CMB weak-lensing constraints on the stochastic GW background

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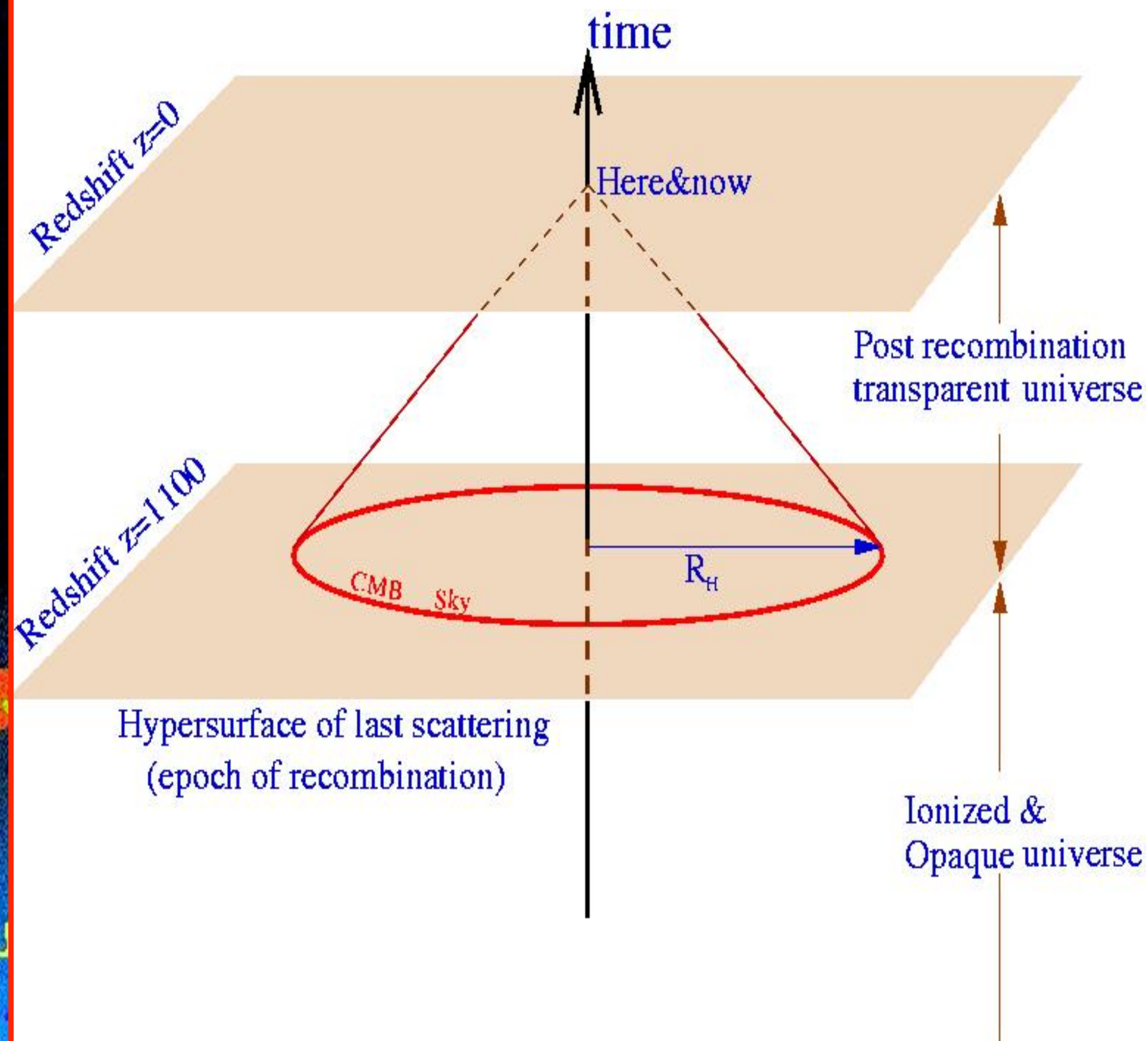
Cosmic Microwave Background

Pristine relic of a hot, dense & smooth early universe - Hot Big Bang model

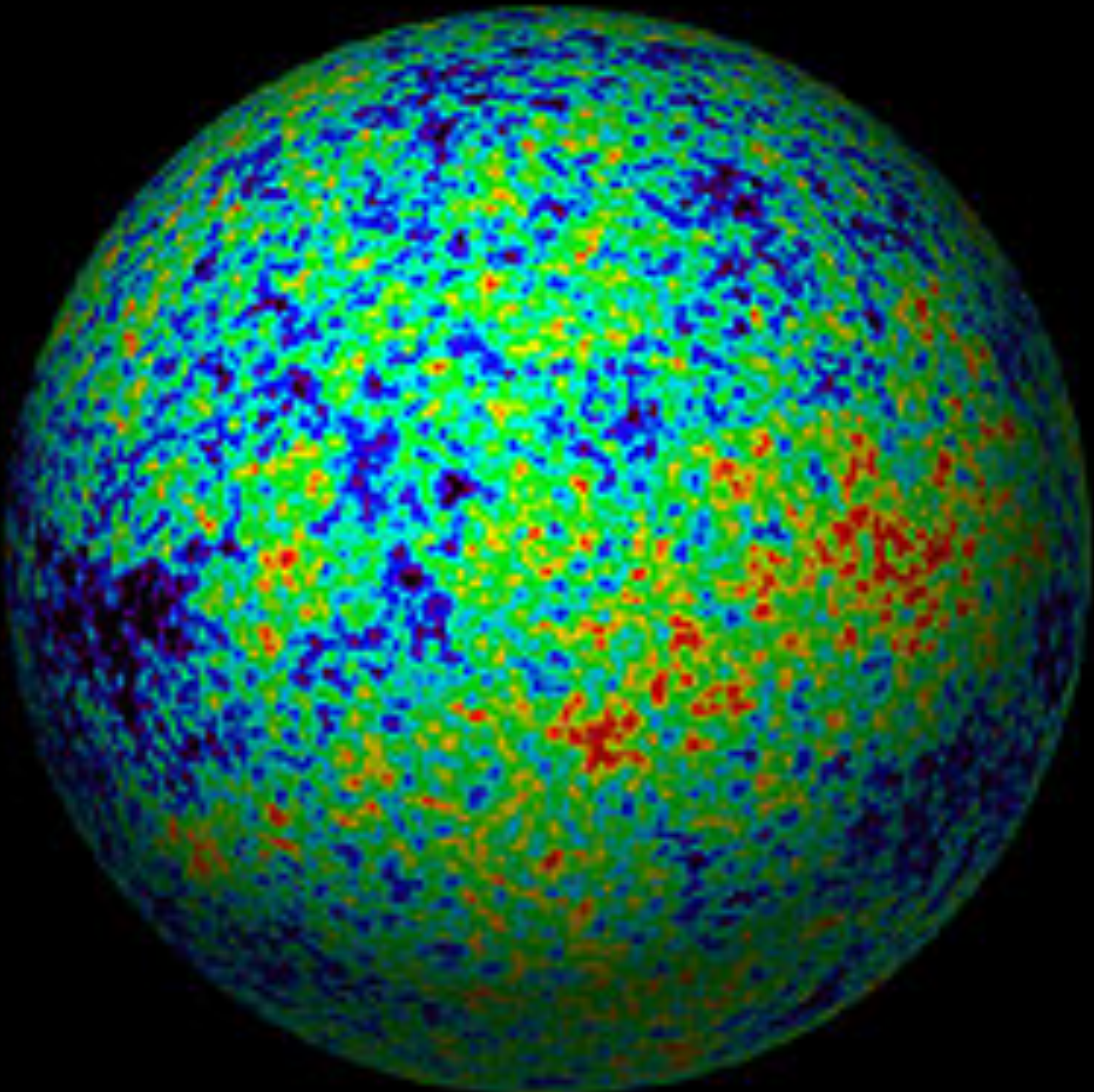
Post-recombination :Freely propagating through (weakly perturbed) homogeneous & isotropic cosmos.

Pre-recombination : Tightly coupled to, and in thermal equilibrium with, ionized matter.

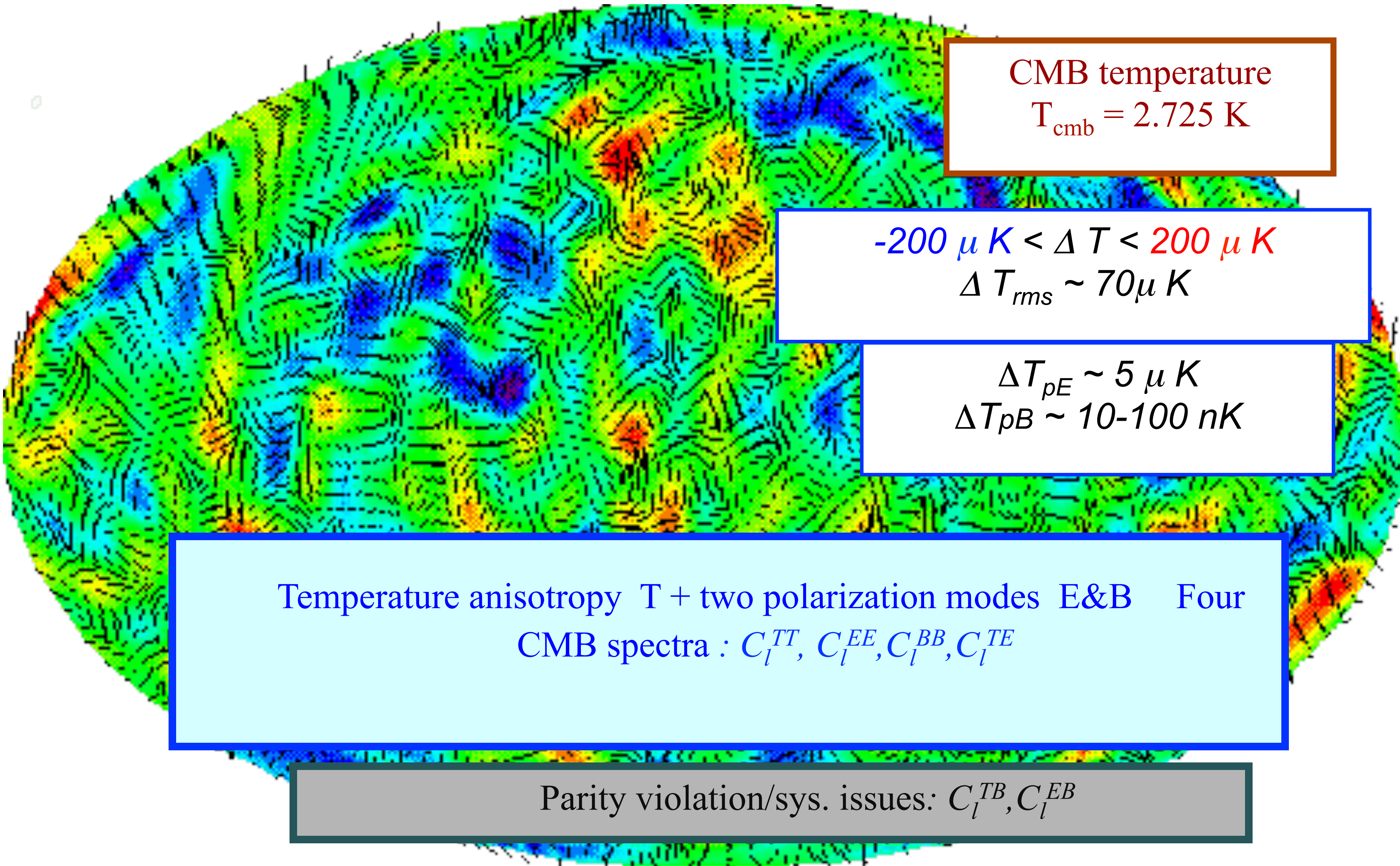
(text background: W. Hu)



