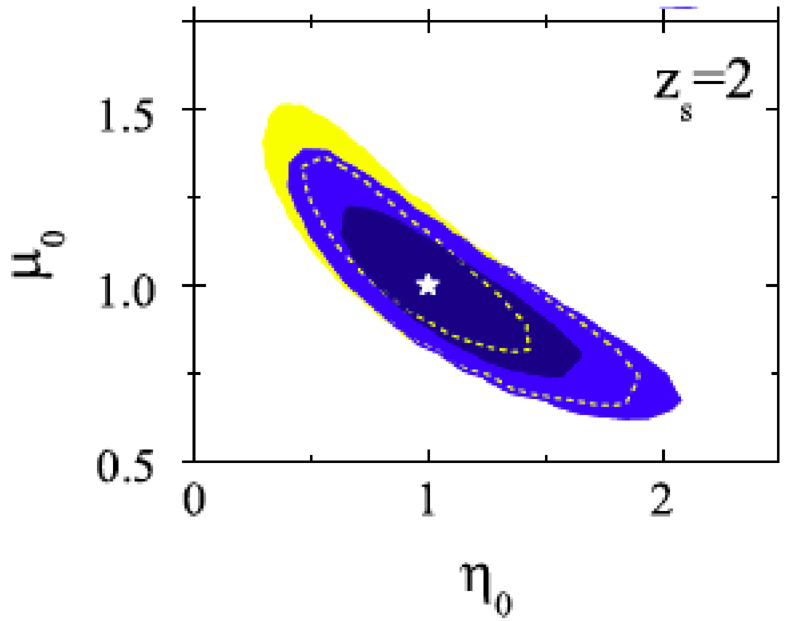
If we set the transition to $z_s = 2$ and $\Delta z = 0.05$ ,
the modification of gravity is characterized by $\mu_0$ and $\eta_0$ .

# MG parameters

$$\vartheta_\alpha = \{\Omega_m, \Omega_b, n_s, \sigma_8, \mu_0, \eta_0\}$$




Zhao’s result
(dark blue region: 68% C.L.
with WMAP5+SNe+ISW)




- DECIGO is much sensitive.
- Shape of error ellipse is completely different !!

# Summary

Gravitational lensing is considered as noise so far.

However, magnification of GW standard sirens brings us crucial information about large-scale structures of the universe.

GW standard siren can be a powerful observational tool to probe for not only dark energy but also inhomogeneities of the universe and gravity theory at cosmological distance.

It should be emphasized that redshift identification of binary sources is quite difficult (as discussed in Nishizawa et al. 2012) but is significantly important for GWs to be a sensitive probe for cosmology.

Now we are considering the application to eLISA and ET.