Cosmic "Super–IMAX" theater



CMB Anisotropy & Polarization



CMB temperature
$T_{cmb} = 2.725$ K

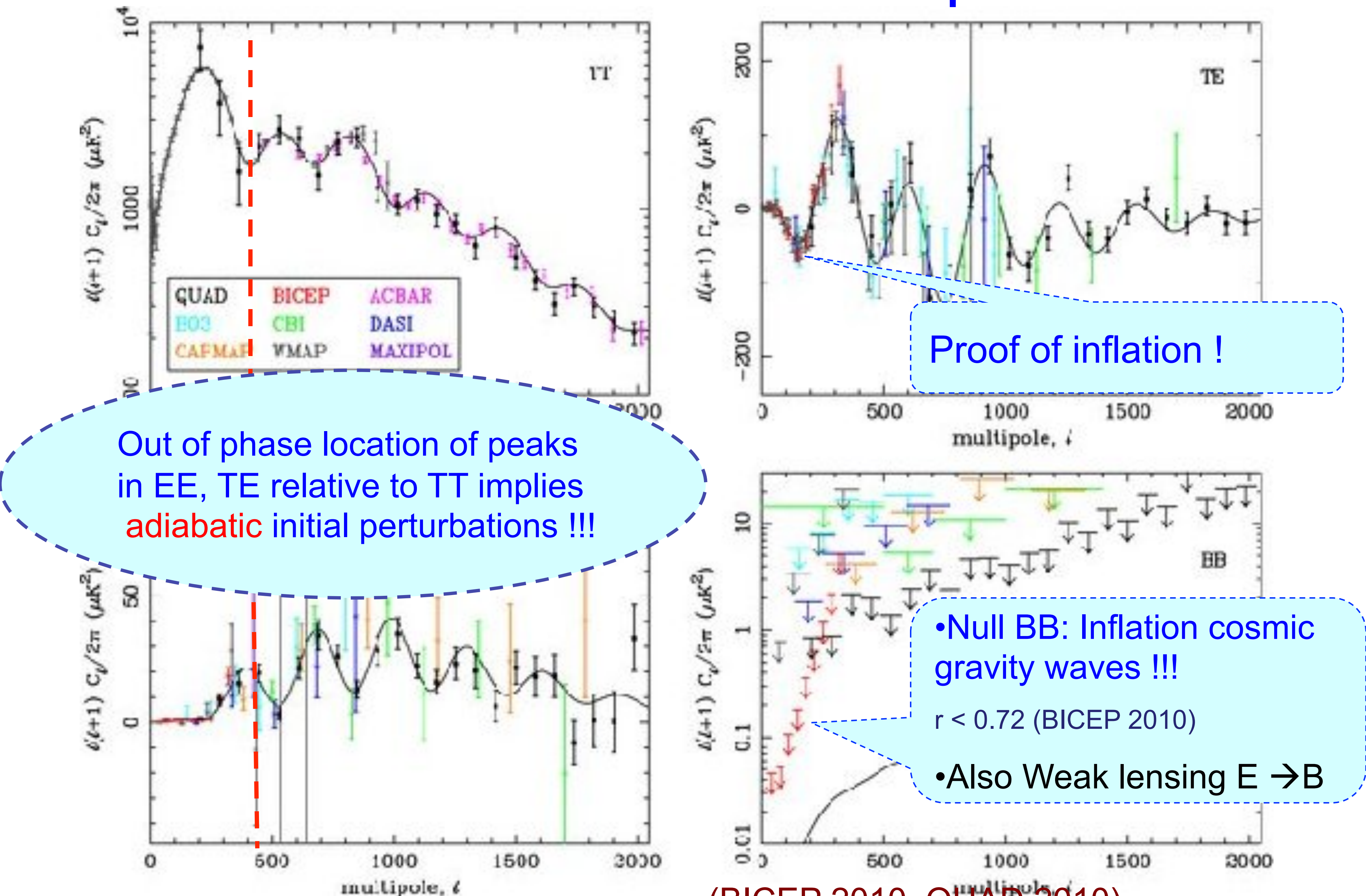
$-200\ \mu K < \Delta T < 200\ \mu K$
$\Delta T_{rms} \sim 70 \mu K$

$\Delta T_{pE} \sim 5\ \mu K$
$\Delta T_{pB} \sim 10\text{-}100$ nK

Temperature anisotropy T + two polarization modes E&B Four CMB spectra : $C_l^{TT}, C_l^{EE}, C_l^{BB}, C_l^{TE}$

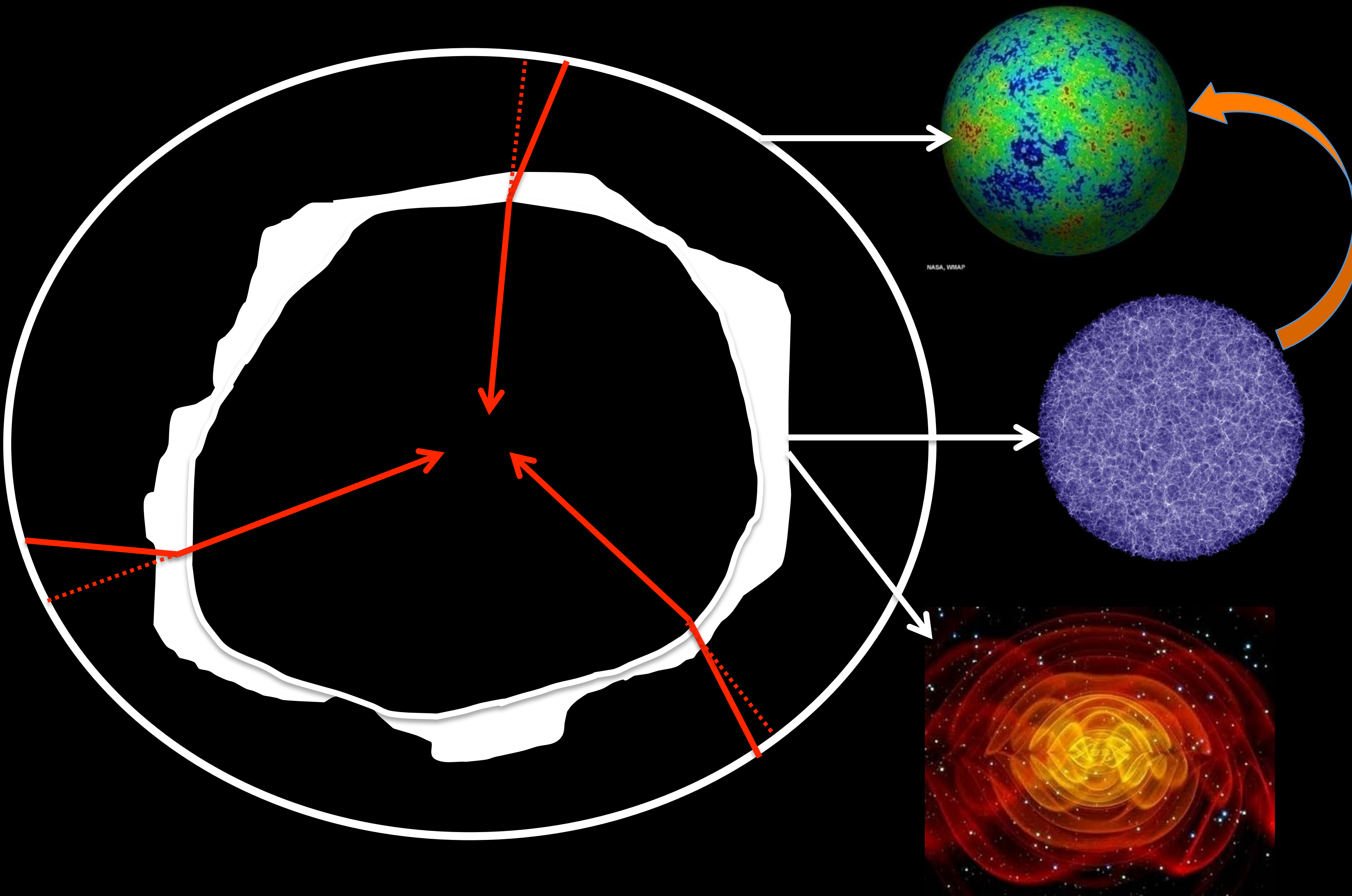
Parity violation/sys. issues: C_l^{TB}, C_l^{EB}

Current status of CMB Spectra

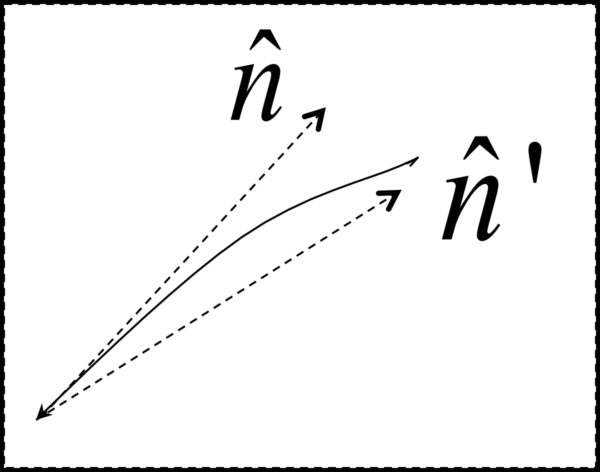


(BICEP 2010, QUAD 2010)

Weak Lensing



Deflection field



$$T(\hat{n}') = T(\hat{n} + \vec{\Theta}) = T(\hat{n}) + \vec{\Theta} \bullet \vec{\nabla} T(\hat{n})$$

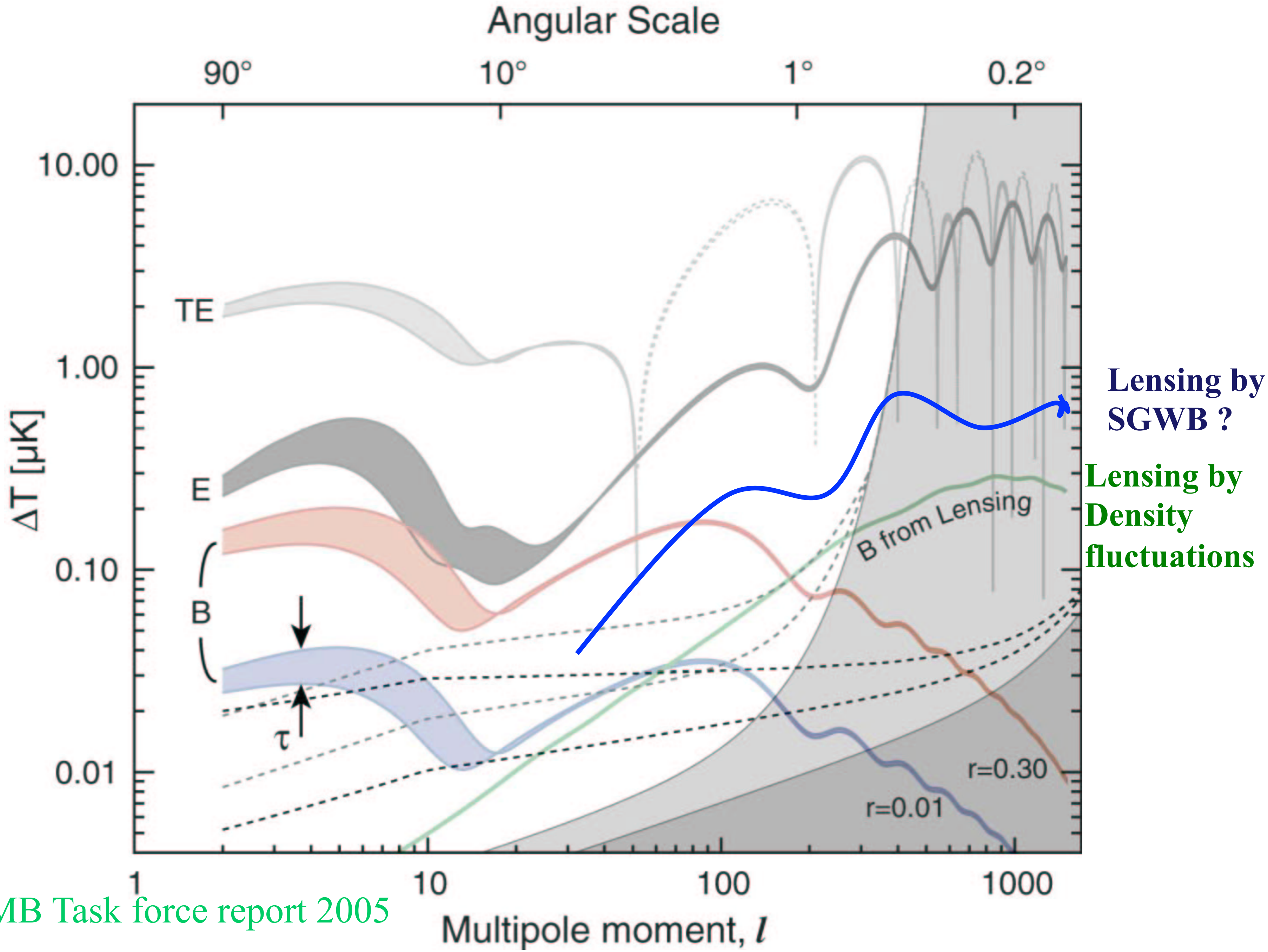
$$\vec{\Theta} = \vec{\nabla}\phi(\hat{n}) + \vec{\nabla} \times \Omega(\hat{n})$$

$$= \nabla_i \phi(\hat{n}) + \varepsilon_{ij} \nabla_j \Omega(\hat{n})$$

Gradient Curl

WL:scalar WL: tensor/GW

Predicted Future Satellite Sensitivities



CMB Task force report 2005

Spectral energy density of SGWB

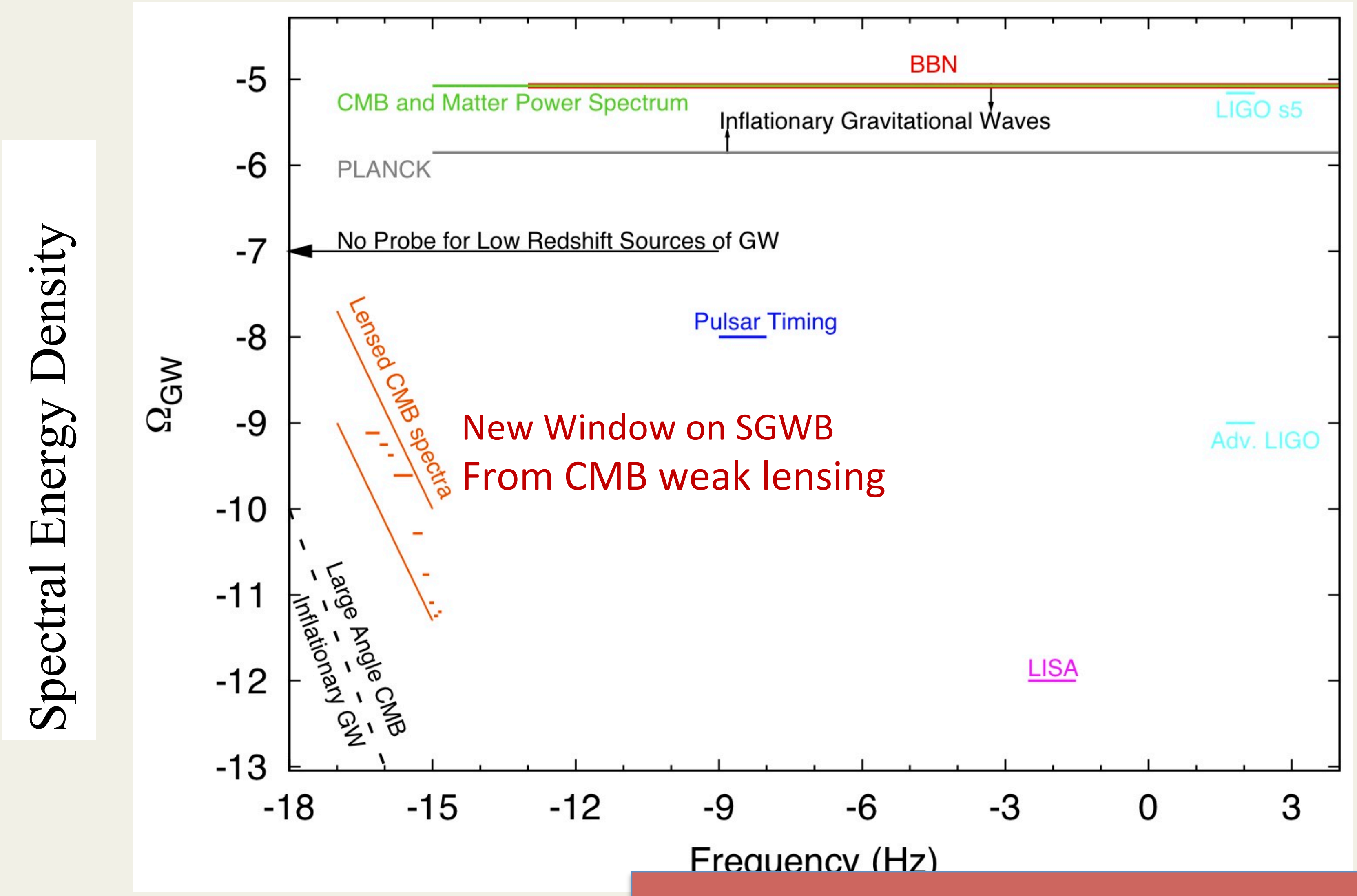
- SGWB : $$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_{\mathrm{crit}}} \frac{\mathrm{d}\rho_{\mathrm{GW}}(f)}{\mathrm{d}\ln f}$$

  – Critical (charactersitic)
  density for universe: $$\rho_{\mathrm{crit}} = \frac{3\,c^2\,H_0^2}{8\pi G}$$

  – total SGWB energy density: $$\Omega_{\mathrm{GW}} = \int_{-\infty}^{\infty} \Omega_{\mathrm{GW}}(f)\,\mathrm{d}\ln f$$

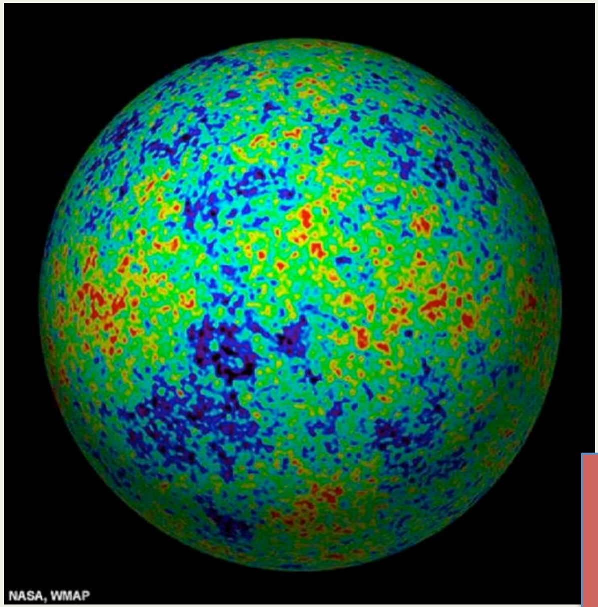
$$\Omega_{GW}(k) = \frac{4\pi}{3}\left(\frac{c}{H_0}\right)^2 k^3 P_T(k)[k\mathcal{T}^{'}]^2$$

The Current Landscape



Aditya Rotti &Tarun Souradeep, astro-ph : 1112.1689

Lensing (Harmonic space)



Direction of photon arrival changed

$$(\theta_0, \phi_0) \rightarrow (\theta_0 + \delta\theta, \phi_0 + \delta\phi)$$

Let Δ denote the displacement on the sphere.

$$\Delta_a = -\sum_{lm} \left(h^{\oplus}_{lm} Y_{lm:a} + h^{\otimes}_{lm} Y_{lm:b} \epsilon^{b}{}_{a} \right)$$

Angular power spectrum of photon displacements

$$C_l^{h^{\oplus}} = \frac{1}{2l+1} \sum_{m=-l}^{m=l} h^{\oplus}_{lm} h^{\oplus *}_{lm}$$

Gradient Spectrum

$$C_l^{h^{\otimes}} = \frac{1}{2l+1} \sum_{m=-l}^{m=l} h^{\otimes}_{lm} h^{\otimes *}_{lm}$$

Curl Spectrum

A. Stebbins, arXiv:astro-ph/p609149

Lensing Modifications to CMB power spectra

Lensing induces power transfer between the two polarization spectra

$$\begin{pmatrix} \tilde{C}_l^{TT} \\ \tilde{C}_l^{EE} \\ \tilde{C}_l^{BB} \\ \tilde{C}_l^{TE} \end{pmatrix} = \underbrace{\begin{pmatrix} A_{11} & 0 & 0 & 0 \\ 0 & A_{22} & A_{23} & 0 \\ 0 & A_{32} & A_{33} & 0 \\ 0 & 0 & 0 & A_{44} \end{pmatrix}}_{\mathcal{F}(C_l^{\phi\phi}, F^{\oplus}, {}_2F^{\oplus}, C_l^{\Omega\Omega}, F^{\otimes}, {}_2F^{\otimes})} \begin{pmatrix} C_l^{TT} \\ C_l^{EE} \\ C_l^{BB} \\ C_l^{TE} \end{pmatrix}$$

Explicit lensed power spectra

$$\tilde{C}_l^{TT} = C_l^{TT} - l(l+1)RC_l^{TT} + \sum_{l_1 l_2} \frac{C_{l_1}^{TT}}{2l+1} \left[C_{l_2}^{\oplus}(F_{ll_1l_2}^{\oplus})^2 + C_{l_2}^{\otimes}(F_{ll_1l_2}^{\otimes})^2\right]$$

$$C_l^{\tilde{E}} = C_l^{E} - (l^2+l-4)RC_l^{E} + \frac{1}{2(2l+1)}$$

$$\times \sum_{l_1 l_2} \left[C_{l_2}^{h\oplus}({}_2F_{ll_1l_2}^{\oplus})^2 + C_{l_2}^{h\otimes}({}_2F_{ll_1l_2}^{\otimes})^2\right] \left[(C_{l_1}^{E} + C_{l_1}^{B}) +/- (-1)^L (C_{l_1}^{E} - C_{l_1}^{B})\right]$$

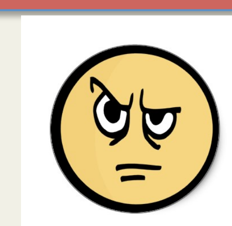
$$C_l^{\tilde{B}} = C_l^{B} - (l^2+l-4)RC_l^{B} + \frac{1}{2(2l+1)}$$

$$\times \sum_{l_1 l_2} \left[C_{l_2}^{h\oplus}({}_2F_{ll_1l_2}^{\oplus})^2 + C_{l_2}^{h\otimes}({}_2F_{ll_1l_2}^{\otimes})^2\right] \left[(C_{l_1}^{E} + C_{l_1}^{B}) +/- (-1)^L (C_{l_1}^{E} - C_{l_1}^{B})\right]$$

$$\tilde{C}_l^{TE} = C_l^{TE} - (l^2+l-2)RC_l^{TE} + \frac{1}{2l+1}$$

$$\times \sum_{l_1 l_2} \left[C_{l_2}^{h\oplus}(F_{ll_1l_2}^{\oplus})({}_{+2}F_{ll_1l_2}^{\oplus}) - C_{l_2}^{h\otimes}(F_{ll_1l_2}^{\otimes})({}_{+2}F_{ll_1l_2}^{\otimes})\right] C_{l_1}^{TE}$$

CORRECTED
C. Li and A. Cooray, PRD 74, 023521(2006)



Aditya Rotti, Hamsa Padmanabhan and Tarun Souradeep, In preparation.

Striking similarity of lensing kernels !

$$F^{\oplus/\otimes}_{l l_1 l_2} = \frac{1}{2}\sqrt{l_1(l_1+1)l_2(l_2+1)}\sqrt{\frac{(2l+1)(2l_1+1)(2l_2+1)}{4\pi}}\begin{pmatrix} l & l_1 & l_2 \\ 0 & -1 & 1 \end{pmatrix}\left[1+/-(-1)^{l+l_1+l_2}\right]$$

Scalar & tensor lensing is a Parity difference : Missed in previous literature!!!

$$|{}_2F^{\oplus/\otimes}_{l l_1 l_2}| = \sqrt{\frac{l_2(l_2+1)(2l+1)(2l_1+1)(2l_2+1)}{8\pi}}$$
$$\times\left(\sqrt{\frac{(l_1+2)(l_1-1)}{2}}\begin{pmatrix} l & l_1 & l_2 \\ 2 & -1 & -1 \end{pmatrix} +/- \sqrt{\frac{(l_1-2)(l_1+3)}{2}}\begin{pmatrix} l & l_1 & l_2 \\ 2 & -3 & 1 \end{pmatrix}\right)$$

Aditya Rotti, Hamsa Padmanabhan and Tarun Souradeep, In preparation.

Recall:

Deflection decomposes into grad &curl parts

$\Delta \rightarrow C_\ell^{h^{\oplus}} \rightarrow$ Scalar Perturbations (LSS)

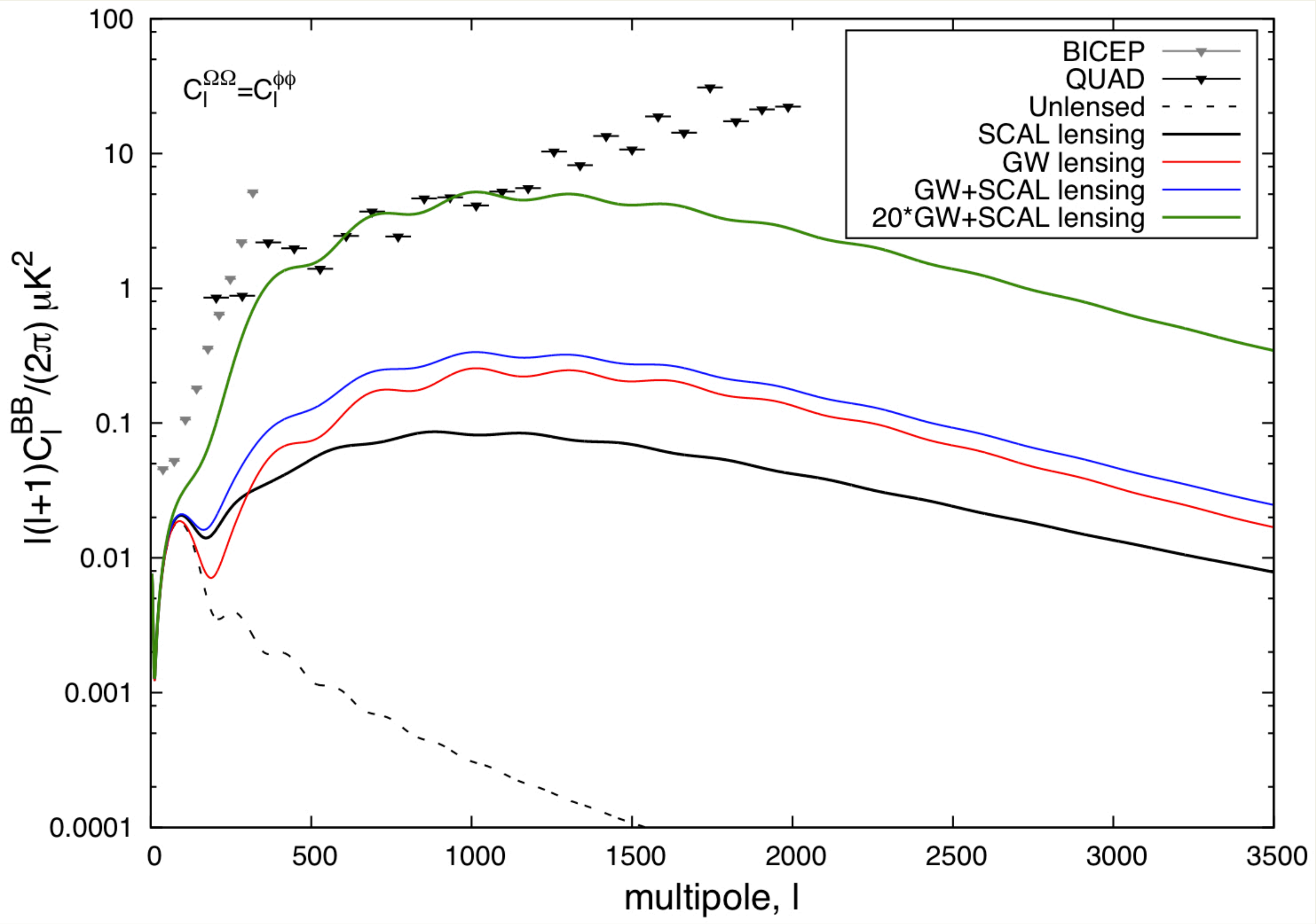
$\Delta \rightarrow C_\ell^{h^{\otimes}} \rightarrow$ Tensor Perturbations (Gravity Waves)

For fair comparison :

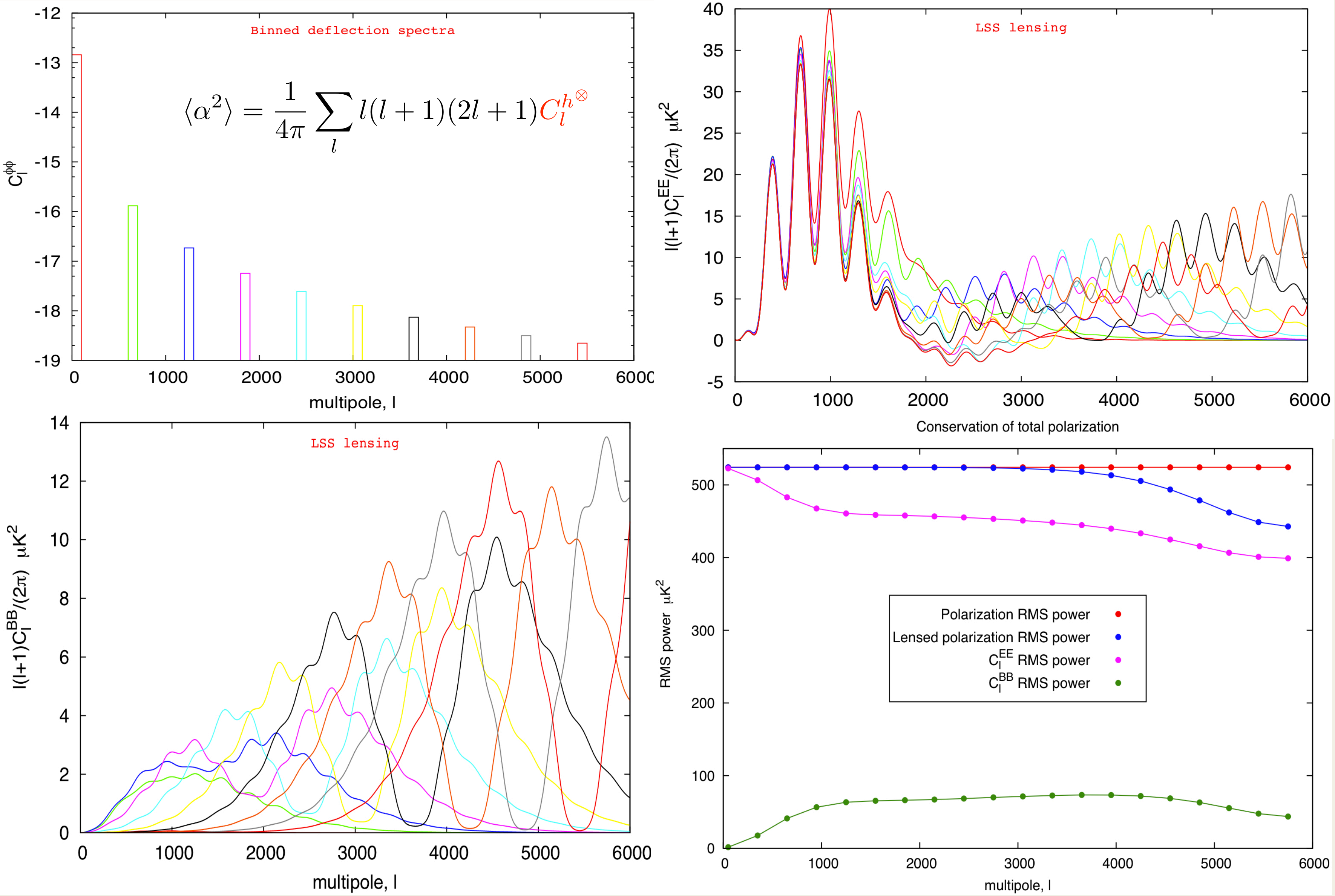
$$C_l^{\otimes} = C_l^{\oplus}$$

Power Spectrum of the projected lensing potential $C_\ell^{\psi\psi}$

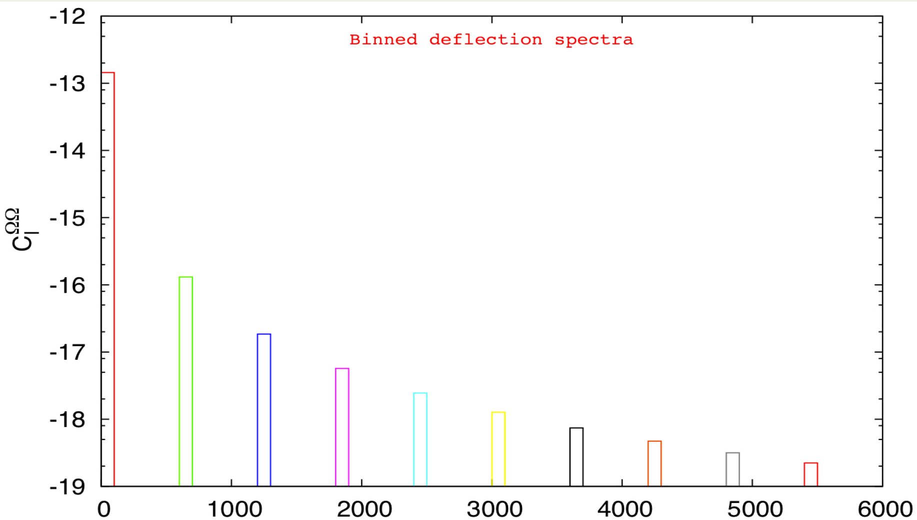
Lensed Polarization Power Spectra

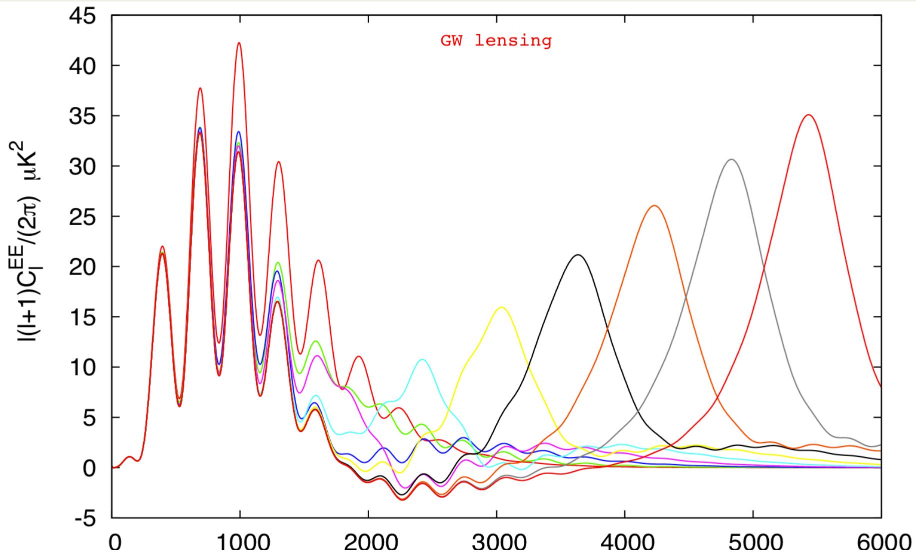


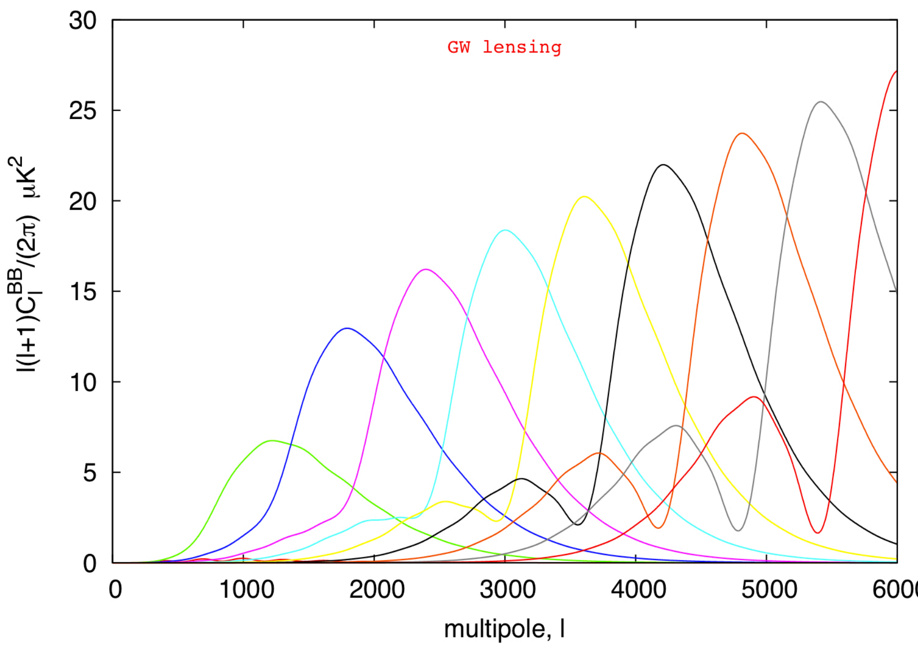
Scalar Lensing

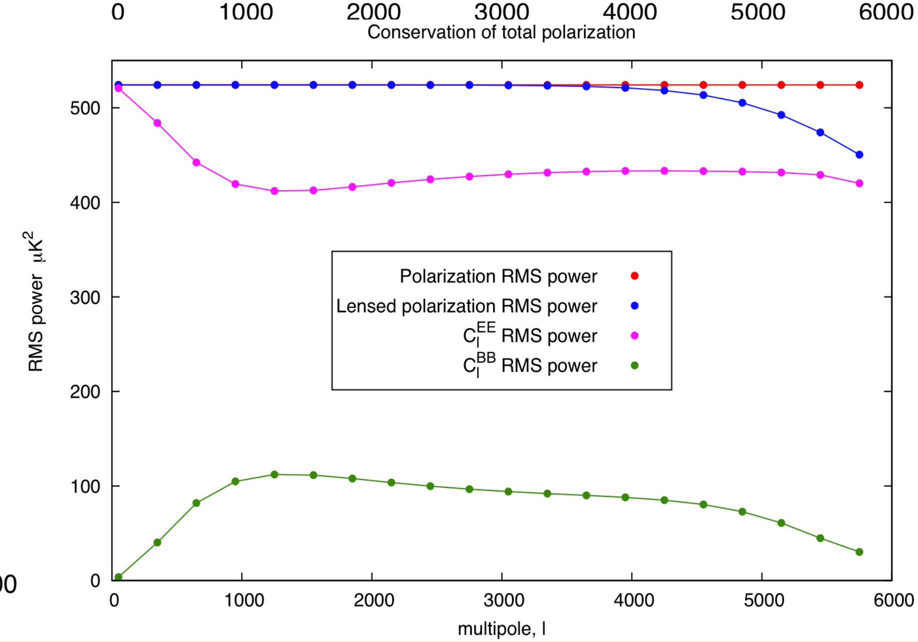


Tensor Lensing

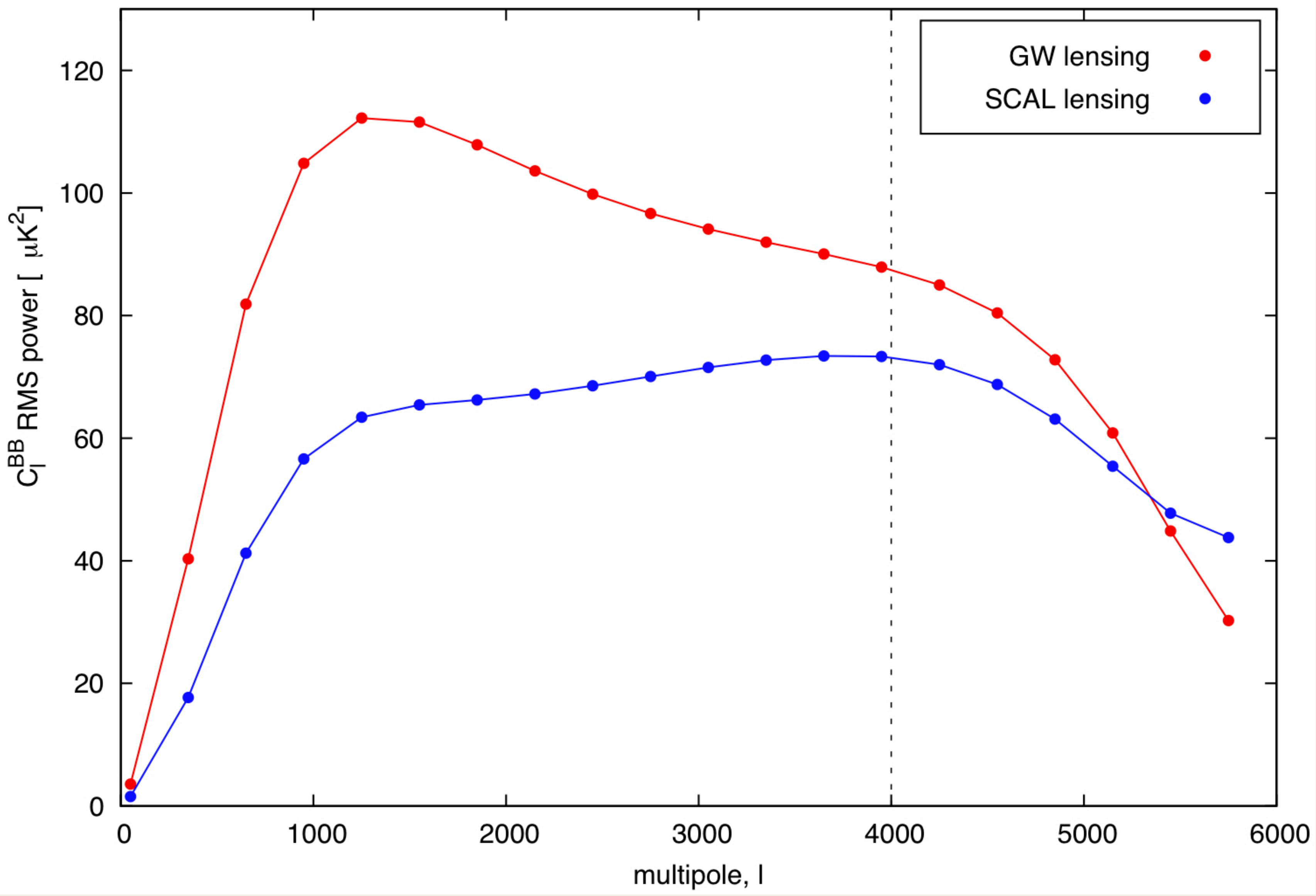








Scalar Lensing Vs. Tensor Lensing



GW Power Spectra ⟷ Deflection Spectra

$$C_l^{h^{\otimes}} = \frac{\pi}{l^2(l+1)^2}\frac{(l+2)!}{(l-2)!}\int d^3\mathbf{k} P_T(k,z)|T_{eff}|^2$$

$$T_{eff} = 2k\int_{\eta_z}^{\eta_0} d\eta' T(k,\eta')\left(\frac{j_l(x)}{x^2}\right)_{x=k(\eta_0-\eta')}$$

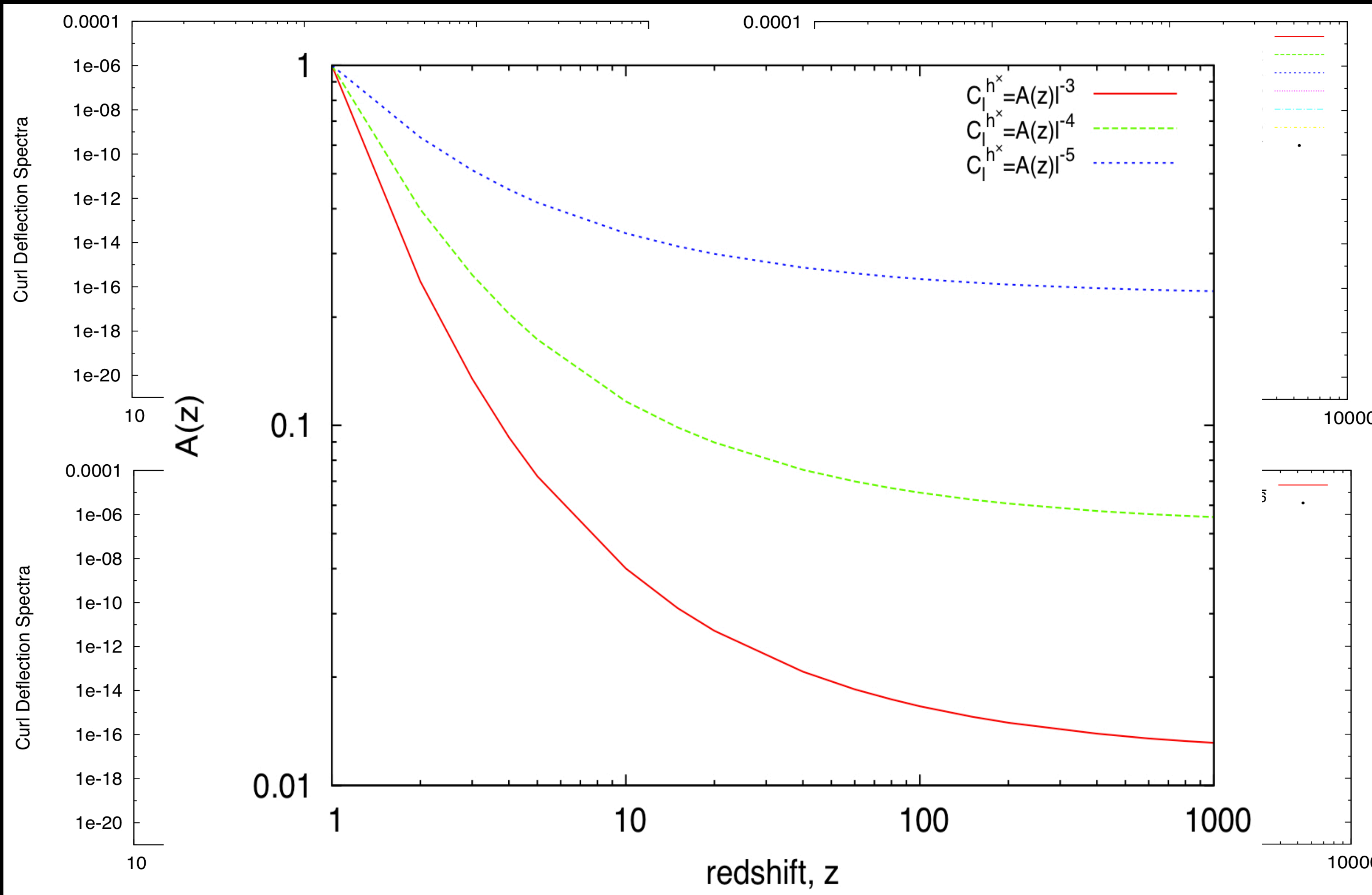
$\left(\frac{j_l(x)}{x^2}\right)$ → Projection on-to the sphere.

$T(k,\eta')$ → GW transfer function

- Gravitational waves also induce gradient type displacements, but the RMS deflection power is much lower.
- We are completely justified in ignoring their effect as we have demonstrated that they are not as efficient as curl type displacements in inducing E → B power transfer.

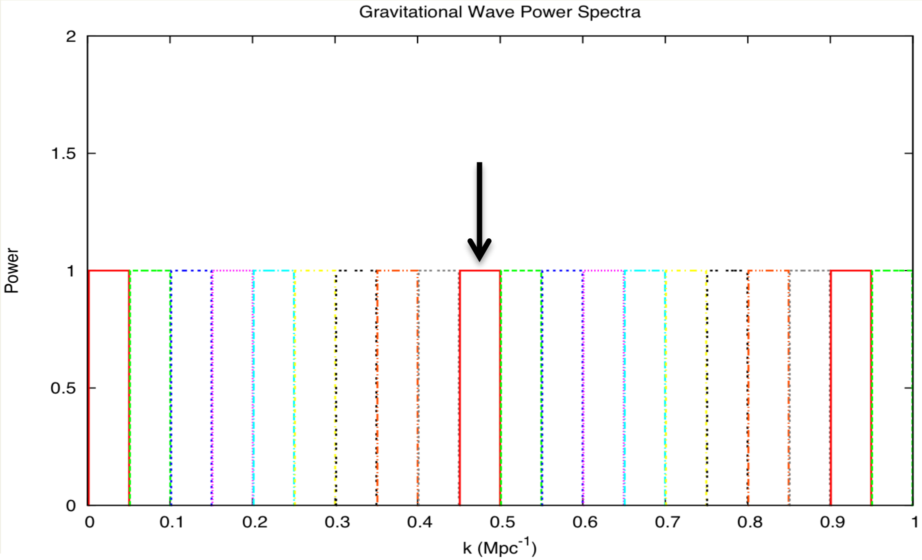
C. Li and A. Cooray, PRD 74, 023521(2006)

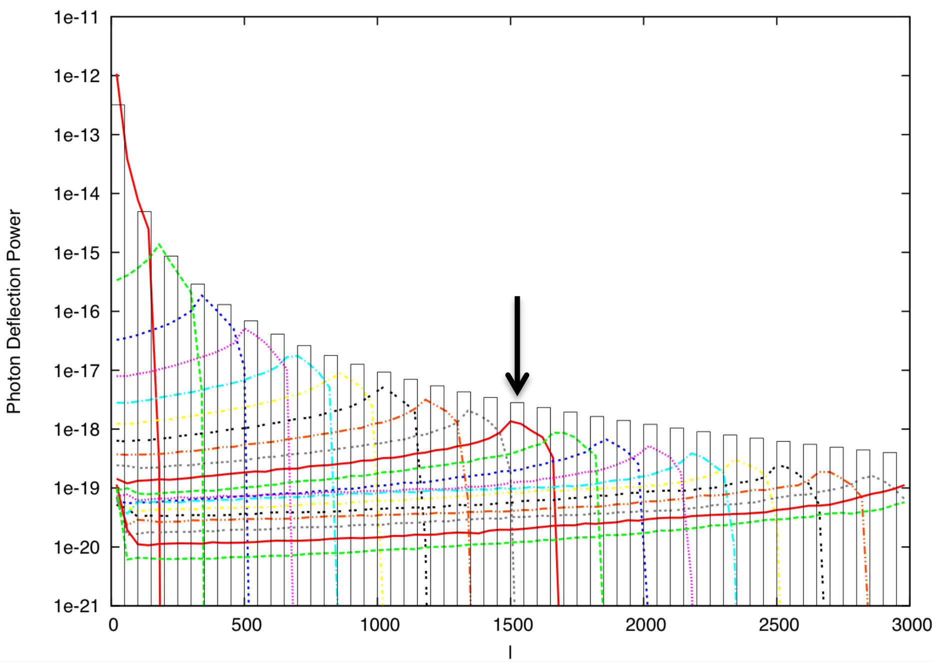
Related by a simple scaling relation !

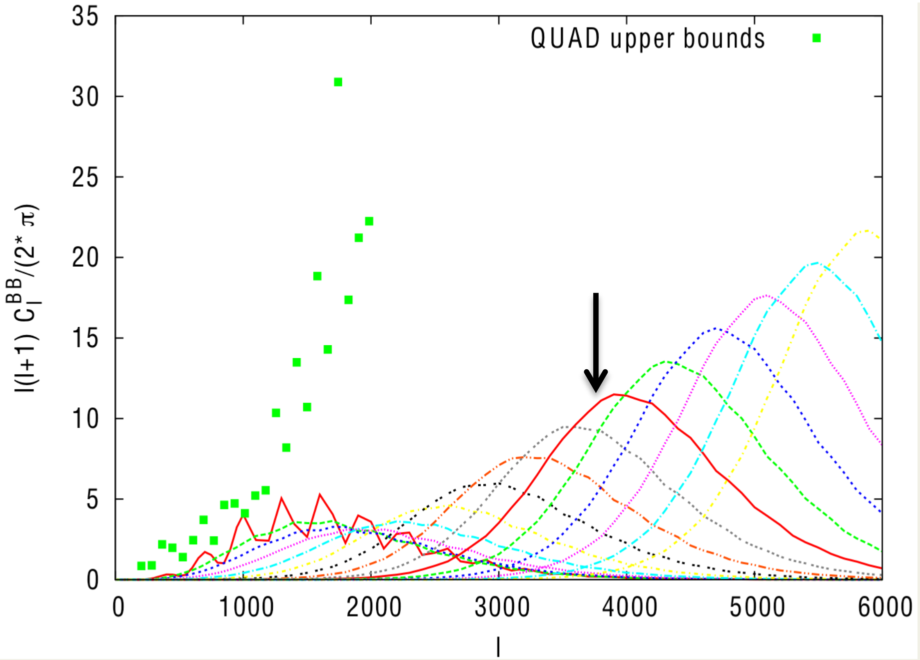


Constraining procedure

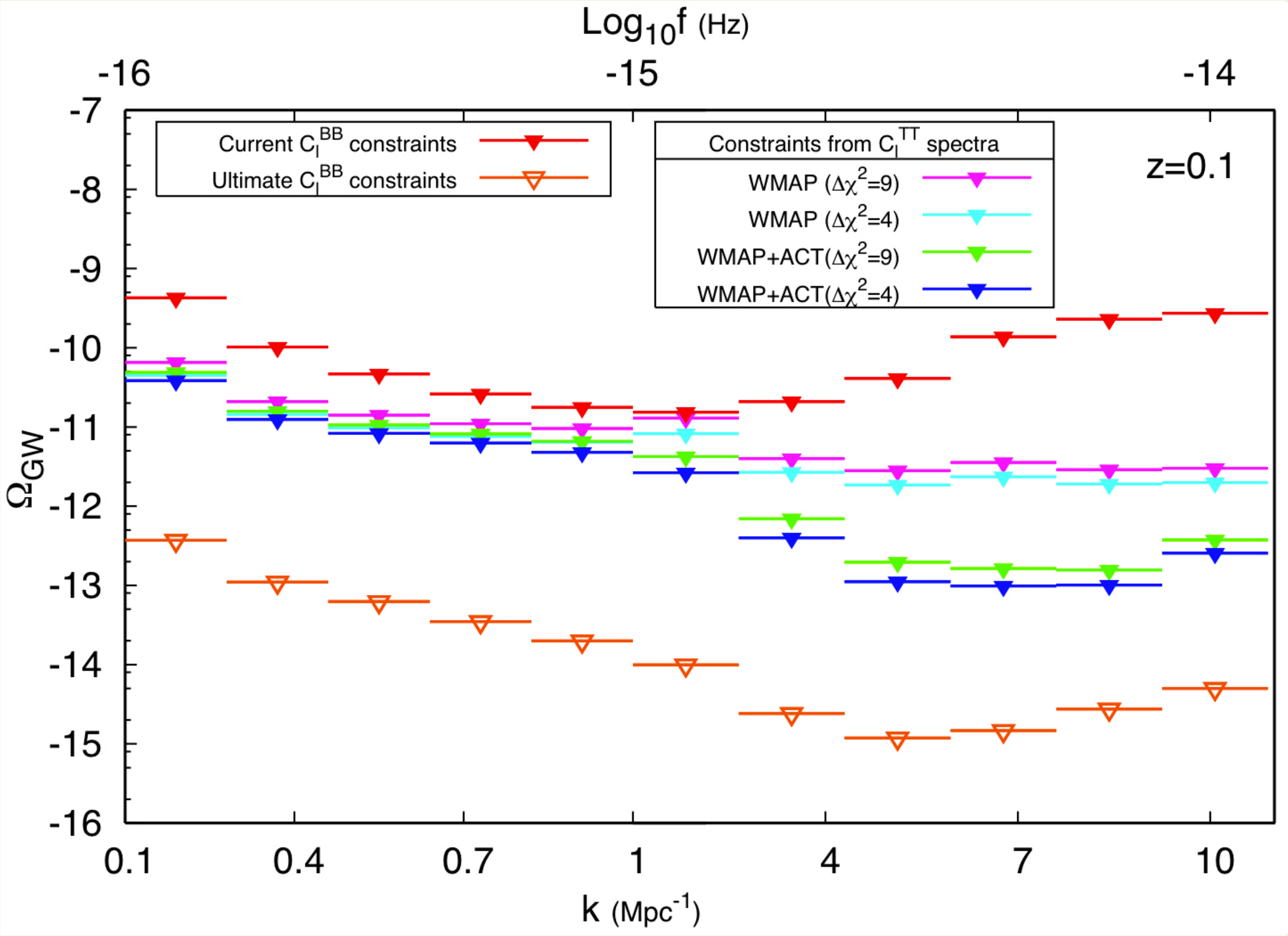
- Choose a form for the GW power spectra.
- Divide the power spectra in bins and evaluate the deflection spectra.
- Use the deflection spectra to evaluate the lensed CMB spectra.
- Keep the cosmological parameters fixed and vary the amount of power in each bin till the CMB spectra are consistent with current data.







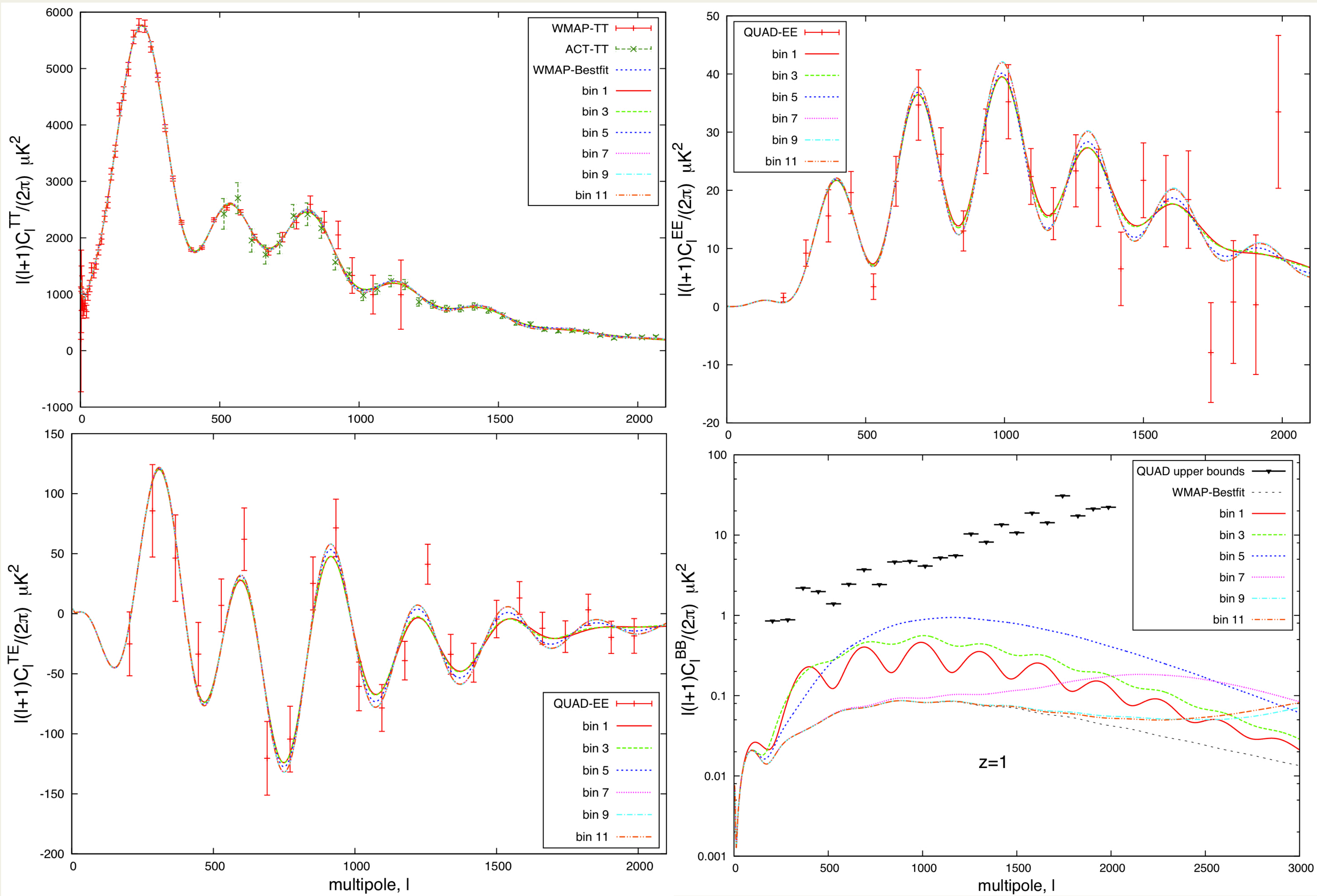
Constraints for GW sourced at different z



AR and Tarun Souradeep, In preparation.

CMB spectra lensed by GW

z=1



AR and Tarun Souradeep, astro-ph : 1112.1689

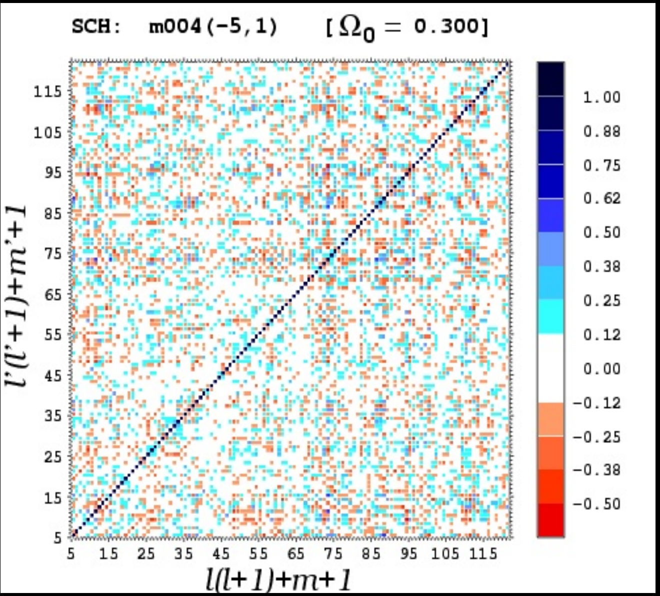
Can we distinguish weak lensing effect on CMB by Scalar density perturbations (LSS) & tensor SGWB?

Look beyond angular power spectrum

Weak lensing $\rightarrow$ SI violation:

Non-diagonal covariance matrix

$$\langle a_{lm}\, a^*_{l'm'} \rangle \neq C_l\, \delta_{ll'} \delta_{mm'}$$



$$A^{LM}_{ll'} = \sum_{mm'} \langle a_{lm} a_{l'm'} \rangle \;\; C^{LM}_{lml'm'}$$

BipoSH measure cross correlation in a_{lm}

(Hajian & Souradeep, ApJL 2003)

BipoSH: Bipolar Spherical harmonic

BIPOLAR measurements by WMAP-7 team

(Bennet et al. 2010)

$$\frac{A^{(+)LM}_{l_1 l_2}}{C^{L0}_{l0l'0}}$$

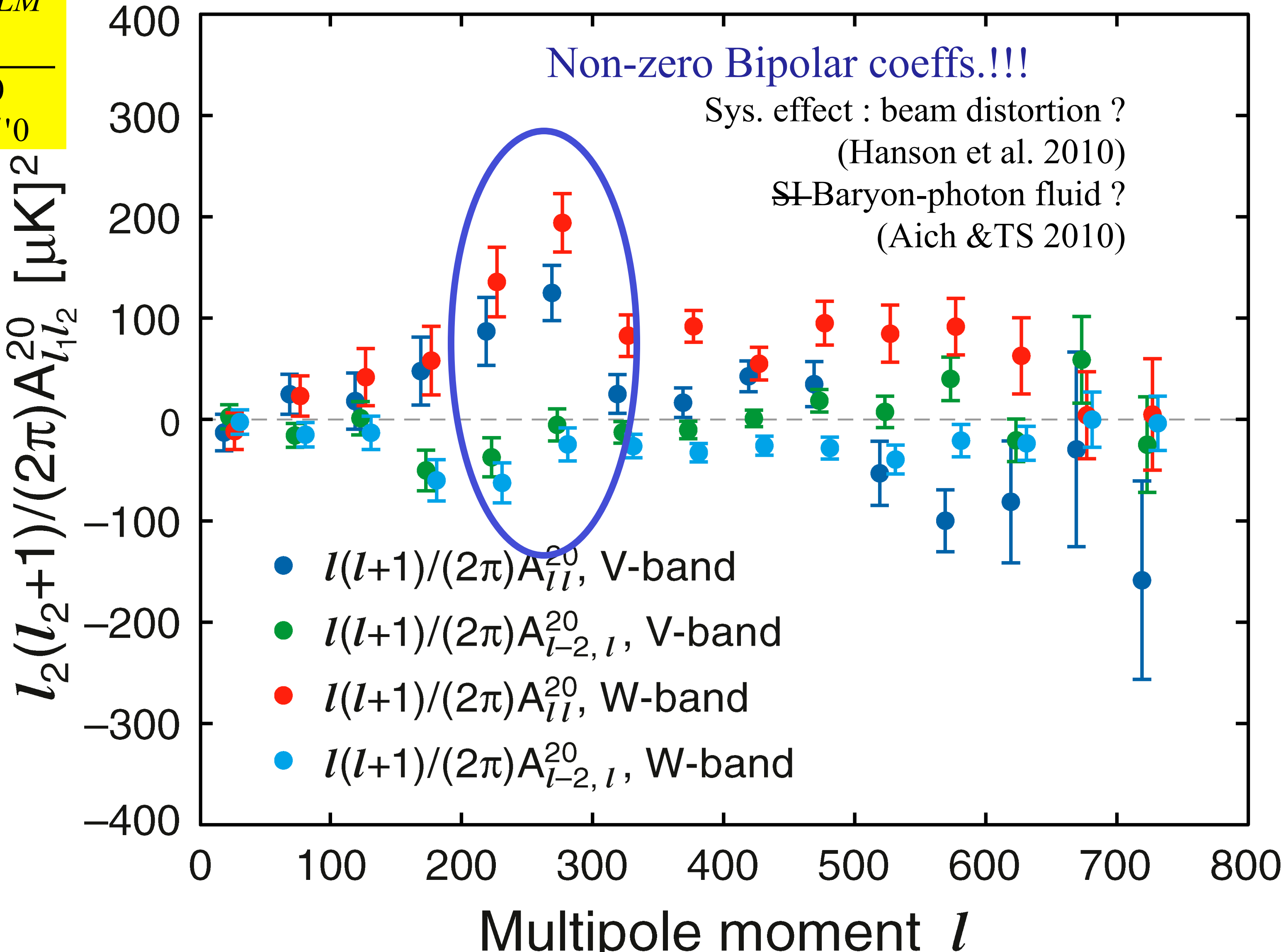


Image Credit: NASA / WMAP Science Team

Even & odd parity BipoSH

$$A_{l_2 l_1}^{(+)LM} = A_{l_1 l_2}^{(+)LM}$$ symmetric

$$A_{l_2 l_1}^{(-)LM} = -A_{l_1 l_2}^{(-)LM}$$ antisymm.

$$[A_{l_1 l_2}^{(+)LM}]^* = (-1)^M A_{l_1 l_2}^{(+)L,-M}$$ Even parity

$$[A_{l_1 l_2}^{(-)LM}]^* = (-1)^{M+1} A_{l_1 l_2}^{(-)L,-M}$$ Odd parity

Deflection field: Even & Odd parity BipoSH

Book, Kamionkowski & Souradeep, PRD 2012

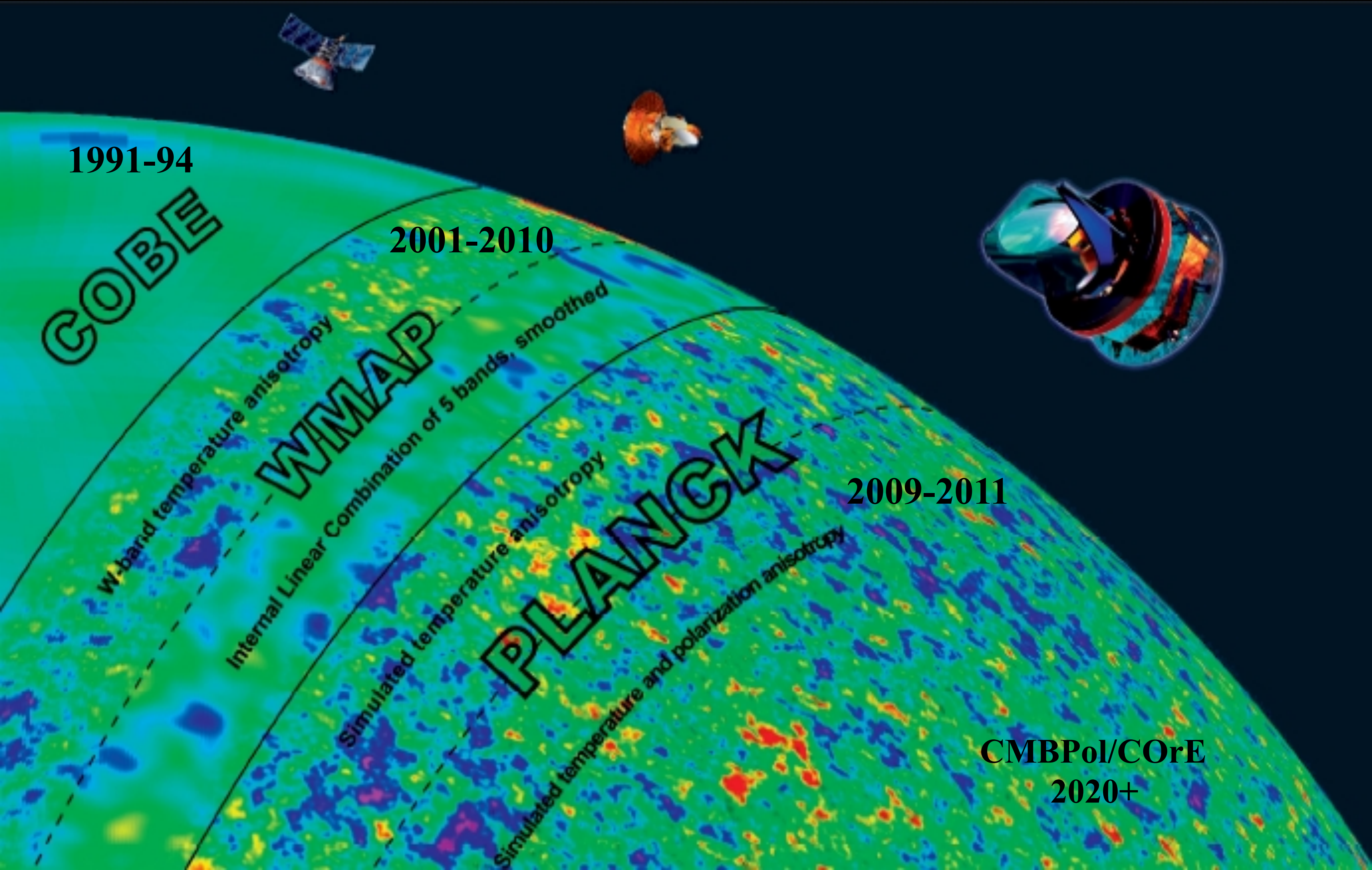
$$A_{ll'}^{(+)LM} = \phi_{LM}\left[\frac{C_l G_{l'l}^L}{\sqrt{l'(l'+1)}} + \frac{C_{l'} G_{ll'}^L}{\sqrt{l(l+1)}}\right]$$

WL: scalar

$$A_{l_2 l_1}^{(-)LM} = i\Omega_{LM}\left[\frac{C_l G_{l'l}^L}{\sqrt{l'(l'+1)}} - \frac{C_{l'} G_{ll'}^L}{\sqrt{l(l+1)}}\right]$$

WL: tensor

CMB space missions



Summary : CMB, A New probe of SGWB.

- The lensing kernels for scalar and tensor lensing have been shown to have a very similar form.
- The correct lensing kernel has been derived for lensing due to GW.
- Tensors are seen to be more efficient at transferring power from EE → BB spectra.
- Current best constraints come from measurements of the angular power spectrum of CMB temperature anisotropies.
- Future bounds/measurements of the CMB polarization spectra can provide better constraints.
- This probe provides a new window into Gravitational Waves which has not been previously explored.

Weak Lensing due to Gravitational Waves

Lensing remaps the CMB anisotropies,

$$\tilde{T}(\hat{n}) = T(\hat{n} + \vec{\Delta})$$

The deflection field can be decomposed into a gradient part and a curl part, analogous to the electromagnetic field,

$$\vec{\Delta} = \nabla\psi + \nabla \times \Omega$$

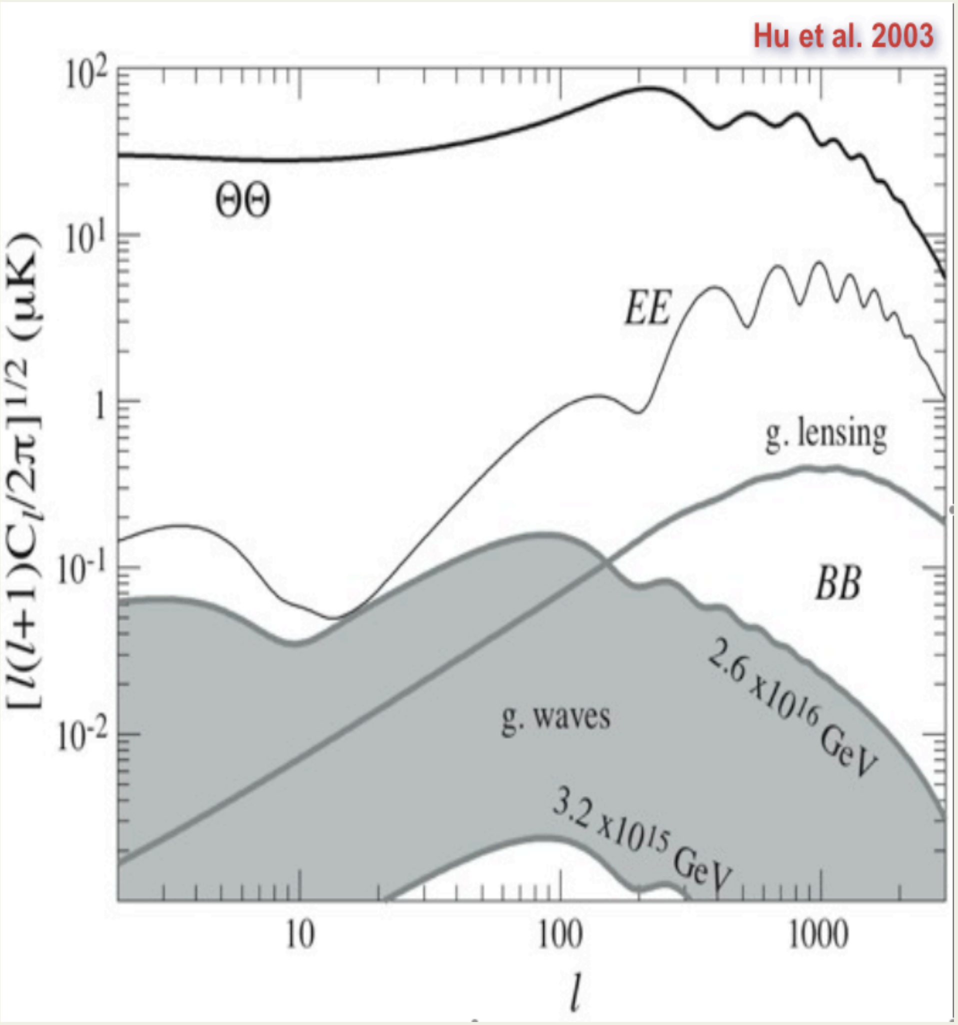
The spectrum for this field is not predicted as the GW background is not well known.

In an isotropic universe, accurate measurements of the BipoSH coefficients (EVEN) could be used to infer the projected lensing potential on the largest angular scales.

Before we try to map this field we need to constrain the amount of power in this field. The current and future CMB observations will allow us to do exactly that !!

Lensing of CMB by LSS

- Current lensing considerations are only due to scalar perturbations (LSS).
- The scalar power spectrum is already well measured.
- A huge difference between current upper limits on the BB spectra and the expected signal.



Bipolar Power spectrum (BiPS) :

A Generic Measure of Statistical Anisotropy

- Correlation is a *two point function* on a sphere

$$C(\hat{n}_1,\hat{n}_2) = \sum_{l_1 l_2 LM} A^{LM}_{l_1 l_2} \{Y_{l_1}(\hat{n}_1)\otimes Y_{l_2}(\hat{n}_2)\}_{LM}$$

BiPoSH

Bipolar spherical harmonics.

$$C(n_1 \bullet n_2) = \sum \frac{2l+1}{4\pi} C_l P_l(n_1 \bullet n_2)$$

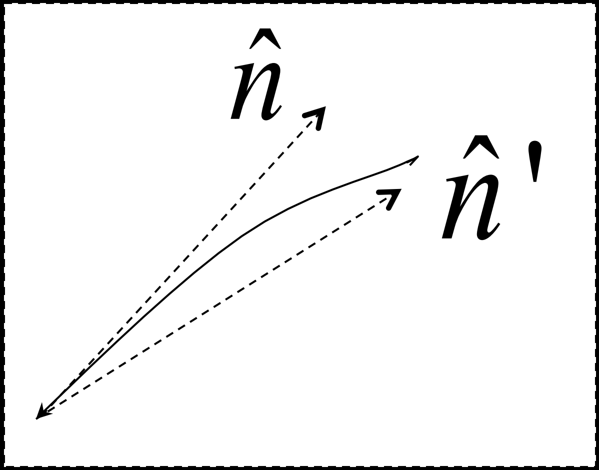
- Inverse-transform

$$A^{LM}_{l_1 l_2} = \int d\Omega_{n_1} \int d\Omega_{n_2} C(\hat{n}_1,\hat{n}_2)\{Y_{l_1}(\hat{n}_1)\otimes Y_{l_2}(\hat{n}_2)\}^*_{LM}$$

$$= \sum_{m_1 m_2} \langle a_{l_1 m_1} a_{l_2 m_2} \rangle \; C^{LM}_{l_1 m_1 l_2 m_2}$$

Linear combination of off-diagonal elements

SI violation : Deflection field



$$T(\hat{n}') = T(\hat{n} + \vec{\Theta}) = T(\hat{n}) + \vec{\Theta} \bullet \vec{\nabla} T(\hat{n})$$

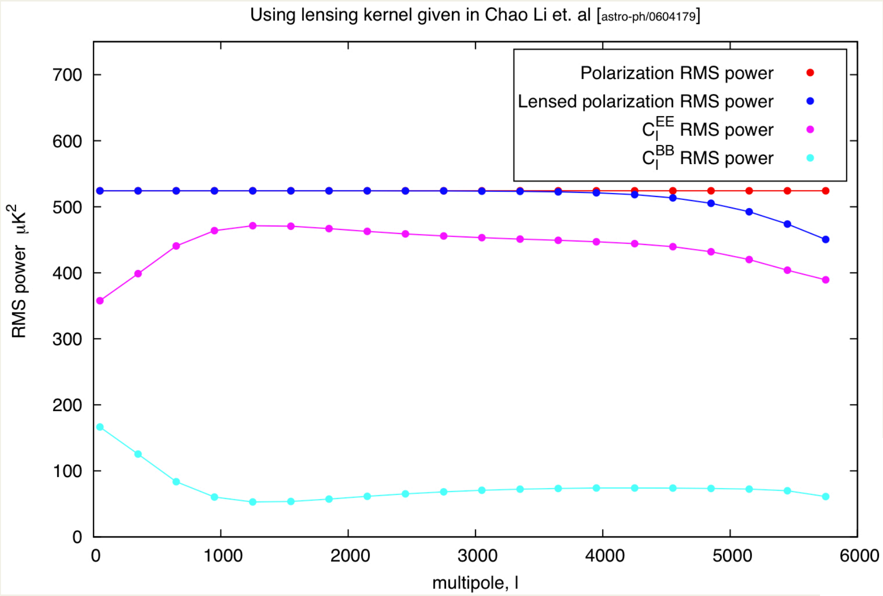
$$\vec{\Theta} = \vec{\nabla}\phi(\hat{n}) + \vec{\nabla} \times \Omega(\hat{n})$$

$$= \nabla_i \phi(\hat{n}) + \varepsilon_{ij} \nabla_j \Omega(\hat{n})$$

Gradient — Curl

WL:scalar — WL: tensor/GW

A closer look at lensing of polarization power spectra



Only ensures that the lensing kernels are properly normalised. The correctness of the kernels is not ensured.